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(summary)

THE USE OF FIRST-GRADE **STUDENT-CREATED GRAPHIC VISUAL REPRESENTATIONS** IN MATHEMATICS. APPLICATIONS FOR WORD **PROBLEM-SOLVING ABILITY** IN PRIMARY SCHOOL

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Argument

Problem-solving is a fundamental cognitive skill required for developing mathematical competence and competence in science, technology, and engineering, which is one of the key competencies necessary for future graduates to integrate successfully into the social and professional environment. Acquired during the fundamental acquisition learning cycle, problem-solving skills are described in official curriculum documents as one of the main objectives of primary education, particularly in the study of Mathematics and environmental exploration.

Problem-solving was assessed during international standardised tests, Romanian students' performances in PISA in 2018 and 2022 (OECD, 2023), evidencing reading comprehension and mathematical and scientific skills below the OECD average. Similar performances were recorded by Romanian students in the 2019 TIMSS (Trends in International Mathematics and Science Study) international assessment. Dragoș Iliescu, one of the coordinators of the Romanian section of the 2019 TIMSS, noted that Romanian students successfully solve word problems formulated in mathematical language, having difficulties in solving similar word problems that describe everyday life contexts (Ion, 2020). Although Romanian fourth-grade students demonstrated significantly improved performances in the 2023 TIMSS test compared to previous editions of the assessment (Ministry of Education and Research, 2024; von Davier et al., 2024), current observations of educational activities highlight multiple difficulties students face when required to solve different types of mathematical word problems. Therefore, it is necessary to gain an in-depth understanding of the cognitive behaviour and reasoning used by students, as well as the causes underlying students' errors that occur during mathematical word problem solving.

Based on the above considerations, this paper aims to offer a complementary perspective on assessing the accuracy of first-grade students' understanding of mathematical word problems, as well as a suitable tool for primary school teachers that enables them to identify errors that may occur at different stages of the problem-solving process.

CHAPTER I
COMPETENCIES AND COMPETENCY-BASED CURRICULUM

I.1. The importance of competency-based curriculum

Over time, the term “curriculum” has been used to describe the educational trajectory, a course of study one follows in the formal setting of an educational institution (Bocoș & Jucan, 2019). “Traditional education” referred to the curriculum as official school documents that indexed the subject matter (Crețu, 1994).

American educator John Dewey (1902/1977) argued against the fragmented knowledge approach in education, which delivers ready-made knowledge organised according to adult reasoning principles rather than children's limited experiences. Dewey emphasised that a meaningful curriculum should be more than just its parts; it should reflect the experiences through which students actively acquire knowledge. Franklin Bobbitt (1918) described a curriculum that focuses on the specific skills one needs to acquire in order to perform successfully in professional activities. The acquisition of the skills, attitudes, habits, values, and knowledge needed to perform specific professional tasks will be achieved through learning experiences organised and conducted within the school education system. These operational acquisitions were identified as competencies and became the objectives of the curriculum. Therefore, the curriculum will comprise a series of experiences that children and young people will undergo to achieve these objectives. The design and structure of the educational path described by Franklin Bobbitt emphasise the idea of a competency-based curriculum.

Decades later, societal changes resulting from scientific and technological advances, globalisation, and the expansion of knowledge domains have transformed the nature of professional activities. The advancements resulting from technological development have facilitated a gradual transition from algorithmic tasks to heuristic tasks, which necessitate experimentation, critical thinking, and innovative skills (Senge, 2000/2016).

The practical approach of the competency-based curriculum ensures a direct link between content and educational outcomes. Therefore, learning becomes more meaningful, stimulating students' motivation, and shifting teaching activity far from the dull transmission of ready-made knowledge (Bocoș & Jucan, 2019).

I.1.1. Competence – the central element of formal curriculum

In the 1950s, the behaviourist approach to learning focused on the student's (cognitive) behaviour as a response to external stimuli, which, in school education, is represented by the knowledge taught in the classroom. Thus, observable behaviour became the indicator of academic achievement. The educational objectives measured the extent to which cognitive acquisition was achieved, outlining the behavioural sequence that could be directly observed and assessed (Cucoș, 1996). Simple objectives describe cognitive behaviours such as recalling and understanding information, while those with higher degrees of complexity aim to apply specific knowledge in practice, analyse, synthesise, and evaluate information. The objective description of the changes to be achieved in students' behaviour replaced the rigid listing of the concepts transmitted to the students and the skills to be developed (Planchard, 1969/1976). The process of operationalisation of educational objectives was often overemphasised in teaching practice; the behaviours indicated became evaluation criteria that reflected the quantitative results of learning, rather than the qualitative ones.

In the 1960s, cognitivism viewed behaviour as a consequence of mental processes, not just a response to external stimuli (Doron & Parot, 1991/2006).

Émile Planchard (1969/1976) identified three types of student acquisitions: cognitive, affective, and attitudinal, as part of an operational structure that facilitates suitable responses in various situations, based on expected behavioural changes resulting from the teaching-learning process. This operational structure, which systematically combines a cognitive component, a practical part utilising cognitive elements, and attitudes and values, has been referred to in the literature as competence. Completing a work task requires activating cognitive, practical, and attitudinal resources to address a specific problem (Dulamă, 2010). The components of competence are acquired holistically, not in fragments, and cannot be separated or treated as isolated parts. Competence itself does not exist outside the contexts in which it is activated. Approaching learning from the perspective of competence development requires contextualising learning content according to specific situations that demand the use of competence, thus enabling students to connect with their own concrete experiences (Manolescu, 2010; Badea, 2010; Dulamă, 2010).

In literature, competencies are described as operational frameworks that combine knowledge, skills, and attitudes. These elements are interconnected and can be effectively applied in various contexts to complete tasks or to solve problems efficiently (Bocoș, 2021; Bocoș & Jucan, 2019; Ionescu, 2007; Niculescu, 2005; Parry, 1996; Perrenoud, 1998).

As objectives of the formal curriculum, competences are outlined in official documents that steer educational policies at both European and national levels. For example, the Recommendation of the European Parliament and of the Council of 23 April 2008 on the establishment of the European Qualifications Framework for lifelong learning, along with Pre-university Educational Law No. 198/2023, Art. 85, para. (1), describe competences as the proven ability to apply a multifunctional and transferable set of knowledge, skills, and attitudes, necessary for adapting, integrating, and actively engaging in social, cultural, and political domains.

In summary, we will further refer to competence as a comprehensive set of transferable knowledge, practical skills, abilities, attitudes, and social and collaborative skills, acquired through a systemic and holistic approach, and used to complete specific tasks or activities.

I.1.2. The structural components of competencies

Described as an integrated and operational set of components, competence encompasses:

• cognitive/intellectual component: to know;

• psychomotor structures – practical skills and abilities: to do;

• affective-attitudinal elements: motivation, attitudes, values, and emotional dispositions: to be, to become, etc.

These components can be acquired or formed before they are assembled into a whole, as they are organised and structured according to the practical demands that emerge at a specific moment, in a particular context, often akin to problem-solving (Bocoș & Jucan, 2019; Bîrzea, 2010).

The cognitive or intellectual component encompasses more than just storing declarative or factual knowledge (pre-existing information). Dan Badea (2010) explained that procedural knowledge involves practical methods of operation, while strategic knowledge enables decision-making regarding which declarative or procedural knowledge to employ in specific situations.

Practical action (skills and abilities) is guided by knowledge and the intended purpose. Without a solid knowledge foundation, practical work skills are less effective.

Attitudes and values form the reflexive part of competence, guiding practical activity by holding the individual responsible for the beliefs and norms in personal, social, and professional life (Badea, 2010).

I.1.3. Competence classification

Starting in 2011, competencies were incorporated into Romanian school curricula, becoming the goals that steer the teaching and learning process. They were divided into *general competencies*, formulated per subject for several years of study, and *specific competencies*, derived from general competencies, structured per subject for one year of study. From the perspective of the scientific field whose achievements they encompass, competencies can be disciplinary, focusing on achievements specific to a particular discipline, or interdisciplinary, combining achievements from multiple disciplines (Dulamă, 2010).

Eliza Dulamă (2010) described *virtual competences* which combine declarative and procedural knowledge along with attitudes that have not yet been applied in practice (for example, a student who knows how a compass works but has never used one) and *in-action competences*, which enable decision-making and action in various situations through one's cognitive acquisitions. Additionally, Dulamă (2010) mentions *reproductive competences*, which involve performing a task based on a plan or scheme, and *productive competences,* which involve tackling challenging tasks without a pre-established plan, such as problem-solving that requires creativity and flexible, divergent thinking rather than algorithmic approaches.

I.2. Forming and developing competences

All human activities are driven by specific goals, which, to be accomplished, require mental anticipation of the expected outcome and organised step-by-step planning. Depending on the goal set, different degrees of expertise in the same skill are involved. For example, although making an origami figurine requires the same mental and practical resources for both novices and experts, the two will produce different products. Although the novice will strictly follow the work stages in creating the paper figure, the expert will notice specific characteristics of the material or shape, utilising mental models of execution gained through repeated origami experiences. The novice overlooks these secondary qualitative aspects, instead focusing solely on following the steps that lead to the final product. Therefore, to effectively teach someone how to use the origami technique, it is essential to identify the primary knowledge and skills that need to be acquired and developed.

In addition to gaining fragmented knowledge and developing practical skills, Roegiers (1998) recommends engaging in *integrative activities*. Through problem-solving tasks, the student will incorporate the cognitive, actional, and attitudinal components learned sequentially. This prevents the passive reproduction of knowledge or mechanical application of work stages, fostering students to develop new connections between existing concepts in their cognitive repertoire.

Integrative activities differ from learning situations in that the latter aim to acquire parts of a competence in a sequential manner. In contrast, integrative activities address the competence as a whole, with a higher degree of complexity. In integrative activities, several types of competences can be structured and developed simultaneously (Dulamă, 2010).

I.3. European curriculum outlook: essential skills for 21st-century graduates

The beginning of the 21st century was marked by significant technological progress, including the development of the internet and social networks, which brought about profound changes in communication and human relationships. The socio-cultural diversity, along with the freedom of movement and employment of European citizens, has led to educational policies that focus on identifying a set of skills aimed at ensuring personal and professional success. The *Recommendation of the European Parliament and of the Council* of 23 April 2008 outlined a set of eight key competences, transferable skills applicable in various socio-cultural contexts, which can be acquired by both young people completing compulsory education or higher education and by adults through education and vocational training.

The socio-economic context, influenced by the COVID-19 pandemic's restrictions and the rapid advancement of artificial intelligence (AI), has shifted perspectives on professional activities. Analyses by the OECD (2019, 2020) and the World Economic Forum (2023) have shown a decline in routine tasks and a notable rise in creative and analytical tasks, which AI would struggle to manage, requiring humans to possess high levels of flexible thinking and creativity to solve problems in various professional and personal situations. The curriculum approach proposed by the OECD (Future of Education and Skills: Education 2030) emphasises the importance of acquiring specific skills as essential components of competencies (skills that go beyond work skills and habits, playing a crucial role in mobilising the cognitive resources needed to achieve a goal). The OECD Learning Compass 2030 outlines the key skills that future graduates must develop to attain well-being and self-fulfilment:

- **cognitive and metacognitive:** critical thinking, creative thinking, the ability to learn how to learn, and cognitive self-regulation;
- **socio-emotional:** empathy, self-efficacy, responsibility, and collaborative skills;
- **practical-applicative:** focused on the effective use of new information and communication technology devices.

The new curricular perspective outlined by the OECD (2020) for the decade 2020–2030 highlights the role of the individual, shifting the focus from simply accepting and promoting multiculturalism to emphasising meaningful learning and personal well-being. In this framework, personal well-being is distinct from professional success or social status; instead, it is a personal construct recognised and valued by society.

I.4. Critical thinking, creative thinking and problem-solving: essential skills

The widespread use of artificial intelligence has redefined labour market requirements, with critical thinking, creativity, and problem-solving being the most sought-after skills among employees (Vincent-Lancrin et al., 2019).

Higher-level cognitive processes, both critical thinking (the ability to question, analyze, interpret, and evaluate different information or aspects of reality) and creativity (the ability to innovate, conceive, and create original products) are involved in identifying and solving different types of problems in novel ways (Bocoș, 2021; Mih, 2018; Murawski, 2014; Ruggiero, 2012). Problem-solving has been described as the cognitive ability to identify and critically analyse an unknown situation, followed by adopting a decision to solve and the actual solving process (Vincent-Lancrin et al., 2019). Thus, individual cognitive and metacognitive resources are mobilised by critical and creative thinking for problem solving.

The formal curriculum encompasses problem-solving within scientific disciplines, including mathematics, physics, and chemistry. The PISA 2022 Mathematics Framework (OECD, 2022) presents mathematics as a crucial subject for acquiring and developing the skills needed for the social and professional integration of 21st-century young people, with problem-solving dedicated a separate chapter, examined in extensive research and detailed in specialised literature.

CHAPTER II
PROBLEM SOLVING: A FUNDAMENTAL SKILL IN MATHEMATICS

II.1. Mathematics: the science of quantitative relationships

Mathematics is generally defined as the science that investigates quantities, their relationships, and spatial forms through deductive reasoning (Academia Română. Institutul de Lingvistică "Iorgu Iordan-Al. Rosetti", 2016). In Ancient Greece, the Pythagorean philosophers and other scholars studied numbers as representations of quantities and amounts, helping to establish mathematics as a scientific discipline (Câmpan, 1978).

As a branch of mathematics, arithmetic studies the fundamental properties of rational numbers. Although the term is less frequently used in current technical language, arithmetic holds an important place in the primary school curriculum, in the study of Mathematics and environmental exploration within the fundamental acquisition learning cycle (preparatory, first, and second grades), as well as in the study of Mathematics during 3rd and 4th grades, in the developmental learning cycle. Arithmetic encompasses counting, the four fundamental mathematical operations and their properties, divisibility rules, and operations involving fractional numbers, all of which are essential for everyday activities (Vălcan, 2018).

Fuchs et al. (2006) identified algorithmic computation, the ability to use mathematical calculus, and problem-solving skills as the primary cognitive abilities that influence students' performance in arithmetic. These skills are employed in solving various types of problems and form the core of mathematical competence and competence in science, technology, and engineering.

II.2. Problem solving – an essential skill for graduates and workers in the 21st century

Mathematical competence and competence in science, technology, and engineering was described in the Recommendation of the European Parliament and of the Council of December 18, 2006, and is structured in three components from which the specific contents of the subjects belonging to the Mathematics and Natural Sciences curriculum area were derived:

- **mathematical competence** encompasses particular mathematical thinking skills, which are activated to solve a broad spectrum of problems faced in different everyday contexts;
- **science competence** involves applying knowledge from the cognitive repertoire to recognise problem situations and formulate questions that guide the development of conclusions based on empirical evidence;
- **technology and engineering competence** refer to the ability to apply scientific knowledge and methodology to find practical solutions for people's needs.

The OECD (2019) has identified the key skills required for mathematical literacy, specifically the development of mathematical reasoning used to solve problems in various real-life contexts. The PISA 2022 Mathematics framework (OECD, 2022) defines mathematical reasoning as the ability to reason logically and present arguments that will lead to valid conclusions. The OECD (2022) has identified six key understandings providing structure and support to mathematical reasoning: (1) understanding quantities, numbers and their algaebric propertie (2) appreciating symbolic representation and abstracting quantitative relationships between the components of real world, (3) transposing problem situations into mathematical structures and regularities, (4) identifying functional relationships between quantities using graphs, (5) using mathematical modelling in the study of other disciplines, (6) acquiring specific statistical concepts to analyze data collected from the environment.

The use of mathematical reasoning in problem solving was outlined in the PISA 2022 Mathematics framework as a three-step cycle: (1) identifying the problem, (2) solving the problem mathematically, (3) evaluating and interpreting the mathematical solution within the context of the real-world problem.

Data from international assessments, such as PISA, TIMSS, and NAEP, has provided the foundation for numerous studies and analyses of the strategies students use to solve mathematics word problems. This research highlights common errors students make at different problem-solving stages and identifies teaching approaches that promote and support the development of mathematical reasoning.

II.3. Developing problem-solving skills in mathematics during primary education

The development of problem-solving skills is outlined in the school curricula for *Mathematics and Environmental Exploration* (covering preparatory class, first grade, and second grade), as well as *Mathematics* (for third and fourth grades), encompassing both general and specific competencies. Problem-solving skills develop throughout primary school, beginning with solving simple problems using concrete supports such as intuitive materials, and then advancing to a mental level, fostering abstract thinking and formal thought processes. Problem-solving is gradually refined in primary school, starting with the fundamental acquisition learning cycle, where students are required to sort and represent data, and then further develop these skills in the subsequent development cycle by solving a wide range of problems in familiar situations.

The learning activities in the preparatory, first, and second-grade school curriculum are designed to develop problem-solving skills by using concrete, intuitive materials, including symbolic representations of problem data, images, visual aids, and object support. These assist in creating mental representations and promote a clear understanding of the relationships between quantitative data in the problem statement.

Solving word problems requires the application of various cognitive skills. For many primary school students, interpreting problem data and developing an appropriate solution strategy can be challenging. The literature has identified, described, and analysed specific ways in which students engage with formal problem-solving activities.

II.4. A problem and a mathematical word problem

II.4.1. What is a "problem"?

A problem describes an issue involving unclear or vague aspects (Academia Română. Institutul de lingvistică "Iorgu Iordan-Al. Rosetti", 2016). From a psychological perspective, the term refers to a situation that arises in the pursuit of a goal, for which the individual does not have an appropriate behaviour or response stored in their memory (Doron & Parot, 1991/2006; Miclea, 1999). The relationship between the known and the unknown created by the problem situation gives rise to a cognitive conflict, which can be resolved through various types of reasoning (Bocoş, 1997).

The solving process improves attention, observational skills, critical thinking, and creative thinking, helping the solver analyse, synthesise, and compare data. Finding new solutions by abstracting, contextualising, and generalising the results characterises problem-solving as a creative process (Mih, 2018), with the final solution being the outcome of this process.

From a creative thinking perspective, problem-solving is regarded as an innovative activity that engages the complete spectrum of cognitive, affective, and psychomotor skills, thereby facilitating the attainment of highly complex educational objectives.

II.4.2. Mathematical word problem

A mathematical word problem depicts a real-life situation translated into numerical relationships, where an unknown quantity must be determined using known related values presented in the problem (Neacșu, 1988).

In primary school, word problems present familiar situations. Since primary school pupils are situated in the concrete operations stage (cf. Piaget, 1936/1973), using familiar contexts helps them understand hypotheses, mathematical reasoning, and analogies between physical coordinates of the environment and quantitative relationships in problem statements, which are often expressed abstractly through numbers.

II.4.3. General classifications of word problems

According to the number of computations performed, mathematical problems can be classified as **simple word problems** (solving process involves only one computation) and **complex word problems** (solving process requires multiple computations) (Aron, 1977; Neacșu, 1988; Roșu, 2006; Petrovici, 2014; Vălcan, 2018).

Complex word problems can be **typical** when a specific solution strategy is used for each type of problem (Aron, 1977), but they can be **atypical** when solving them demands critical thinking and the full range of cognitive skills (Mărcuț, 2006; Vălcan, 2018).

Depending on the **solving process**, typical problems can be solved using the graphical method, the reduction to unity method, the false hypothesis method, the comparison method, the simple three rule, or the reverse method (Neacșu, 1988; Petrovici, 2014; Vălcan, 2018).

Depending on the type of specific computations required for solving the problem, we distinguish between word problems that can be solved by addition, subtraction, multiplication, or division (Vălcan, 2018; Neacșu, 1988) and word problems that involve the use of all mathematical operations or at least two types of them.

Depending on the language used to express the quantitative relationships between data, word problems can be either **abstract** (the relationships between data are presented using mathematical abstract language) or **concrete** (the language expressing the relationships between data refers to aspects of concrete, objectual reality) (Vălcan, 2018; Petrovici, 2014).

Depending on the specific reasoning required for developing the solving strategy, we distinguish between the **analytical method** (which uses deductive reasoning, moving from the unknown to the known, from what is required in the problem to what needs to be revealed) and the **synthetic method** (which uses inductive reasoning, moving from parts of the problem to the whole, from the known to the unknown) (Neacșu, 1988; Vălcan, 2018). Compared to the synthetic method, which demands less mental effort from students, the analytical method requires a greater degree of reasoning, facilitating a more comprehensive approach to the problem as a whole, while always keeping in mind the guiding question that informs the solution process. The two methods combine to varying extents in the strategy for solving each problem, depending on the specific relationships between known and unknown data.

II.4.4. Phases of the word problem solving process

The word problem-solving process was described based on the phases that the student's reasoning goes through to solve a word problem successfully. The main phases in solving a word problem involve:

1. Gaining knowledge of the problem content involves students or the teacher (in the case of first-grade students) rereading the problem statement multiple times. At this stage, students familiarise themselves with the content of the problem — including all the information about known and unknown data — and its purpose, which will guide the process of developing the solution strategy (Ana et al., n.d.; Lupu, 2014; Magdaș, 2022; Mărcuț; Neacșu, 1988; Petrovici, 2014; Roșu, 2006; Vălcan, 2018).

2. Understanding the problem and identifying important information: known and unknown data, and the relationships between them, i.e., the condition of the problem (Polya, 1945/1965). To support students' understanding of the problem's content and identify the relevant data necessary for the solving process, the teacher can organise the important information in the problem by synthesising its statement (using dots instead of unnecessary verbal expressions and vertical arrangements of numeric data).

3. Analysing the problem and constructing the mathematical model involves a coherent presentation of the data necessary for the solving process, using inductive or deductive reasoning to transform the specific problem context into abstract mathematical relationships (Lupu, 2014; Mărcuț, 2006; Neacșu, 1988). In this phase, the quantitative relationships between known and unknown data are converted into mathematical relationships and algebraic expressions (Ana et al., n.d.; Petrovici, 2014; Reusser, 1990; Vălcan, 2018; Verschaffel et al., 1994; Verschaffel & De Corte, 1993).

4. Writing and performing the calculations derived from the mathematical model, resulting in specific mathematical outcomes.

5. Additional activities after solving the problem (Neacşu, 1988; Petrovici, 2014; Roşu, 2006) include verifying and assessing the mathematical result within the context of the problem. This step involves identifying alternative solving strategies and formulating similar problem statements based on the current mathematical model. Interpreting and evaluating the results within the context of the problem facilitates the communication of the problem solution (Polya, 1945/1965; Vălcan, 2018).

II.5. Structural components of word problems

Riley et al. (1983) defined word problems from a structural perspective, focusing on the relationships between the quantitative data in the problem statement. The word problem consists of:

- **a text** (the problem statement), which presents **quantitative relationships** between multiple types of **information** within the text (Greer et al., 2002)
- and a **question or request**, whose answer can be derived through mathematical calculation from the values provided in the text and the relationships established between them (Mellone et al., 2014; Vălcan, 2018).

II.5.1. Information embedded in the word problem: characteristics and implications in the solving process

Moreau & Viennot (2003) and Voyer (2011) identified three categories of information word problem texts: solving information, situational information, and explanation information.

Solving information provides essential data required for solving strategies (Moreau & Viennot, 2003; Voyer, 2011). It refers to both known and unknown data, as well as the relationships between them (Vălcan, 2018). Understanding and mentally representing the relationships between data in a problem are crucial for a successful solving strategy (Verschaffel et al., 1994; Verschaffel & De Corte, 1993).

Situational information does not influence the solving strategy. It anchors the solving information in a real-life situation through descriptions and details often related to students' familiar experiences with the situation described (Voyer, 2011). Evoking familiar aspects, situational information facilitates deductions about the specific context described in the statement (Kintsch & Dijk, 1978; Kintch & Rawson, 2005).

Explanation information details the other types of information in the problem statement (solving information and situational information).

Cummins et al. (1988) and Stern & Lehrndorfer (1992) demonstrated that situational and/or explanation information has a positive influence on students' solving performance. Situational and explanation information do not necessarily coexist in problem statements.

II.5.2. The relatioships between known and unknown data: structural components of word problems

Riley et al. (1983) classified simple word problems, whose solution is determined by computing a single addition or subtraction operation, depending on the action that the solver has to perform on the sets of elements presented in the problem statement: change, combine or compare the two sets (Table 3.II.).

1. Change problems, which involve modifying the number of elements in a given set (Riley et al., 1983). The problem solution represents the set of elements obtained after modifying an initial set, resulting from a cause-and-effect relationship (Nesher et al., 1982).

initial set ± change = result set

Depending on the change executed on the initial set (increase or decrease), Riley et al. (1983) described word problems whose solution is obtained by:

- **increasing** the number of elements in a set (join word problems):

Radu has 3 apples. Vlad gives 4 apples to Radu. How many apples does Radu have now?

- **decreasing** the number of elements in a set (separate word problem):

Radu has 8 apples. He gives Vlad 5 apples. How many apples does Radu have left?

- **equalizing** the number of elements (equalizing word problems) of the two sets (Riley et al., 1983):

Radu has 3 apples and Vlad has 8. How many apples does Radu need to receive to have as many apples as Vlad? / Radu has 3 apples and Vlad has 8. What does Vlad need to do to have as many apples as Radu?

2. Combine problems involve combining two or more sets of elements. These word problems describe a part–whole relationship, where the unknown set can be either the whole or one of the parts. In these problems, the quantitative relationships between sets of elements are static; therefore, no set of elements presented in the problem statement is modified:

Radu has 3 apples, and Vlad has 5 apples. How many apples do they have together?

The solving strategy represents the whole as the sum of its parts:

part 1 + part 2 = whole set (sum, total)

3. Compare problems require the problem solver to compare sets of elements or quantities. To solve the task, the solver must identify and use the difference between the two sets or quantities. In comparison problems, the quantitative relationships between data are static, with one set being compared to another, known as the *reference set* (Nesher, 1982; Riley et al., 1983).

Radu has 8 apples, and Vlad has 5. How many more apples does Radu have than Vlad?

Usually, compare word problems can be transposed into mathematical relationships as follows:

large set – small set = difference.

A subset of compare word problems where the number of elements in the compared set is unknown, expressed by the difference in the number of elements in the reference set:

Radu has 3 apples, and Vlad has 5 more or fewer (than Radu). How many apples does Vlad have? / Radu has 8 apples, which is 3 more than Vlad. How many apples does Vlad have?

Usually, the algaebric expression underlying the solving process of compare problems is:

reference value ± difference = compared value

Table No. 3.II. Classification of simple word problems by the relationship between known and unknown data (Riley et al., 1983)

Dynamic relationships	**Static relationships**
CHANGE (increase or decrease)	**COMBINE**
Result unknown	**Combine value unknown**
1. Radu has 3 apples. Vlad gives Radu 4 apples. How many apples does Radu have now?	1. Radu has 3 apples, and Vlad has 5 apples. How many apples do they have altogether?
2. Radu has 8 apples. He gives Vlad 5 apples. How many apples does Radu have left?	**Subset unknown**
Change unknown	2. Radu and Vlad have 8 apples altogether. Radu has 3 apples. How many apples does Vlad have?
3. Radu has 3 apples. Vlad gives him some of his apples. Now Radu has 8 apples. How many apples did Radu receive?	**COMPARE**
4. Radu has 8 apples. He gives some apples to Vlad and has 3 apples left. How many apples did Radu give to Vlad?	**Difference unknown**
Start unknown	1. Radu has 8 apples, and Vlad has 5. How many apples does Radu have more than Vlad?
5. Radu has some apples. After receiving 5 apples from Vlad, Radu has 8 apples. How many apples did Radu have in the beginning?	2. Radu has 8 apples, and Vlad has 5. How many apples does Vlad have less than Radu?
6. Radu has some apples. After giving Vlad 5 apples, Radu has 3 apples left. How many apples did Radu have in the beginning?	**Compared quality unknown**
EQUALIZING	3. Radu has 3 apples, and Vlad has 5 more apples than Radu. How many apples does Vlad have?
1. Radu has 3 apples, and Vlad has 8 apples. How many apples does Radu need to have as many apples as Vlad?	4. Radu has 8 apples, and Vlad has 3 apples less than Radu. How many apples does Vlad have?
2. Radu has 8 apples, and Vlad has 3 apples. What does Radu have to do to have as many apples as Vlad?	**Referent unknown**
	5. Radu has 8 apples, which is 3 more than Vlad. How many apples does Vlad have?
	6. Radu has 3 apples, which is 5 less than Vlad. How many apples does Vlad have?

Research on students' solving performance in each of the three main problem categories of problems has indicated that compare problems are the most difficult for primary school students to solve (Giroux & Ste-Marie, 2001; Hegarty et al., 1995; Nesher & Teubal, 1975; Stern & Lehrndorfer, 1992).

II.5.3. Solving task: the request that guides the solution process

Polya (1945/1965) categorised word problems based on the specific nature of the solving task: finding an unknown value ("problems to be found") or demonstrating a statement ("problems to be demonstrated"). "Problems to be found" include the known and unknown data, and the problem condition (the relationship between known and unknown quantitative data/condition), while the main components of "problems to be proven" are the hypothesis and the conclusion(s).

II.6. Understanding word problems

To solve a word problem, it is necessary to identify the solving information (Reusser, 1985; Voyer, 2011). Understanding the information contained in a word problem requires the solver to possess *(1) real-world knowledge* and *(2) logico-mathematical knowledge* (mental relationships and connections derived from ordering, categorisation, abstraction, and conceptualisation of experiential knowledge). Logico-mathematical knowledge is essential for understanding the quantitative relationships between data in the problem (Kamii & Joseph, 2004; Nesher et al., 1982) and is gained through the mental internalisation of actions performed on objects. Piaget (1964) described the *operation* as an internalised action, performed mentally on the object. An operation relates to other operations, forming mental operational structures that enable new knowledge to be acquired and organised (Piaget, 1964, 1936/1973). Variations in students' problem-solving performance have been attributed to differences in the development of logico-mathematical reasoning, which can lead to faulty representations of problem content and difficulty in mentally manipulating sets of objects presented in problem statements (Riley et al., 1983; Verschaffel et al., 1994).

The difficulties experienced by some students when solving word problems have been attributed to the language of the problem statements, which does not always have a referent in the cognitive repertoire of young children (e.g., "with... greater than", "as many... as...", "each... has as many..." etc.) (Cummins et al., 1988; Hudson, 1983; Nesher & Teubal, 1975; Schoenfeld, 1991; Stern & Lehrndorfer, 1992).

Hudson (1983) investigated the role of language in understanding and solving problems by asking some first-grade students to solve the following compare word problem, whose task contained the expression "how many more... than..." specific to mathematical language: *Here are 5 birds and 3 worms. How many more birds are there than worms?* The problem was solved correctly by 64% of the children. Reformulating the task in more accessible language improved the students' performance: *Here are 5 birds and 3 worms. Suppose the birds all race over, and each one tries to get a worm. Will every bird get a worm?... How many birds won't get a worm?* Once the question was rephrased, all subjects provided the correct problem solution. Similar results were obtained in a replicative, quasi-experimental (pre-test-posttest) study on a single sample of 45 Romanian first-grade students. The significant amount of incorrect solutions to the problem whose task was formulated using the expression "How many children are there more than balls?" compared to the problem whose task was reformulated a more familiar language

("How many children won't have a ball?") evidenced that inappropriate interpretation of the first expression led to a large number of incorrect solutions.

II.6.1. Keywords – verbal cues of mathematical operations required for the solving process

Giroux & Ste-Marie (2001), Hegarty et al. (1995), Stern & Lehrndorfer (1992), and Nesher and Teubal (1975) identified and described the main types of "mistakes" that students make when solving compare word problems. The most common error involves associating specific keywords in the problem with the suggested mathematical operation. For example, "more" or "with... more" are frequently associated with addition, and "less" or "with... less" with subtraction. In some cases, similar expressions describe different relationships between the data in the problems, and the operation necessary to determine the problem solution is not congruent with the basic meaning of the expression, as in the following example: *There are 9 boys and 14 girls in the third grade. How many* ***more*** *girls are there than boys?*

An analysis of the strategies used by primary school students to solve compare word problems evidenced that participants perform better on problems where some expressions in the statement are congruent with the mathematical operation that must be performed to determine the solution (Giroux & Ste-Marie, 2001; Hegarty et al., 1995; Nesher & Teubal, 1975; Riley et al., 1983; Stern & Lehrndorfer, 1992), such as the following problem: *Elena has 15 beads, and Mihaela has 7* ***more****. How many beads does Mihaela have?*

The differences in the solving strategies employed by students for problems involving both congruence and incongruence in the expression, which indicate the comparison of the sets described in the statement, were analysed in a quasi-experiment (pre-test-posttest) conducted on a sample of 42 Romanian first-grade students, where the two mentioned problems were used.

The analysis of solutions to the two problems (Table No. 8.II.) indicates that most students correctly solved the problem where the basic meaning of the expression "with... more" is congruent with the operation required to determine the problem solution. The incongruence between the basic meaning of the verbal indicator and the mathematical operation required to solve the other problem led to an increase in the number of incorrect problem solutions obtained by performing an addition.

Table No. 8.II. Analysis of problem solutions and solving strategies

	Problem 1 (verbal expression, "*how many* ***more***" is congruent with solving operation)		**Problem 2** (verbal expression "***more***" is incongruent with solving operation)	
Solving strategy	Addition 15 + 7	Incorrect solution/ no response	Subtraction 14 – 9	Addition 9 + 14
Students No. (%)	38 (90,4%)	4 (9,5%)	33 (78%)	9 (21,4%)

The solving performance of Romanian students in the sample analysed corresponds to the results obtained in similar studies (Giroux & Ste-Marie, 2001; Hegarty et al., 1995; Nesher & Teubal, 1975; Riley et al., 1983; Stern & Lehrndorfer, 1992). The superficial approach to specific expressions in the statement can be attributed to either poorly developed reading comprehension skills or the complex ways in which the data is presented in the problem (Boonen et al., 2016; Hegarty et al., 1995; Reusser, 1988). Research analysing the relationship between reading comprehension skills and students' solving performance (Bjork & Bowyer-Crane, 2013; Boonen et al., 2016; Can, 2020; Pongsakdi et al., 2020; Timario, 2020) has demonstrated a strong correlation between the two variables, with the development level of reading comprehension skills being a predictor of students' problem-solving performance.

II.6.2. Factors influencing the understanding of compare word problems

During preparatory and first-grade classes, teaching addition and subtraction depends on children's representations of concrete actions performed on various sets of objects (Piaget, 1964). Using real actions to represent addition and subtraction supports children's transition from physical action to abstract mathematical symbols, which explains why most first-grade students successfully solve change and combine word problems (Nesher et al., 1982; Riley et al., 1983).

Unlike the dynamic action that produces change, the static nature of comparing two sets of elements makes it difficult for some students to represent the comparison to a concrete action, thereby hindering their understanding and application of this relationship as an abstract mathematical operation (Giroux & Ste-Marie, 2001; Nesher, 1980). Translating a comparison between two sets of elements into an abstract mathematical operation is achieved after acquiring specific mental schemas, together with the ability to mentally represent the difference between the two sets presented in a problem as an independent set (Kamii & Joseph, 2004; Stern & Lehrndorfer, 1992). Stern & Lehrndorfer (1992) explained that the difficulties in understanding compare relationships are related to children's practical experience, which lacks quantitative comparisons in their current activities. Most solving tasks require indicating the set values of differences, unlike the current children's activities, where most comparisons are qualitative.

First-grade students' difficulties in understanding compare word problems often manifest as measurable cognitive behaviours observed by the teacher. We describe the case of T., a first-grade student who was asked to solve a simple compare problem, indicated as having the highest success rates among all categories of compare word problems (Carpenter et al., 1981; Giroux & Ste-Marie, 2001; Nesher et al., 1982; Riley et al., 1983): *There are 6 light bulbs and 2 more batteries in a box. How many batteries are in the box?*

The solving success of the above-mentioned problem can be attributed to the congruence of the keyword “more” with the operation required for the solving process. Nonetheless, the degree of logico-mathematical reasoning is crucial for the accuracy of the solving strategy. We present the discussion with student T., which indicates their particular understanding of the problem content.

Figure No. 1.II. Student's T. representation of both sets in the problem

After reading the problem, T. is asked to identify the known data in the statement, guided by a few additional questions to help him understand the problem content:

Researcher: What is the problem about? How many sets of objects are there in the box?

T.: There are two sets of objects.

R.: What kind of objects are those?

T.: Light bulbs and batteries.

R.: What do we know about them, how many of each are there?

T.: There are 6 light bulbs.

R.: And batteries?

T.: There are 2 batteries.

R.: Read the problem once more, carefully! (T. reads the statement aloud.) *What does it say about the batteries?*

T.: That there are two more.

R: So, which of the two objects are there more of: light bulbs or batteries? (qualitative comparison of sets, in order to ensure proper understanding of the relationships between data)

T.: Batteries.

R.: How did you figure that out?

T.: Because the problem says there are more of them.

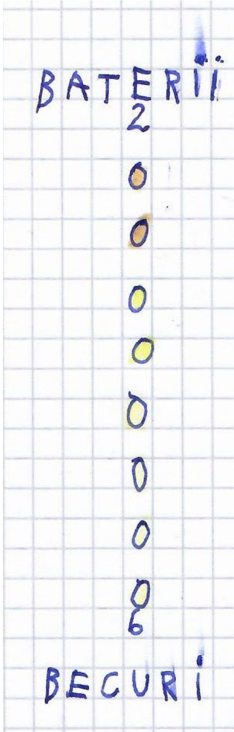
R.: What operation do you think you need to perform to solve the problem?

T.: 6+2

R.: Please draw a picture of the light bulbs and batteries in the problem, as indicated in the statement.

T.: You mean, draw 6 light bulbs and 2 batteries? (Rephrasing the problem statement from one's understanding reveals the actual level of comprehension of the content, as evidenced by Cummins et al., 1988.)

R.: Read the problem statement once again, carefully, then draw the light bulbs and batteries as indicated in the problem (T. was told that he could represent the two sets of objects using circles of different colours) (Figure No. 1.II.).



* The student's T. drawing illustrates the accuracy of mental representation of the two sets of elements: light bulbs and batteries.

Student T's answers reveal a lack of logical-mathematical knowledge necessary to understand the compare relationship between the data in the problem and to interpret the expression "2 more" accurately. Rewording the problem provides clues about students' understanding of the quantitative relationships between the data, as Cummins et al. (1988) also observed. Additionally, the child's drawing highlights the comprehension gaps in understanding compare relationships (Paquette et al., 2007). Although the student performs the correct mathematical operation, this is not due to an accurate understanding of the relationships between the data but to associating them with keywords. Despite the incorrect reasoning, the accuracy of the problem solution highlights that performing the correct mathematical operation does not guarantee an understanding of the problem or the validity of the solving strategy.

CHAPTER III
VISUAL REPRESENTATIONS – WORD PROBLEM SOLVING TOOLS

Student T.'s drawing on the problem content reflected their understanding of the quantitative relationships between the data. Children-created drawings serve as a form of communication specific to children, illustrating their psychological development (Cambier, 1990/2008), as the richness of spoken language indicates cognitive development. Language comprehension or thought expression through spoken or visual forms, such as words or drawings, was explained with the help of mental visual representations.

III.1. Simulation theory

The role of mental images in understanding message expressed through verbal, oral, or written language was described using simulation theory. Verbal language is understood by mentally simulating the presented situation or context, which involves evoking one's mental images related to the action or specific aspects described (Gallese & Lakoff, 2005; Glenberg, 2011; Glenberg & Robertson, 1999; Piaget & Inhelder, 1966/2011).

Explaining text comprehension based on simulation theory emphasises the specific and contextual nature of understanding, where the meaning of each statement in the text involves recalling mental representations and real-life knowledge of the presented context (Glenberg & Robertson, 2009).

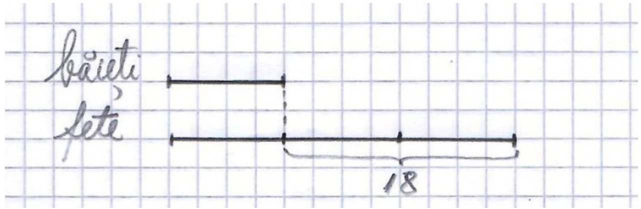
III.2. Mental images, visualisation and visual reasoning: conceptual boundaries

Like any text, understanding word problems requires mentally simulating the problem content. Research exploring text comprehension has demonstrated the importance of mental images as reasoning tools. The use of visual representations in understanding and solving word problems is referred to as "visualisation" and "visual reasoning" (Arcavi, 2003; Bishop, 1988; Dreyfus, 1991; Geçici & Türnüklü, 2021; Lean & Clements, 1981; Presmeg, 1986a; Presmeg, 1986b).

Visualisation is described as the ability to represent, organise, and logically synthesise data in a visual format, such as graphs, diagrams, charts, or other visual representations (internal or external to the subject). These visual tools facilitate connections between information, deductions, and other relationships within data that are not readily accessible through direct perception (Ahmad, 2010; Arcavi, 2003; Zimmerman & Cunningham, 1991). In their works, Arcavi (2003), Bishop (1989), Herskowitz et al. (1989), Presmeg (1986b), Polya (1945/1965), and Zimmerman & Cunningham (1991) referred to the potential of representing visually what is not explicitly presented in the statement of mathematical problems (inferences, deductions), describing visualisation as the ability and process of recalling, creating, using, and interpreting mental or concrete visual representations to understand information and formulate new ideas and meanings, leading to global understanding of the problem.

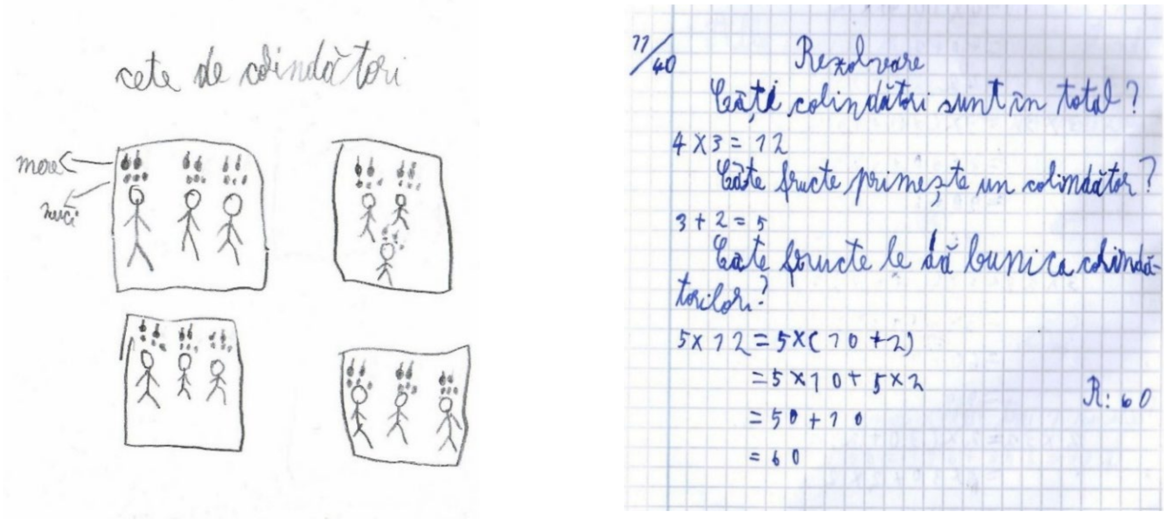
Creating a visual representation of the content of the following word problem through schematic drawing ensures the 'visibility' of the quantitative relationships between the data, highlighting the relationship between the two sets described in the statement (Figure No. 1.III.): *In a class, the number of girls is three times more than the number of boys. Knowing that there are 18 more girls than boys, determine the number of girls and boys in the class* (Mogoş, 2024).

Figure No. 1.III. Outlining the relationships between data in the problem using a graphic representation



Ried et al. (2022) concluded that the solving performance of individuals with limited ability to generate mental images is significantly lower than that of those with advanced mental representation skills. The solving process based on generating and analysing spatial and/or visual representations of the problem content by the solver has been referred to as "visual reasoning" (Bishop, 1989; Dreyfus, 1991; Geçici & Türnüklü, 2021; Herskowitz et al., 2001). Visual reasoning is described as one's ability to organise visual representations into functional structures used for solving word problems. The ability to use visual reasoning is developed through repeated reflection on visual representations that emphasise the extent to which the information in the problem statement is processed and understood (Dreyfus, 1991; Hershkowitz et al., 2001; Rosken & Rolka, 2006). Employing visual reasoning in problem-solving includes finding solutions by creating and using drawings, graphs, diagrams, or charts during the development of the solving strategy (Suwarsono, 1982). The following problem generates difficulties for second-grade students because it presents multiple subsets included in a larger set, making it difficult for the children to organise the data and construct an accurate solving strategy: *Grandma gives each carol singer three walnuts and two apples. How many walnuts and apples did grandma give to all the children if four groups of three carol singers each arrived?* (Mogoş, 2024). The accurate depiction of the problem's content through a drawing by one of the students, who was unable to solve the problem due to the difficulty of coordinating visual representations of each semantic structure from the statement into a comprehensive overall view, demonstrated the role of graphic representation in organising and structuring the data and in developing the solving strategy (Figure No. 2.III.).

Figure No. 2.III. Developing the solving strategy using the student-created drawing



Once the problem content was organised into a coherent graphical representation, developing the solving strategy became straightforward, with the problem solution emerging from visual reasoning (Bishop, 1989; Geçici & Türnüklü, 2021; Herskowits et al., 2001; Presmeg, 1986a; Suwarsono, 1982).

III.3. Mental images: relevant factor involved in reading comprehension

Mental images are cognitive constructs recalled in the absence of the object of perception. Depending on the context that generated them, mental images can be reproductive (evoking previously perceived objects or actions) or anticipatory (obtained by mentally combining other types of representations). The capacity to operate with mental representations manifests in individual behaviour through the emergence of symbolic play, verbal language, and drawing (Piaget & Inhelder, 1966/2011; Miclea, 1999).

III.4. Visual representations: categorisation and implications in the solving process

Russel (1997) classified the representations of mathematical problems according to the environment in which they are generated: internal representations (mental products resulting from perceptions and interactions with the environment) and external representations (physical materialisations of internal representations: paper and pencil drawings, illustrations, diagrams, graphs, charts, graphic organisers, tables) (Bishop, 1989; Boonen, 2014; Presmeg, 1986b; Russel, 1997).

Presmeg (1986a, 1986b) described five categories of imagery involved in representing and solving mathematical problems:

(1) concrete pictorial imagery: presents in visual language the illustrative elements described by contextual information in the problem statement, without relevance to solving strategy;

(2) pattern imagery: spatial representations that describe the relationships between the data in the problem, resulting in visual representations of the solving information, essential for solving strategy (Boonen et al., 2014; Hegarty & Koshevnikov, 1999);

(3) memory images of formulae: photographic visual representations of mathematical formulas;

(4) kinaesthetic imagery: representations that the solver "explores" mentally;

(5) dynamic imagery: spatial and visual representations that the solver mentally transforms to understand and solve the problem (Hegarty & Kozhevnikov, 1999; Presmeg, 1986a).

Boonen et al. (2014) identified three types of graphic visual representations of word problems, which correspond to different levels of solving success: illustrative representations and schematic representations (visual representations of solving information), which may be either correct or incorrect. Correct schematic representations depict the relationships between data and solving information, such as student-created drawings, sketches, and diagrams (see Figure No. 2.III.). Incorrect schematic representations (of solving information) suggest the student's misunderstanding and reflect their inappropriate grasp of the quantitative relationships between data in the problem (see the student T. drawing, Figure No. 1.II.). Boonen et al. (2014) and Gros et al. (2025) highlighted an association between each type of graphic representation and the success rate in the solving procedure. Students who generated correct schematic representations performed better than those who produced illustrative representations or incorrect schematic representations of the solving information. Likewise, Boonen et al. (2014) demonstrated that text comprehension and the ability to process spatial relationships are strong predictors of students' solving performance.

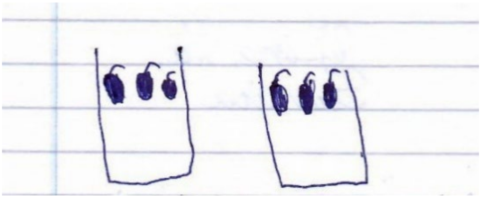
III.5. Student-created graphic representations – visual expression of word problem understanding

The relationship between the accuracy of visual representations and one's knowledge concerning a specific subject or concept was demonstrated through the Talking Drawings strategy. Talking Drawings involves translating mental images into graphic representations, specifically student-created drawings (McConell, 1993), which serve as an indicator of understanding a particular content expressed in visual language (Bainbridge, 2022).

Paquette et al. (2007) compared drawings made by several primary school students before and after reading an informative text on a specific topic. The graphic representations created after reading the text presented additional details and increased accuracy of the topic addressed, compared to the graphic representations created before reading the text. Bainbridge (2022), Nielsen-Hibbing & Rankin-Erickson (2003), Paquette et al. (2007), and Peeck (1987) demonstrated that students' drawings reflect the quality of their knowledge and their level of understanding of a subject or concept. At the same time, incorrect graphic representations highlight potential gaps in understanding the content (Carotenuto et al., 2021; Paquette et al., 2007) (see Figure No. 1.II and Figure No. 3.III).

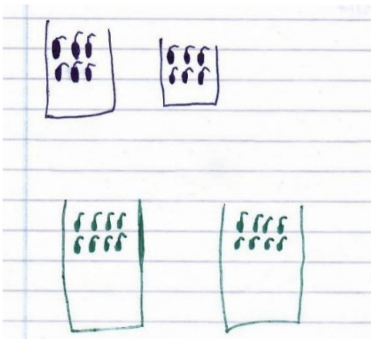
The accuracy of understanding the relationships between data in the following problem was evaluated using the Talking Drawings strategy: *"A shop sold two boxes of six blue globes each and two boxes of eight green globes each. How many globes were sold?"* (Mogoș, 2022, p. 51) The difficulties a second-grade student experienced in describing what he did not understand about the relationships between the data in the problem were eliminated once the student drew what he pictured regarding the problem content (Figure No. 3.III).

Figure No. 3.III. Graphic representation of verbal misunderstood expression *'two boxes of 6 globes each'*



The initial graphic representation of the problem indicated a misunderstanding of the expression *'two boxes of 6 globes each'*. After clarifying the distributive meaning of the expression '… of … each …' (which indicates that there are 6 globes in each box), the quality of the drawing improved based on the new understanding (Figure No. 4.III.).

Figure No. 4.III. Graphic representation of word problem after clarifying the verbal expression *'two boxes of 6 globes each'*



Graphic representations, as a means of expressing students' understanding, can serve as a formative assessment tool for teachers (Scott & Weishaar, 2008; Gros et al., 2025), enabling them to readily intervene and correct errors that occur during the learning process.

III.6. "Draw a picture!" – additional phase of the solving process

The difficulties that primary school students encounter in understanding and solving compare word problems (subchapters II.5., II.6.) have been discussed in the literature, with most studies analysing solutions and visual representations of other types of word problems involving secondary or high school students. Therefore, it is necessary to explore how primary school students' drawings can serve as a tool for solving a specific type of word problem, frequently encountered in the first-grade mathematics curriculum, which involves comparing and combining two sets of elements. In a pilot quasi-experimental study (pretest–posttest, one sample), 45 first-grade students solved two similar compare-combine word problems (Purcar et al., 2024). The following questions guided the research:

1. How will the Talking Drawings strategy provide insights into first-grade students' understanding of compare-combine word problems?

2. To what extent will first-grade students use their drawings to develop a solving strategy?

In the pretest, students read and solved word problems as usual. In the posttest, they read the problem, drew a picture of the problem content, and then solved it.

Problem 1 (pretest): Radu has 3 pencils, and Tudor has 4 more pencils. How many pencils do the two children have?

Problem 2 (posttest): There are 5 frogs on a water lily leaf. On another lily leaf, there are 3 frogs less. How many frogs are on the water lily leaves?

Analysis of the solutions provided by students revealed three types of results: correct problem solutions, incorrect problem solutions, and no answer (Table No. 1. III.).

Table No. 1.III. Problem solutions in pretest and posttest

Results	Pretest	Posttest
Correct problem solutions	12 students (26.6%)	19 students (42.2%)
Incorrect problem solutions	29 students (64.4%)	17 students (37.7%)
No answer	4 students (8.8%)	9 students (20%)

The most *correct problem solutions* in the pretest and posttest were obtained by performing two mathematical operations: first, determining the number of elements in the compared set through addition or subtraction, and then adding the elements of the two sets.

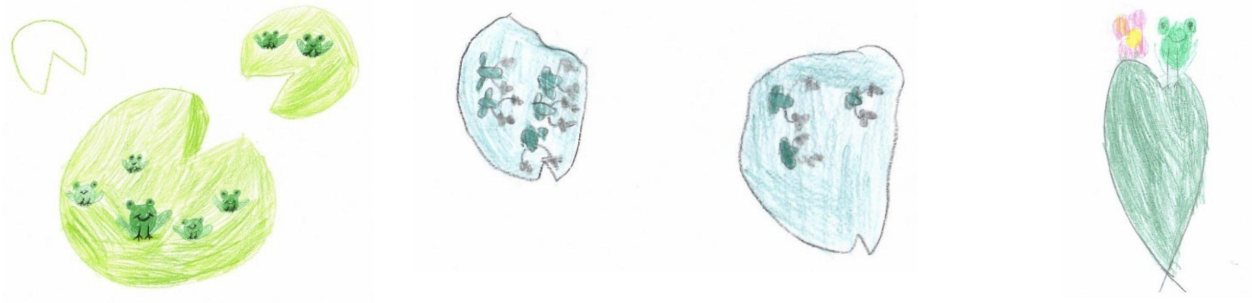
All *incorrect problem solutions* resulted from performing a single operation—addition or subtraction—based on the keywords in each statement (more in the pretest or less in the posttest). The higher number of correct solutions in the posttest compared to the pretest can be attributed to the support provided by the graphic representation of the problem content, which helped students develop their solving strategies. Differences between students' correct solutions in the pretest and posttest were analysed using the paired sample T-test. The results indicated a statistically significant, low-intensity correlation between the mean values used to assess solution correctness in the two testing phases ($p = 0.04 < 0.05$; $r = 0.36$; mean difference = -0.31). An analysis of student-created drawings identified three categories of graphic representations, corresponding to those described by Boonen et al. (2014): accurate and inaccurate graphic representations of solving information, and illustrative graphic representations (Table No. 2.III.).

Table No. 2.III. Classification of graphic representations of word problems

Graphic representations	
Accurate	33 students (73.3%)
Inaccurate	7 students (15.5%)
Illustrative	4 students (8.8%)
No representation	1 student (2.2%)

The accurate graphic representations accurately present the two sets of elements described in the problem statement, while the inaccurate graphic representations consist of presenting two sets with as many elements as indicated by the numbers in the problem statement (Figure No. 5.III.).

Figure No. 5.III. The main categories of student-created graphic representations of problem 2 (posttest)



a) Correct graphic representation b) Incorrect graphic representation c) Illustrative representation

Accurate graphic representations were associated with a large number of correct problem solutions (Figure No. 6.III.); in contrast, inaccurate graphic representations were associated with a small number of incorrect problem solutions and only one correct solution. A small number of students who produced inaccurate graphic representations did not solve the problem.

Figure No. 6.III. Problem solutions of accurate graphic representations in posttest

a) Accurate graphic representation associated with correct problem solution

b) Accurate graphic representation associated with incorrect problem solution

In the pretest, the number of correct problem solutions was associated with the overall understanding of the problem content. In the posttest, the numerous accurate graphic representations indicated an increased understanding of the problem content. However, a significant number of students who created accurate graphic representations of the problem content still arrived at incorrect solutions in the posttest by performing the subtraction suggested by the keywords in the statement, indicating ‘2 frogs...’ as problem solution. In an attempt to identify the causes of this superficial problem-solving behaviour, individual interviews were conducted with several students (see Figure No. 7.III.), which provides an example of student explanations.

Figure No. 7.III. Dialogue with one of the students who created an accurate graphic representation of the problem content but determined an incorrect problem solution by performing a single subtraction

Researcher: You have correctly represented the two sets of frogs, but you have only performed the subtraction 5–3. Is this the final solution? (n.a. 2 frogs)

Student: Yes.

Researcher: What does the problem ask us to find out? Please read the question once again!

Student (reading the question aloud): How many frogs are on the lily leaves?

Researcher: So, what is the answer to the question?

Student: …5–3=2 …

Researcher: Look at the drawing, you realised. I explained to you that it is correct. What is the solution to the problem?

Student (looking at the confusing drawing): 5–3…

Researcher: Please carefully examine the drawing. How many frogs are there on the lily leaves *altogether*?

Student: Oh, there are 7 frogs!

The analysis of student-created graphic representations of the problem content and the problem solutions evidenced that understanding a problem does not necessarily lead to the correct solving strategy. The student-created drawings can be used as tools to assess understanding of the problem content and to support elaborating the solving strategy. The small sample involved in the pilot study does not allow us to generalise the conclusions, and further investigation is needed to examine the effects of graphic representation of word problem content on the quality of the solving strategy and the problem solution, involving a larger and more diverse sample of participants, with various work skills, cognitive abilities, and problem-solving approaches, depending on different teaching styles they are exposed to.

CHAPTER IV
PRESENTATION OF THE PEDAGOGICAL RESEARCH "THE IMPLICATIONS OF USING FIRST-GRADE STUDENTS' CREATED GRAPHIC VISUAL REPRESENTATIONS ON UNDERSTANDING AND SOLVING COMPARE – COMBINE WORD PROBLEMS"

IV.1. Research methodology

IV.1.1. Research aim and objectives

Research aim: to investigate the implications of first-grade student-created graphic representations of compare-combine word problems on understanding and developing of solving strategy.

Research objectives:

1. Describing first-grade students' *reading comprehension skills* as assessed by primary school teachers.
2. Describing the compare-combine *word problem-solving strategies* and *solutions* identified by first-grade students;
3. Describing the main categories of first-grade *student-created graphic representations* of compare-combine word problems;
4. Identifying correlations and associations between first-grade *students' reading comprehension skills*, the *correctness of compare-combine problem solutions*, the *specific solving strategies* used, and the *accuracy of student-created graphic representations* of word problems.

IV.1.2. Research questions and hypotheses

I. What are the main types of first-grade student-created graphic representations of compare-combine word problems?

II. To what extent will the first-grade students' created graphic representations of compare-combine word problems positively influence problem-solving strategies and solutions?

Most first-grade students will produce accurate graphic representations of solving information, which will increase the number of correct problem solutions compared to solving problems without a graphic representation of the problem content.

Research question 1: To what extent does *the correctness of compare-combine word problem solutions* correlate with *first-grade students' reading comprehension skills*?

Hypothesis 1: There is a positive correlation between the correctness of compare-combine word problem solutions and first-grade students' reading comprehension skills.

Research question 2: To what extent do *reading comprehension skills* correlate with *the accuracy of first-grade student-created graphic representations of compare-combine word problems*?

Hypothesis 2: There is a positive correlation between reading comprehension skills and the accuracy of first-grade student-created graphic representations of compare-combine word problems.

Research question 3: What is the relationship between *the main types of first-grade student-created graphic representations of compare-combine word problems* and *the correctness of problem solutions* in post-test?

Hypothesis 3: There is a positive correlation between the *accuracy of first-grade student-created graphic representations of word problems* in posttest and the *correctness of the problem solutions* determined by students for these problems.

Research question 4: To what extent do the *accurate student-created graphic representations of compare-combine word problems* increase *the success rate of solving* these problems?

Hypothesis 4: The *accurate student-created graphic representations of compare-combine word problems* will increase *the success rate of solving* the respective problems.

Research question 5: Is there an improvement in *first-grade students' solving performance in posttest* compared to their *solving performance in pretest*?

Hypothesis 5: The *first-grade students' solving performance in posttest* will improve compared to their *solving performance in pretest*.

Research question 6: To what extent does *the explanation information in compare-combine word problems statement* lead to an *increased number of accurate student-created graphic representations of word problems*?

Hypothesis 6: *The explanation information in compare-combine word problems statement* will lead to an *increased number of accurate student-created graphic representations of word problems*.

IV.1.3. Methods

The hypotheses were tested in a ***quasi-experimental study***, employing a ***one-group pretest-posttest design***.

1. Independent variable: student-created graphic representations of compare-combine word problems.

2. Dependent variable: problem solutions. Indicators: (1) correctness of the numeric result (correct problem solutions/ incorrect problem solutions) and (2) components of the solving strategy.

3. Moderator variable: first-grade students' reading comprehension skills.

Table No. 1.IV. Methods and data collection instruments

Variables	Methods	Research instruments
Independent variable *(nominal, dichotomous)*: student-created graphic representations of compare-combine word problems	Quasi-experiment	Self-designed coding sheet for student-created graphic representations *(qualitative analysis of the independent variable)*

Dependent variable ***(nominal, dichotomous)***: problem solutions	Quasi-experiment	Self-designed coding sheet *(qualitative analysis of the independent variable)*
Moderator variable ***(quantitative/ ordinal)***: students' reading comprehension skills	Questionnaire survey	10-point Likert scale (1 – minimum; 10 optimal for the age group *(moderator variable measurement)*

IV.1.4. Research variables and instruments

Depending on the data characteristics, the three variables analysed in the research were classified as follows:

1. Independent variable *(nominal, dichotomous variable)*. Instrument: self-designed coding sheet for student-created graphic representations.

2. Dependent variable *(nominal, dichotomous variable)*. Instrument: self-designed coding sheet.

3. Moderator variable *(ordinal variable).* In some statistical analyses, it was considered a quantitative variable (subchapters IV.2.6.2. and IV.2.6.3.). Instrument: 10-point Likert scale.

Tests and statistical tools used for hypothesis testing:

Hypothesis 1: The correlation between the moderator variable (students' reading comprehension skills – considered as a quantitative variable) and the dependent variable (the correctness of compare-combine word problem solutions) was tested using **biserial correlation**.

Hypothesis 2: The correlation between the moderator variable (students' reading comprehension skills – considered as a quantitative variable) and the independent variable (accuracy of first-grade student-created graphic representations of compare-combine word problems) was tested using **biserial correlation**.

Hypothesis 3: The relationship between the independent variable (the main types of first-grade student-created graphic representations of compare-combine word problems) and the dependent variable (correctness of the problem solutions) was tested using **Chi^2 test**.

Hypothesis 4: The success rate of the dependent variable (correct problem solution in posttest) under conditions of the independent variable (accurate student-created graphic representations of compare-combine word problems) was analysed using **logistic regression.**

Hypothesis 5: The difference between the dependent variable in posttest (first-grade students' solving performance in posttest) and the dependent variable in pretest (solving performance of the same students in pretest) was analysed using the **McNemar Chi^2** test for paired samples.

Hypothesis 6: The difference between independent variable – problem 1 and independent variable – problem 2 (the number of accurate student-created graphic representations of word problems in posttest, depending on the presence of explanation information in the problem statement) was tested using the **McNemar Chi^2** test for paired samples.

IV.1.5. Sample

Participants were selected through non-probabilistic, convenience sampling. The sample consisted of 256 subjects, comprising 137 girls and 119 boys (Figure No. 4.IV.), mean age of 7.4 years.

Figure No. 4.IV. Graphic representation of participant distribution by biological gender

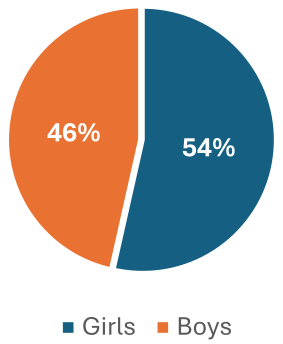
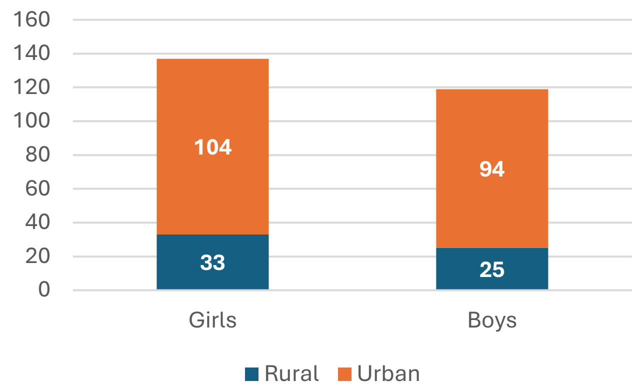


Figure No. 5.IV. Graphic representation of participants' distribution according to their residence (rural/urban)



IV.1.6. Content sample

The content sample consisted of five compare-combine word problems (Table No. 5.IV.)

Table No. 5.IV. Summary presentation of the types of information found in word problems within the content sample

	Problem	Solving information	Situational information	Explanatory information
Pretest	**0.** The first term of an addition is 10, and the second term is 4 less than the first. Find the sum of both terms.	- First term is 10; - The second term is 4 less than the first.	–	–
	1. There are 4 pears and 2 less strawberries on a plate. How many fruits are there on the plate?	- 4 pears; - 2 less strawberries.	On a plate, pears, strawberries, fruits.	–
	2. Ionuț gives his mother a bouquet for her birthday, containing several snowdrops and more violets. Given that there are 7 snowdrops and 3 more violets, determine how many flowers are in the bouquet.	- 7 snowdrops; - 3 more violets.	Ionuț gives his mother a bouquet for her birthday, containing snowdrops and violets.	… several snowdrops and more violets.
Posttest	**1.** There are 4 blue fish and 3 more yellow fish in an aquarium. How many fish are there in the aquarium?	- 4 blue fish ; - 3 more yellow fish.	in an aquarium, blue fish, yellow fish.	–

2. A large family of frogs lives at the edge of a lake. Several frogs sit on one lily leaf, and fewer frogs are on the other lily leaf than on the first. If there are 5 frogs on the first lily leaf and 3 less on the other, find out how many frogs are on the lily leaves.	- 5 frogs; - 3 less.	A large family of frogs lives at the edge of a lake, lily leaves, frogs.	Several frogs sit on one lily leaf, and fewer frogs are on the other lily leaf than on the first.

The correct problem-solving of the five problems involves developing similar solution strategies, consisting of the following steps:

1. determining the number of elements in the compared set;
2. combining the two sets of elements (Table No. 6.IV.).

The strategies for solving problems 0 and 1 in the pretest and problem 2 in the posttest involve performing subtraction and addition operations, while the strategies for solving problems 2 in the pretest and 1 in the posttest involve performing two addition operations.

Tabel No. 6.IV. Correct solving strategies of word problems in pretest and posttest

		Pretest			Posttest	
	Problem	**0**	**1**	**2**	**1**	**2**
Solving strategy	**Determining the number of elements in the compared set**	10 – 4 = 6 (first term)	4 – 2 = 2 (strawberries)	7 + 3 = 10 (violets)	4 + 3 = 7 (yellow fish)	5 – 3 = 2 (frogs on the other lily leaf)
	Combining the two sets	10 + 6 = 16 (sum)	4 + 2 = 6 (fruits)	7 + 10 = 17 (flowers)	4 + 7 = 11 (fish)	5 + 2 = 7 (frogs on the lily leaves)
	Problem solution	16 (sum)	6 fructe	17 flowers	11 fish	7 frogs

IV.1.7. Research stages and investigation procedure

1. *Collecting data on the individual characteristics of participating students* (questionnaire addressed to primary school teachers): biological gender (M/F), age (in years), and residence (rural/urban).

2. *Assessing students' reading comprehension skills* by primary school teachers (10-point Likert scale).

3. *Administering researcher-designed tests* in two different mathematics classes (Table No. 7.IV.). Students received two paper-and-pencil tests containing five compare-combine word problems.

Table No. 7.IV. Summary presentation of the researcher-designed test administration

Pretest	**Posttest**
1. Reading aloud the problem statement to the students once by the primary school teacher;	1. Reading aloud the problem statement to the students once by the primary school teacher;
2. Individual reading the problem statement by the students;	2. Individual reading the problem statement by the students;

–	**3. Graphic representing the problem content through students' drawings;**
3. Solving the problem by performing the calculations to find the solution.	4. Solving the problem by performing the calculations to find the solution.

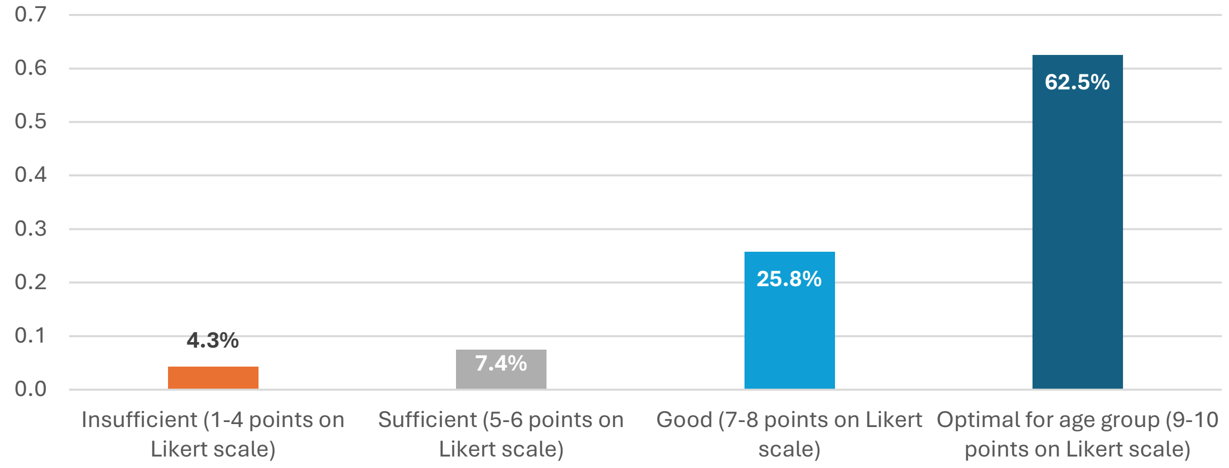
4. Qualitative and quantitative data analysis and interpreting the findings. The data gathered from solving strategies and solutions were analysed using the DATAtab platform: DATAtab - Online Statistics Calculator. DATAtab e.U. Graz, Austria. URL https://datatab.net.

IV.2. Data analysis and results interpretation

IV.2.1. First-grade students' reading comprehension skills analysis

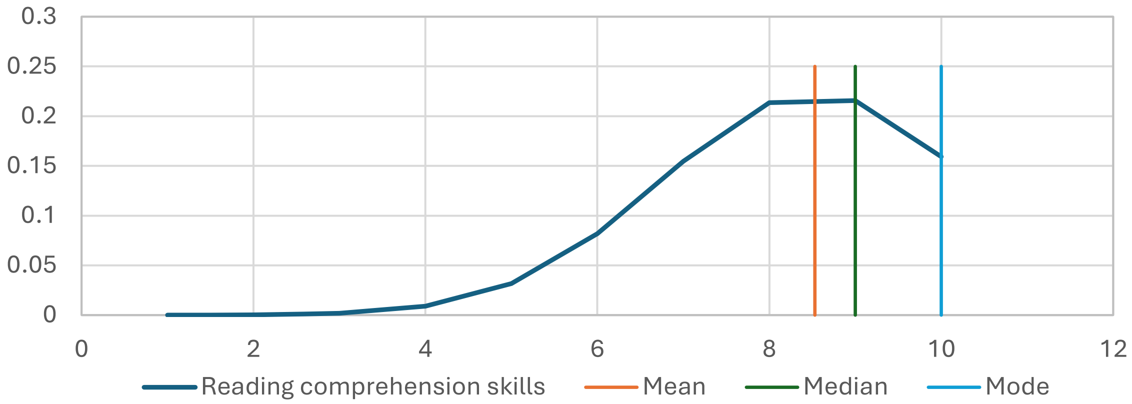
Students' reading comprehension skills were assessed by primary school teachers using a 10-point Likert scale. The scores used to evaluate reading comprehension skills were categorised into four ranges: optimal for the age group, good, sufficient, and insufficient.

Figure No. 9.IV. The graphic representation of students' reading comprehension skills score ranges



Analysis of data provided by primary school teachers showed that more than half of students (62.5%) had reading comprehension skills optimally developed for their age group (Figure No. 9.IV.), indicating a significant deviation from the normal distribution towards high scores on the Likert scale (8–10), which suggests a possible overestimation of students' reading comprehension skills by teachers.

Figure No. 10.IV. The distribution of the moderator variable (students' reading comprehension skills) in the sample



Based on the results obtained by Bjork & Bowyer-Crane (2013), Boonen et al. (2016), Can (2020), Pongsakdi et al. (2020), Timario (2020), which highlight the role of reading comprehension skills as a predictor of students' solving performance, we can infer that at least 62.5% of the research participants, whose reading comprehension skills were rated as optimal for their age group (Figure No. 9.IV.), will correctly solve the word problems in both the pretest and posttest.

IV.2.2. Categorisation of *problem solutions* in pretest and posttest

Problem solutions were categorised as *correct* or *incorrect* based on the accuracy of the numeric result and the written details provided by the students who explained the numeric outcome of the mathematical operations (Table No. 11.IV.).

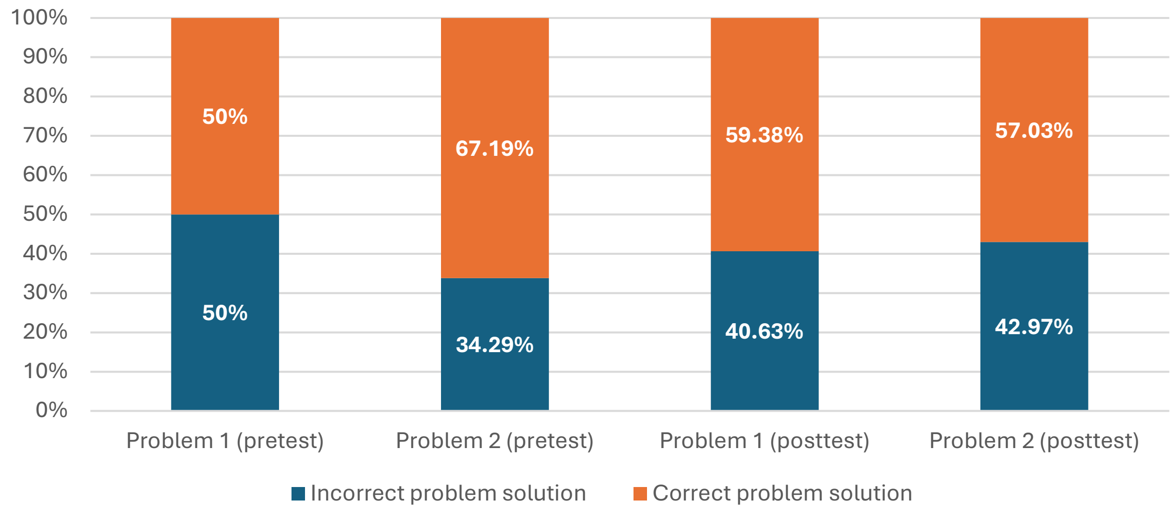
Table No. 11.IV. Categorisation of problem solutions in pretest and posttest based on the accuracy of the numeric result

	Correct problem solution (CS)		**Incorrect problem solution (IS)**		**Total**	
	Students No.	%	Students No.	%	Students No.	%
Pretest						
Problem 0	111	43.4%	145	56.6%	256	100%
Problem 1	128	50%	128	50%	256	100%
Problem 2	84	32.8%	172	67.2%	256	100%
Posttest						
Problem 1	104	40.6%	152	59.4%	256	100%
Problem 2	110	43%	146	57%	256	100%

Since problem 0 was expressed in abstract mathematical language, it has no equivalent in the posttest and was excluded from most statistical analyses. The number of solutions students provided for problem 0 served as a benchmark in some of the quantitative analyses of the solving strategies for problem 1 in the pretest and problem 2 in the posttest, which share a similar structure to problem 0.

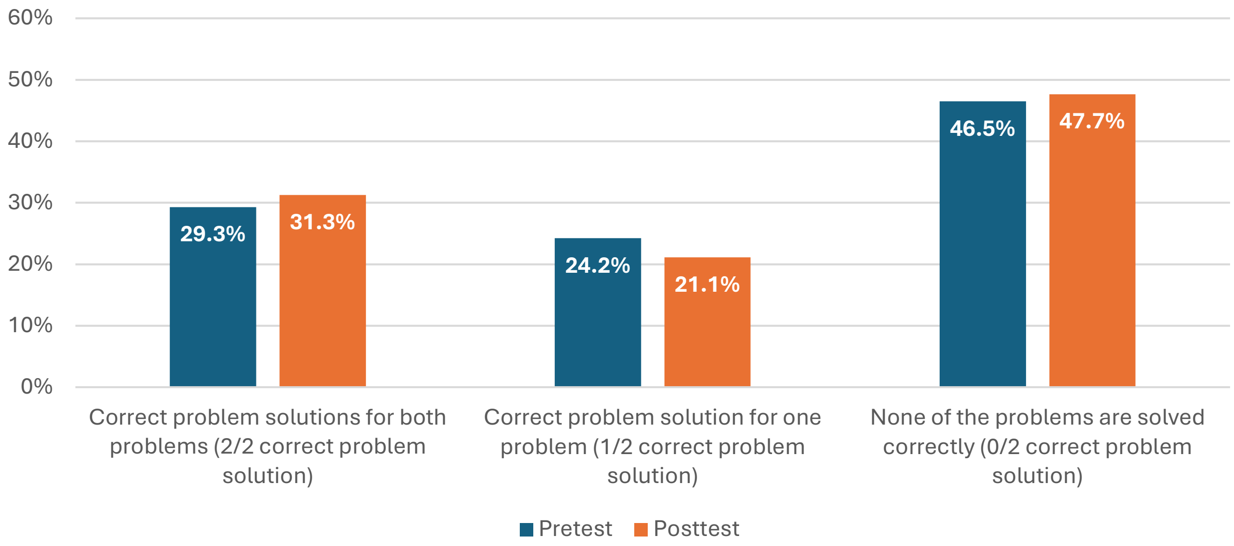
Quantitative analysis of the solutions provided by research participants of the four problems in pretest and posttest indicated a large number of incorrect problem solutions (Table No. 11.IV., Figure No. 11.IV.).

Figure No. 11.IV. Comparative presentation of correct and incorrect solutions to problems 1 and 2 in the pretest and posttest



Although the students whose reading comprehension skills were assessed as being optimally developed for their age group (Figure No. 9.IV.) should have predicted a similar number of correct answers in the pretest and posttest, only 29.3% of students in the pretest and 31.3% in the posttest managed to solve both problems correctly, and 24.2% of students in the pretest and 21.1% of students in the posttest solved only one of the two problems correctly at each stage of the research (Figure No. 12.IV.).

Figure No. 12.IV. Distribution of correct and incorrect problem **solutions** provided by students in pretest and posttest

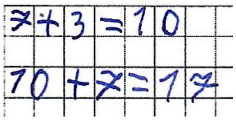
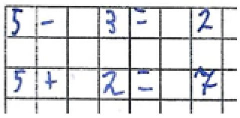
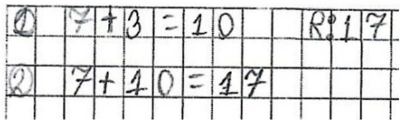
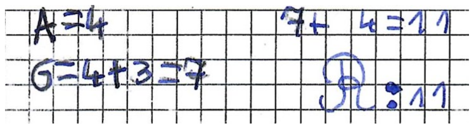
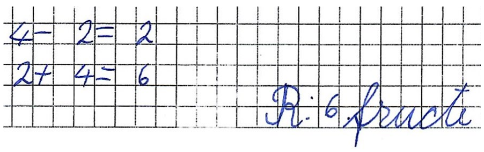
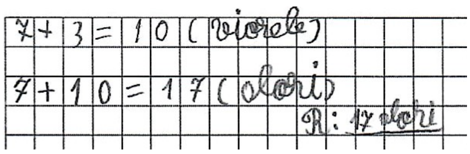


IV.2.3. Analysis of the main solving strategies used by first-grade students to determine the problem solutions in pretest and posttest

Depending on the quality of the solving strategy, the problem solution, and the written details of the numeric result provided by some students, the solving strategies were classified into two categories: *correct solving strategies* (CS) and *incorrect solving strategies* (IS).

1. Correct solving strategies (CS) demonstrate understanding of the problem content (see subsection IV.1.6., Table No. 4.IV.). Strategies were classified as correct if they included either some or all of the following components: properly writing and performing calculations in the logical order of identifying unknown quantities in the problem (Figure No. 13.IV.a.); the correct problem solution – that is, the numeric result obtained from accurately performing the mathematical operations (Figure No. 13.IV.b); and correct written explanations of the problem solution (Figure No. 13.IV.c). Problem solutions were also considered correct if the mathematical operations or results were not accompanied by written explanations or if students did not explicitly indicate the problem solution (Figure No. 13.IV.a.), using the conventional formulation "R(esponse): numeric result + written explanations" (Figure No. 13.IV.b, Figure No. 13.IV.c.), as this way of presenting the problem solution does not exist in the absence of formal training of the student in this regard.

Figure no. 13.IV. Examples of correct solving strategies in pretest and posttest, with different components of the solving strategy

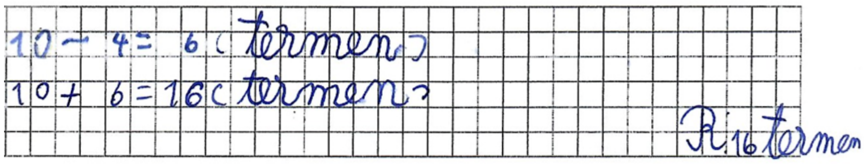
a.	► **Correct solving strategy**	**Problem 2** (pretest) 	**Problem 2** (posttest) 
b.	► **Correct solving strategy** ► **Correct problem solution**	**Problem 2** (pretest) 	**Problem 1** (posttest) 
c.	► **Correct solving strategy** ► **Correct problem solution** ► **Correct written explanations**	**Problem 1** (pretest) 	**Problem 2** (pretest) 

2. Incorrect solving strategies (ISS) were regarded as an indicator of poor understanding of the problem and were categorised based on the solving component where the error occurred: during writing and performing the mathematical operations, when indicating the problem solution, or when explaining the numeric results.

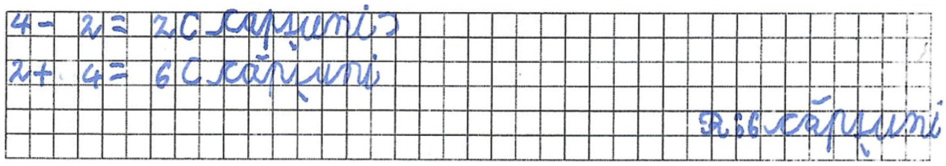
- **correct mathematical operations, correct problem solution, followed by incorrect explanations of the numeric results obtained (ISe)** (Figure No. 14.IV.). This error was identified in problems where the statement describes two sets of distinct elements, subordinate to a higher-level set that defines the category of those elements (e.g., problem 1, pretest – pears and strawberries belong to the set of fruits; problem 2, pretest – snowdrops and violets are flowers; problem 1, posttest – blue and yellow fish belong to the set of fish). This error cannot be recognised without written explanations of numerical results. The presence of incorrect explanations raises questions about the accuracy of problem understanding and the reasoning behind the solving strategy. RIe-type strategies were not identified among the problem 1 solving strategies in post-test (Table No. 13.IV.), as these strategies were developed after graphic representating the problem content by students, which highlights the clarifying potential of the student-created drawings.

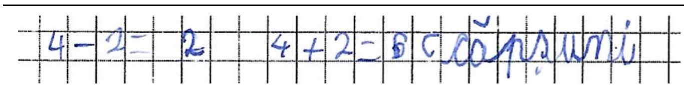
Figure No. 14.IV. Incorrect solving strategies (ISe) (incorrect explanations of the numeric results obtained)

Problem 0
(pretest)

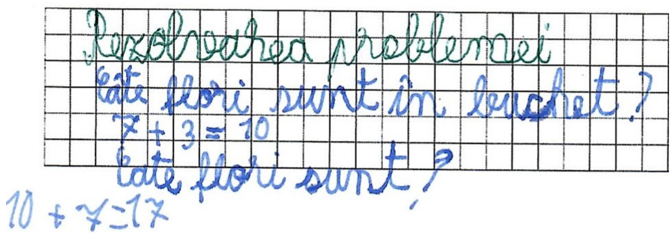


Problem 1
(pretest)



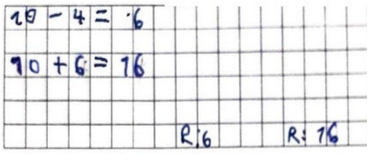
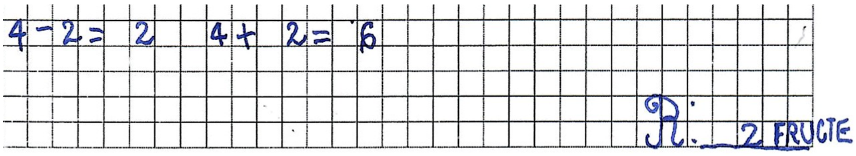


Problem 2
(pretest)



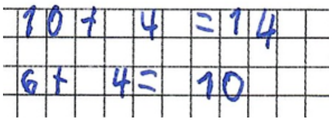
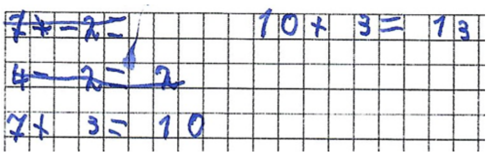
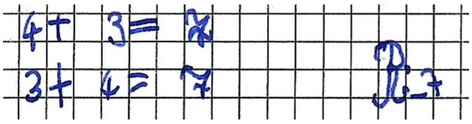
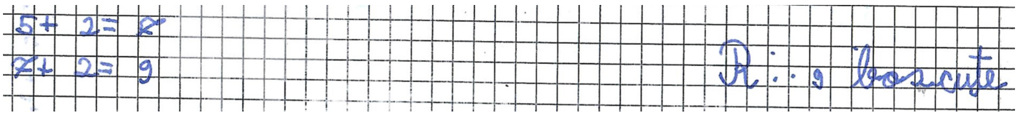
- **correct mathematical operations** followed by **incorrect problem solution (ISs)**. Although the numeric result was correct, several participants incorrectly explained it (Figure No. 15.IV.), highlighting their inaccurate understanding of the problem.

Figure No. 15.IV. Incorrect solving strategy (ISs) (correct mathematical operations followed by incorrect problem solution)

Problem 0 (pretest)	
Problem 1 (pretest)	

- **incorrect solving strategy:** the student writes and performs other mathematical operations or performs the operations in a different order from the logical order required to identify the unknown quantities (Figure No. 16.IV.).

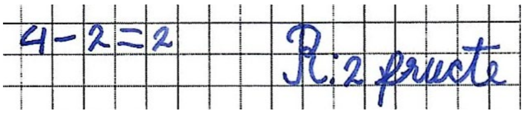
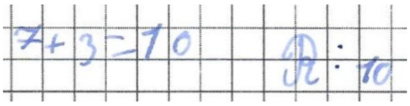
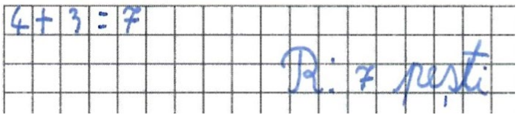
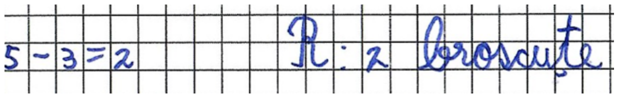
Figure No. 16.IV. Incorrect solving strategy (incorrect mathematical operations)

Problem 0 (pretest)	
Problem 2 (pretest)	
Problem 1 (posttest)	
Problem 2 (posttest)	

Several categories of **incorrect solving strategies** were identified (Figure No. 16.IV.), providing insights into the reasoning behind the subjects' choice of operations to solve the problem:

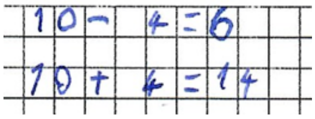
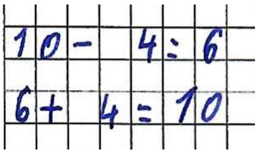
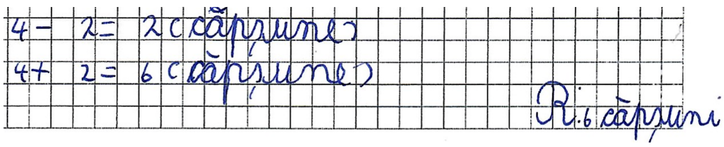
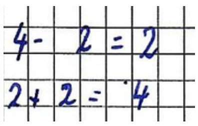
- **incorrect solving strategy (IS3) → performing the operation needed to determine the compared value.** Participants who employed this strategy performed only the operation suggested by the keywords in the problem statement (Figure No. 17.IV.).

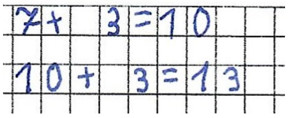
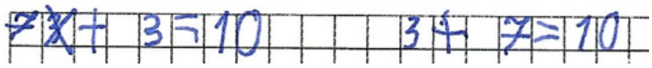
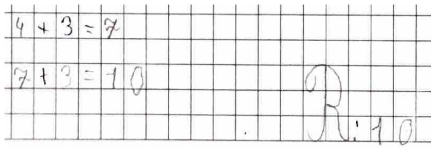
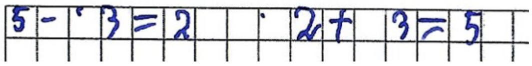
Figure No. 17.IV. Incorrect solving strategy (IS3) – incorrect solving strategy by performing the operation needed to determine the compared value

Problem 1 (pretest)	
Problem 2 (pretest)	
Problem 1 (posttest)	
Problem 2 (posttest)	

- **incorrect solving strategy (IS2) → (1) performing the operation suggested by the keywords in the problem statement**, followed by **(2) adding the numbers from the problem statement** (Figure No. 18.IV.).

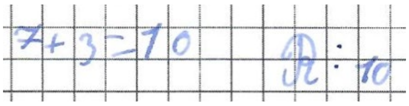
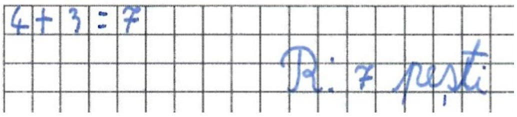
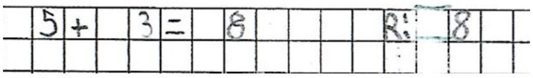
Figure No. 18.IV. Incorrect solving strategy (IS2) – incorrect solving strategy by (1) performing the operation suggested by keywords in the problem statement and by (2) adding the numbers from the problem statement

Problem 0 (pretest) (1) performing subtraction suggested by the keyword '*4 less*' (2) adding the numbers from the problem statement	
(1) performing subtraction suggested by the keyword '*4 less*' (2) adding the numeric result of the first operation to the difference between the compared value and the reference value (the second number from the problem statement)	
Problem 1* (pretest) (1) performing subtraction suggested by the keyword '*2 less*' (2) adding the numbers from the problem statement * In the particular case of this word problem, using faulty reasoning may accidentally lead to the correct numeric result (6).	
(1) performing subtraction suggested by the keyword '*2 less*' (2) adding the numeric result of the first operation to the difference between the	

compared value and the reference value (the second number from the problem statement)	
Problema 2 (pretest) (1) performing addition suggested by the keyword *'3 more'* (2) adding the numbers from the problem statement	
(1) performing addition suggested by the keyword *'3 more'* (2) adding the numeric result of the first operation to the difference between the compared value and the reference value (the second number from the problem statement)	
Problem 1 (posttest) (1) performing addition suggested by the keyword *'3 more'* (2) adding the numbers from the problem statement	
(1) performing addition suggested by the keyword *'3 more'* (2) adding the numeric result of the first operation to the difference between the compared value and the reference value (the second number from the problem statement)	
Problem 2 (posttest) (1) performing subtraction suggested by the keyword *'3 less'* (2) adding the numbers from the problem statement	
(1) performing subtraction suggested by the keyword *'3 less'* (2) adding the numeric result of the first operation to the difference between the compared value and the reference value (the second number from the problem statement)	

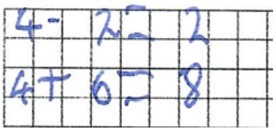
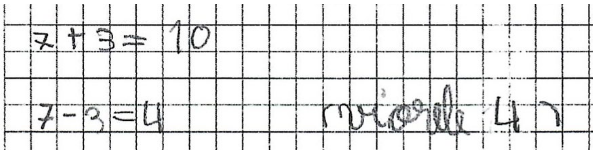
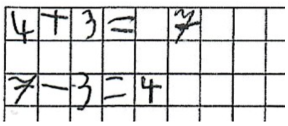
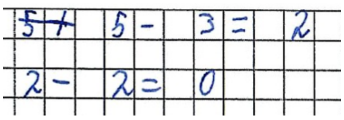
- **incorrect solving strategy (RI1) → adding the numbers from the problem statement.** In problems 0 and 1 in pretest and problem 2 in posttest, where the compared value is derived through subtraction, the subjects' intention to determine the total number of elements by adding the numbers in the problem statement is evident (Figure No. 19.IV.). In those problems where the compared value is calculated by addition, adding the numbers from the problem statement as a solving strategy can be attributed either to association with keywords or to the intention to determine the total number of elements in the two sets, both of which constitute incorrect reasoning leading to performing the same mathematical operation (Figure No. 19.IV.).

Figure No. 19.IV. Incorrect solving strategy (IS1) – incorrect solving strategy by adding the numbers from the problem statement

Problem 0 (pretest)	
Problem 1* (pretest) * The numeric result obtained from adding the numbers from the problem statement is the same as the numeric result of the correct problem solution.	
Problem 2 (pretest)	
Problem 1 (posttest)	
Problem 2 (posttest)	

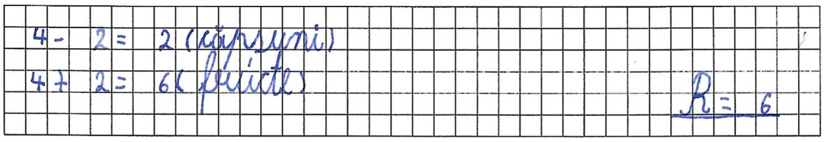
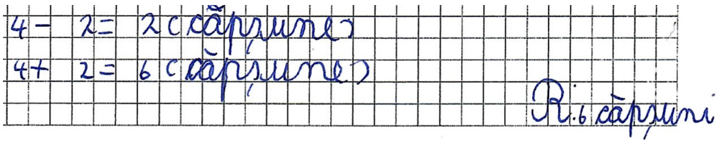
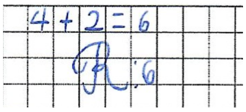
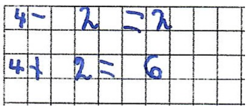
- **incorrect solving strategy (IS0)** → performing any other mathematical operations or the necessary mathematical operations to determine the problem solution in a different order than the logical one required to find the unknown values (Figure No. 20.IV.).

Figure No. 20.IV. Incorrect solving strategy (ISO) – performing any other mathematical operations or the required mathematical operations, but in an incorrect order

Problem 0 (pretest)	
Problem 1 (pretest)	
Problem 2 (pretest)	
Problem 1 (posttest)	
Problem 2 (posttest)	

The analysis of students' solving strategies in the pretest and posttest revealed that there were more correct solutions for problem 1 in the pretest compared to the other problems (Table No. 11.IV.). The high number of correct solutions for problem 1 in pretest is due to the fact that the difference between the compared value (the number of strawberries) and the reference value (the number of pears) is 2, which is the same as the number of elements in the unknown set (the strawberries), also 2. Because of these equalities, students can arrive at the correct answer of 6 (fruits) either by reasoning based on understanding the quantitative relationships between data or through incorrect reasoning using strategies such as ISe, IS2, or IS1, whose numeric result (6) matches the correct solution. The presence of written explanations of mathematical operations and problem solutions helps to distinguish correct reasoning from incorrect reasoning, despite the similar operations involved in all three types of solving strategies (Figure No. 21.IV.).

Figure No. 21.IV. Similarities in the students' solving strategies for problem 1 in the pretest

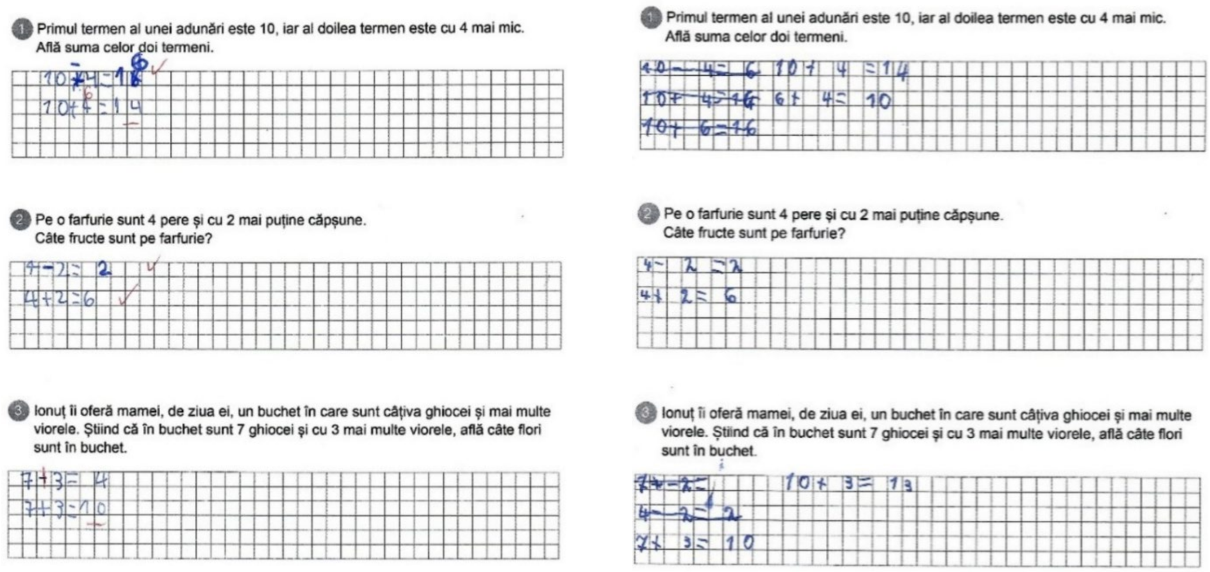
Problem 1 (pretest)	There are 4 pears and 2 less strawberries on a plate. How many fruits are there on the plate?
CS	
ISe/ IS2	
IS1	
CS / IS2	

In the absence of written explanations, the correct solving strategies were identified by referencing the solving strategies of other problems on the worksheet (problem 0 and problem 2), especially by referencing the solving strategy of problem 0, which uses a similar compare relationship to that of problem 1.

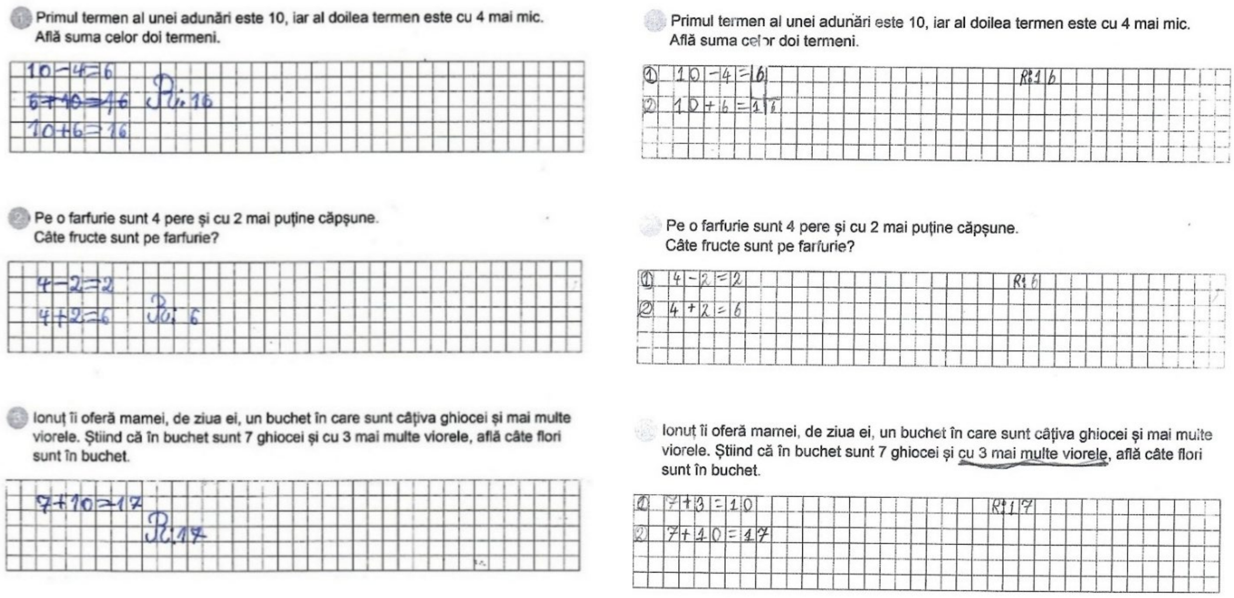
The absence of written explanations for the mathematical operations that underlie some of the correct solving strategies for problem 1 in the pretest makes it difficult to distinguish and assess the accuracy of the reasoning used during the solving process. Therefore, evaluating the correct solving strategies for problem 1 that lack written details was done by referring to the solving strategies of other problems on the worksheet (problem 0 and problem 2), especially by examining the strategy for problem 0, which describes a similar compare relationship to that of problem 1 (Figure No. 22.IV.).

Figure No. 22.IV. Similarities in the participants' problem-solving behaviour in pretest

Incorrecr solving strategies (IS) of the three word problems in pretest



Correct solving strategies (CS) of the three word problems in pretest



Similarly, for problems that require performing two additions to solve (problem 2 in pretest and problem 1 in posttest), it is not possible to distinguish between the reasoning that led to incorrect IS3 solutions (Figure No. 17.IV.) and the reasoning that led to incorrect IS1 solutions (Figure No. 19.IV), as both incorrect solving strategies resemble an addition using the two numbers from the problem statement.

The analysis of the incorrect solving strategies used by students in solving the problems in pretest and posttest evidenced that the RI3 (performing the operation suggested by the keywords in the problem statement) and RI0 (performing other incorrect mathematical operations) solving strategies had the highest frequency. The distribution of different types of incorrect solving strategies used by students, depending on their reading comprehension skills, showed that incorrect RI3 and RI solving strategies were the most prevalent, regardless of the reading comprehension skills (Table No. 13.IV.).

Following the classification and analysis of the students' solving strategies, a large number of incorrect problem solutions were identified (more than half of the solutions to each of the five problems). The most commonly used solving strategies by students with good, adequate, or inadequate reading comprehension skills were IS0, for which no specific reasoning pattern could be identified in selecting the mathematical operations. Additionally, the most frequent incorrect solving strategies employed by students with optimally developed reading comprehension skills were IS-type strategies, where, to determine the solution, the students only performed the operation suggested by the keywords in the problem statement.

Table No. 13.IV. Quantitative analysis of students' solving strategies in pretest and posttest

	Correct solving strategies (CS)	**Incorrect solving strategies (IS)**						**Total**
	► Correct solving strategy ► Correct problem solution ► Correct written explanations	► Correct solving strategy ► Correct problem solution ► Incorrect written explanation **(ISe)**	► Correct solving strategy ► Incorrect problem solution **(ISs)**	► Incorrect solving strategy → performs mathematical operation suggested by the keywords in the problem statement **(IS3)**	► Incorrect solving strategy → performs the mathematical operation suggested by the keywords in the problem statement, followed by adding the numbers in the problem statement **(IS2)**	► Incorrect solving strategy → adding the numbers from the problem statement **(IS1)**	► Incorrect solving strategies → performs any other incorrect mathematical operations/ incorrect problem solution **(IS0)**	
Problem 0 *(pretest)*	*111 students* ***43,4%***	*5 students* ***2%***	*1 student* ***0.4%***	*21 students* ***8.2%***	*16 students* ***6.3%***	*1 student* ***0.4%***	*101 students* ***39.5%***	*256 students* ***100%***
Problem 1 (pretest)	128 students **50%**	6 students **2.3%**	4 students **1.6%**	53 students **20.7%**	8 students **3.1%**	14 students **5.5%**	43 students **16.8%**	256 students **100%**
Problem 2 (pretest)	84 students **32.8%**	3 students **1.2%**	–	81 students **31.6%**	17 students **6.6%**	1 students **0.4%**	70 students **27.3%**	256 students **100%**
Problem 1 (posttest)	104 students **40.6%**	–	2 students **0.8%**	73 students **28.5%**	22 students **8.6%**	–	55 students **21.5%**	256 students **100%**
Problem 2 (posttest)	110 students **43%**	–	–	55 students **21.5%**	24 students **9.4%**	23 students **9%**	44 students **17.2%**	256 students **100%**

IV.2.4. Quantitative analysis of students-created graphic representations of the problem content in posttest

Depending on the **information provided in the problem statement**, the student-created graphic representations of the problem content were classified into *graphic representations of solving information (correct or incorrect)* (Figure 24.IV.a.) and *illustrative representations*, which include elements from the problem content that are irrelevant for the solving strategy, depicted by situational information or related to the specific context they present (Figure No. 24.IV.b.).

Figure No. 23.IV. Comparative presentation of the distribution of graphic representations of solving information and illustrative representations created by students for problems 1 and 2 in the posttest

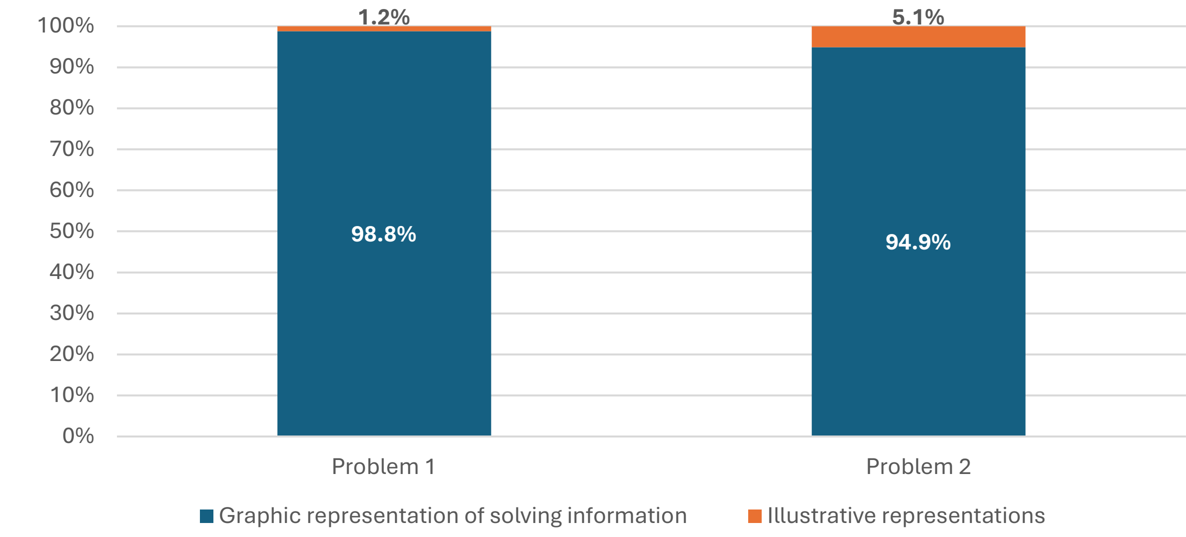
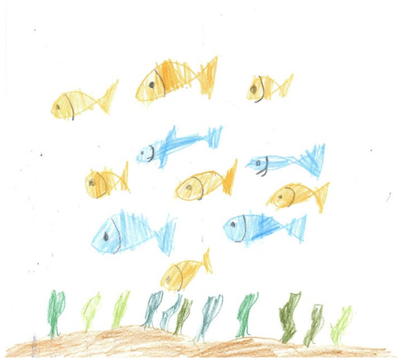
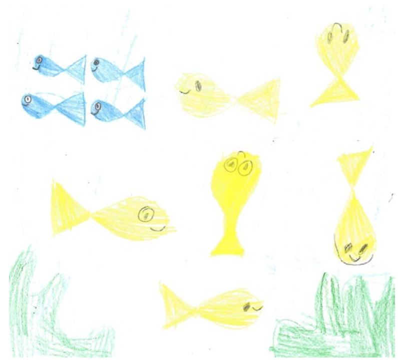


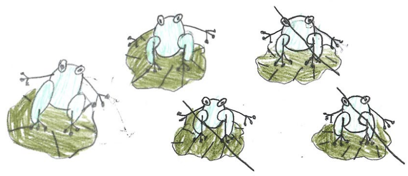
Figure No. 24.IV. Student-created graphic representations of solving information and illustrative representations of the problem content in posttest

Problem 1 (posttest)	There are **4 blue fish** and **3 more yellow fish** in an aquarium. How many fish are there in the aquarium?
a. graphic representations of solving information	
	

b. illustrative representations

Problem 2 (posttest)	A large family of frogs lives at the edge of a lake. Several frogs sit on one lily leaf, and fewer frogs are on the other lily leaf than on the first. If there are **5 frogs** on the first lily leaf and **3 less** on the other, find out how many frogs are on the lily leaves.

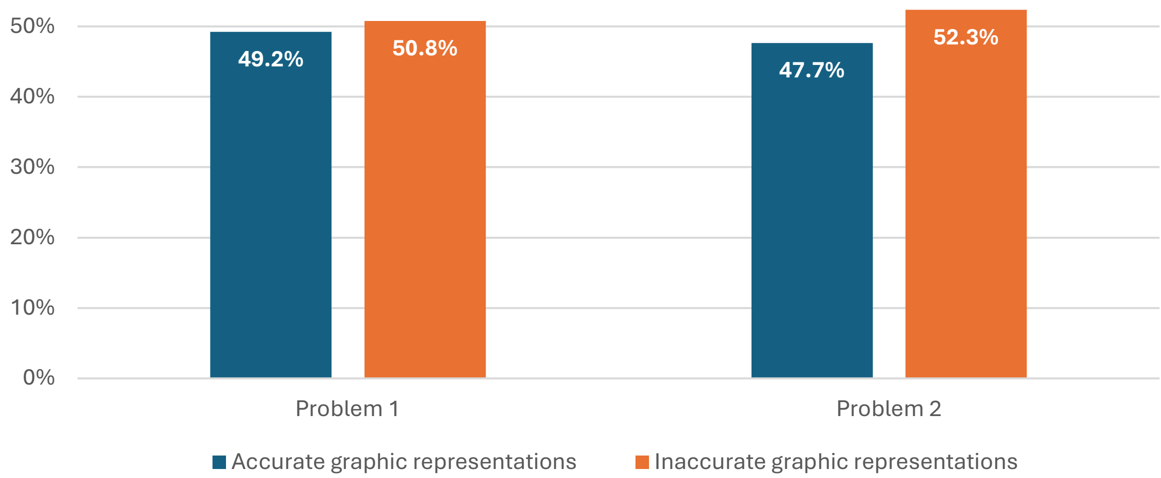
a. graphic representations of solving information



b. illustrative representations

Depending on the **accuracy of representing the solving information**, the student-created graphic representations of the problem content in posttest were classified, as described by Boonen et al. (2014), into *accurate graphic representations* and *inaccurate graphic representations* (Figure No. 25.IV.).

Figure No. 25.IV. Comparative analysis of the distribution of accurate and inaccurate student-created graphic representations of problems 1 and 2 in posttest



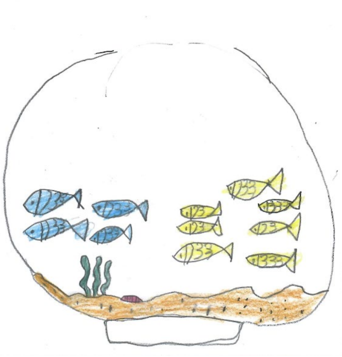
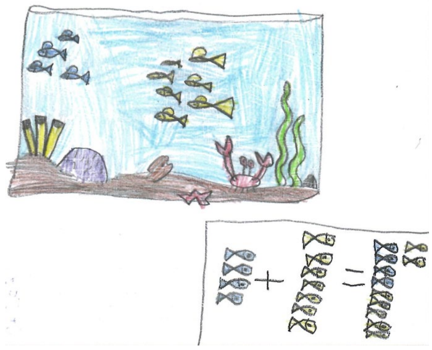
Due to the difficulty in distinguishing between illustrative representations and some incorrect graphic representations, as presented in subchapter IV.2.5., illustrative representations were considered as incorrect graphic representations during data analysis.

IV.2.5. Visual analysis of student-created graphic representations of the problem content

1. Correct graphic representations depict two sets of elements: the reference set (the first numerical value in the statement) and the compared set, whose number of elements is obtained by adding (for problem 1) or subtracting (for problem 2) the difference (the second value in the statement) from the number of elements in the reference set (Figure No. 26.IV.a.). A particular case of correct graphic representations is represented by those that display the correct number of elements using digits, without a quantitative visualisation of the elements within those sets (Figure No. 26.IV.b.).

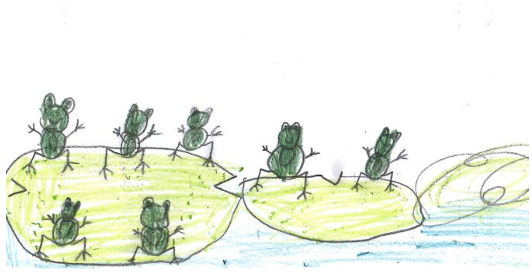
Figure No. 26.IV. Student-created correct graphic representations of problem content in posttest

Problem 1 (posttest)	There are **4 blue fish** and **3 more yellow fish** in an aquarium. How many fish are there in the aquarium?
a. correct representations of the elements of two sets	

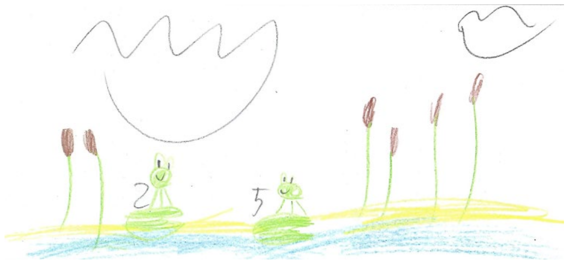



Problem 2 (posttest)	A large family of frogs lives at the edge of a lake. Several frogs sit on one lily leaf, and fewer frogs are on the other lily leaf than on the first. If there are **5 frogs** on the first lily leaf and **3 less** on the other, find out how many frogs are on the lily leaves.

a. correct representations of the elements of two sets

b. indicating the number of elements of two sets using digits without a quantitative visualisation of the number of elements

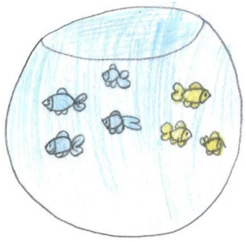


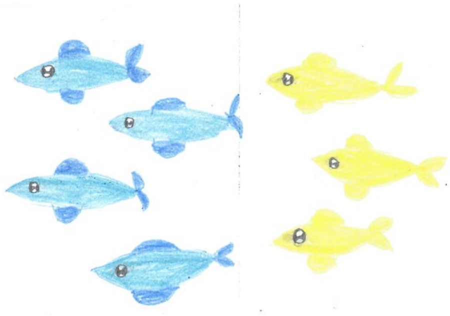

2. The analysis of **incorrect graphic representations** revealed two categories of drawings. One category of incorrect graphic representations illustrates two sets with as many elements as indicated by each number in the problem statement (IGRno) (Figure No. 27.IV.a) and incorrect graphic representations that illustrate the two sets as having any other incorrect number of elements (IGR0) (Figure No. 27.IV.b.).

Figure No. 27.IV. Student-created incorrect graphic representation of the problem content in posttest

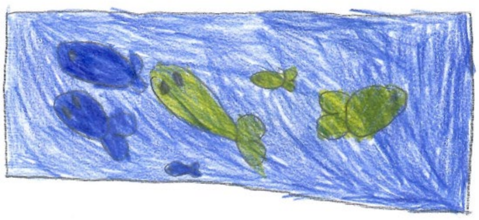
Problem 1 (posttest)	There are **4 blue fish** and **3 more yellow fish** in an aquarium. How many fish are there in the aquarium?

a. representing the two sets with as many elements as indicated by each number in the problem statement (IGRno)



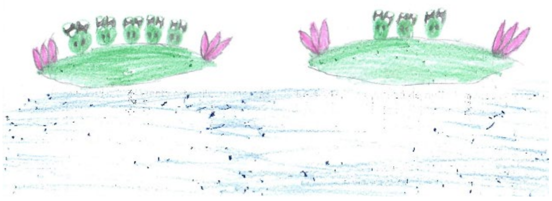


b. representing the two sets as having any other incorrect number of elements (IGR0)



Problem 2 (posttest) A large family of frogs lives at the edge of a lake. Several frogs sit on one lily leaf, and fewer frogs are on the other lily leaf than on the first. If there are **5 frogs** on the first lily leaf and **3 less** on the other, find out how many frogs are on the lily leaves.

a. representing the two sets with as many elements as indicated by each number in the problem statement (IGRno)



b. representing the two sets as having any other incorrect number of elements (IGR0)

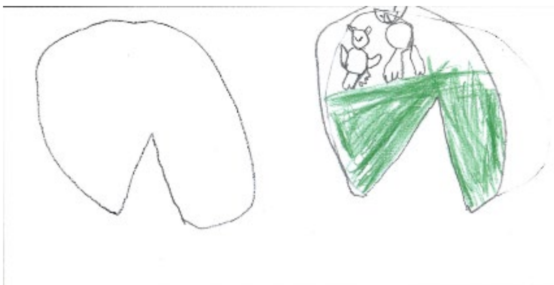


Table No. 20.IV. Qualitative analysis of student-created graphic representations of problem content in posttest

	Accurate graphic representations		**Inaccurate graphic representations**		**Total**
	► Representing the correct number of elements in both sets	► Indicating the correct number of elements in both sets using digits, without providing a quantitative representation of elements.	► Representing the two sets with as many elements as indicated by each number in the problem statement **(IGRno)**	► Other incorrect representations of the number of elements in both sets **(IGR0)**	
Problem 1 (posttest)	126 students **49.2%**	–	88 students **34.4%**	45 students **17.6%**	256 students **100%**
Problem 2 (posttest)	116 students **45.3%**	6 students **2.3%**	83 students **32.4%**	51 students **20%**	256 students **100%**

Based on the accuracy of graphic representations of solving information in the problem, during the data analysis process, the illustrative representations were considered as IGR0 incorrect graphic representations (subchapter IV.2.4.).

IV.2.6. Interpreting research data to answer research questions

IV.2.6.1. Research question 1: To what extent does the correctness of compare-combine word problem solutions correlate with first-grade students' reading comprehension skills?

To verify whether there is a positive correlation between the correctness of compare-combine word problem solutions and first-grade students' reading comprehension skills, the distribution of correct problem solutions in pretest among students with varying reading comprehension skills was analysed.

Figure No. 28.IV. The distribution of correct problem solutions in pretest among students with *optimal reading comprehension skills (9 or 10 points on the Likert scale)*

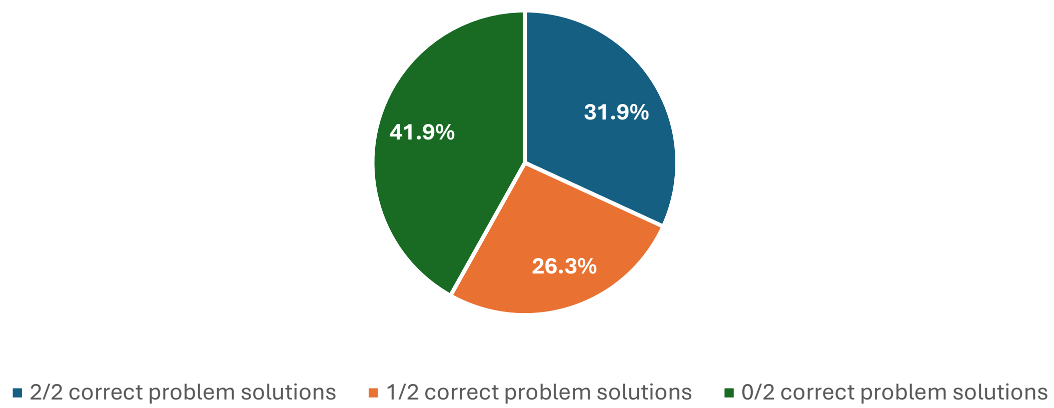
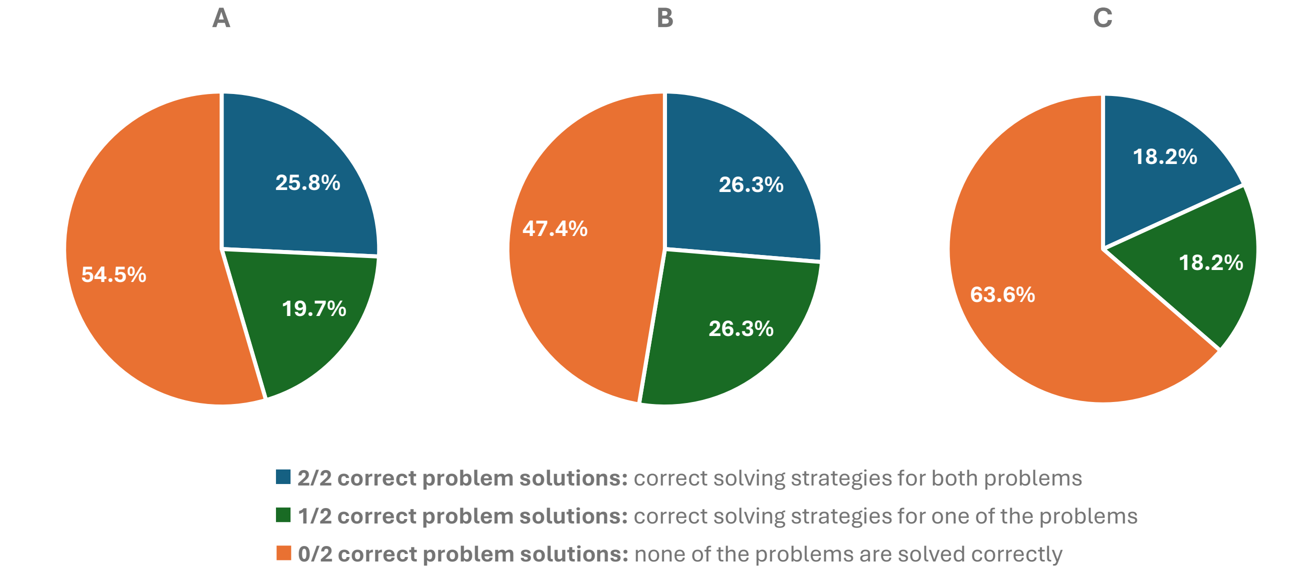


Figure No. 29.IV. The distribution of correct problem solutions in pretest among students with *(A) good*, *(B) sufficient* and *(C) insufficient reading comprehension skills*



Compared to students whose reading comprehension skills were assessed as being optimally developed for their age group, students with good, sufficient, and insufficient reading comprehension skills provided fewer correct problem solutions (Figure No. 29.IV.).

Hypothesis 1 was tested using biserial correlation. Correlation coefficients were calculated for the relationship between students' reading comprehension skills, as assessed by primary school teachers on a 1 to 10 points Likert scale, and the correctness of the solutions to each of problems 1 and 2 in the pretest, assessed as correct or incorrect (Table No. 22.IV.).

Table No. 22.IV. Correlation coefficients (r_{pb}) for the relationship between *students' reading comprehension skills* and *the correctness of the solutions to each of problems 1 and 2 in pretest*

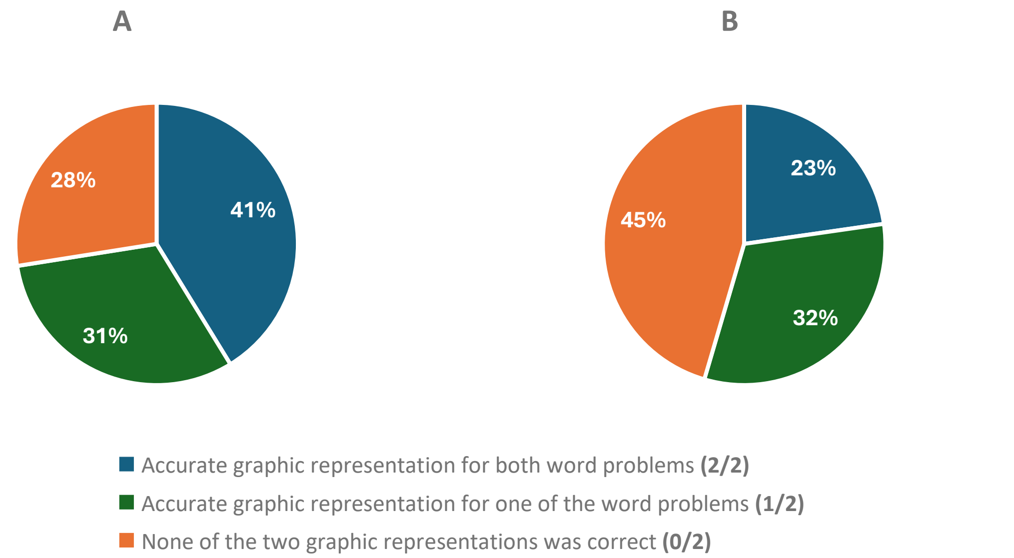
	r_{pb}	df	t	*p*
Student's reading comprehension skills – correctness of ***problem's 1*** *solutions in pretest*	0.13	254	2.04	.043
Student's reading comprehension skills – correctness of ***problem's 2*** *solutions in pretest*	0.12	254	1,97	.05

The correlation coefficients calculated for the relationship between students' reading comprehension skills and the correctness of solutions to each of problems 1 (rpb = 0.13, n = 256, p = 0.043) and 2 (rpb = 0.12, n = 256, p = 0.5) in pretest indicated a positive, low-intensity, statistically significant correlation between the correctness of first-grade students' compare-combine word problem solutions and their reading comprehension skills, which confirms hypothesis 1. These results are consistent with those obtained by Bjork & Bowyer-Crane (2013), Boonen et al. (2016), Can (2020), Pongsakdi et al. (2020), Timario (2020), who highlighted the role of reading comprehension as a predictor of students' solving performance.

IV.2.6.2. Research question 2: To what extent do reading comprehension skills correlate with the accuracy of first-grade student-created graphic representations of compare-combine word problems?

To verify whether there is a positive correlation between students' reading comprehension skills and the accuracy of the student-created graphic representations of compare-combine word problems (hypothesis 2), the distribution of student-created accurate graphic representations of the problem content among students with varying reading comprehension skills, was analysed (Figure No. 30. IV.).

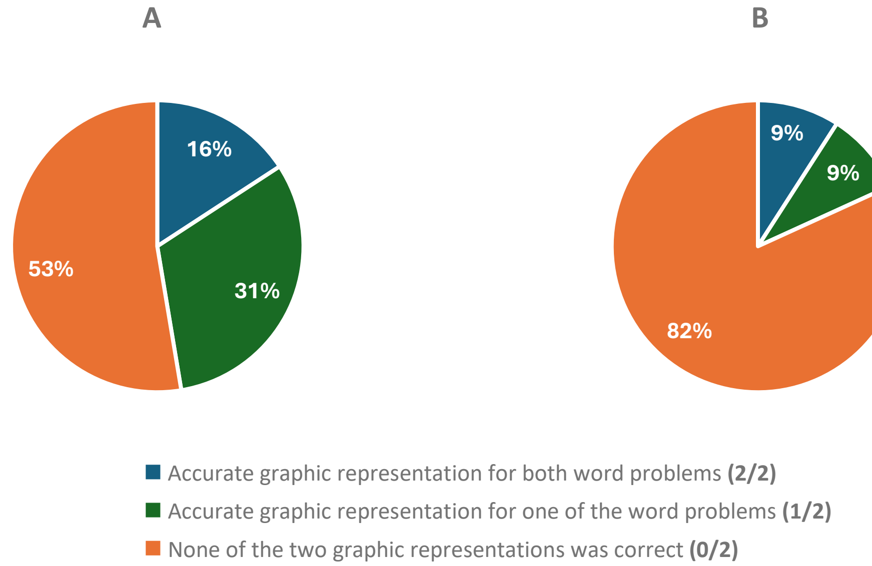
Figure No. 30.IV. The distribution of accurate graphic representation of word problems among students with (A) *optimal reading comprehension skills* and (B) *good reading comprehension skills*



The distribution of accurate graphic representations (Figure No. 30.IV.) among students with optimal and good reading comprehension skills shows that most students with optimal developed reading comprehension skills produced accurate graphic representations for both problems in posttest (Figure No. 30.IV.A.), compared to a much smaller percentage of students with good reading comprehension skills who did the same (Figure No. 30.IV.B.). Incorrect graphic representations are more common among students with good reading comprehension skills, with 45% of them inaccurately representing the solving information, highlighting their limited understanding of the relationships between data.

The analysis of how accurate graphic representations are distributed among participants with sufficient (Figure No. 31.IV.A) or insufficient (Figure No. 31.IV.B) reading comprehension skills shows a decline in the proportion of accurate student-created graphic representations for either one or both problems in the posttest. Therefore, the accuracy of graphic representations decreases as students' reading comprehension skills decline.

Figure No. 31.IV. The distribution of accurate graphic representation of word problems among students with (A) *sufficient* and (B) *insufficient reading comprehension skills*



The relationship between reading comprehension skills and the accuracy of student-created graphic representations of word problems (hypothesis 2) was examined using biserial correlation. The correlation coefficients were calculated for the relationship between students' reading comprehension skills, as assessed by primary school teachers on a 1 to 10 point Likert scale, and the accuracy of student-created graphic representations of word problems, rated as accurate or inaccurate (Table No. 24. IV.).

Table No. 24.IV. Correlation coefficients (r_{pb}) for the relationship between *students' reading comprehension skills* and *the accuracy of student-created graphic representations of* problems 1 and 2 in posttest

	r_{pb}	df	t	*p*
Student's reading comprehension skills – accuracy of student-created graphic representation of ***problem 1*** *in posttest*	0.27	254	4.5	< ,001
Student's reading comprehension skills – accuracy of student-created graphic representation of ***problem 2*** *in posttest*	0.29	254	4.82	< ,001

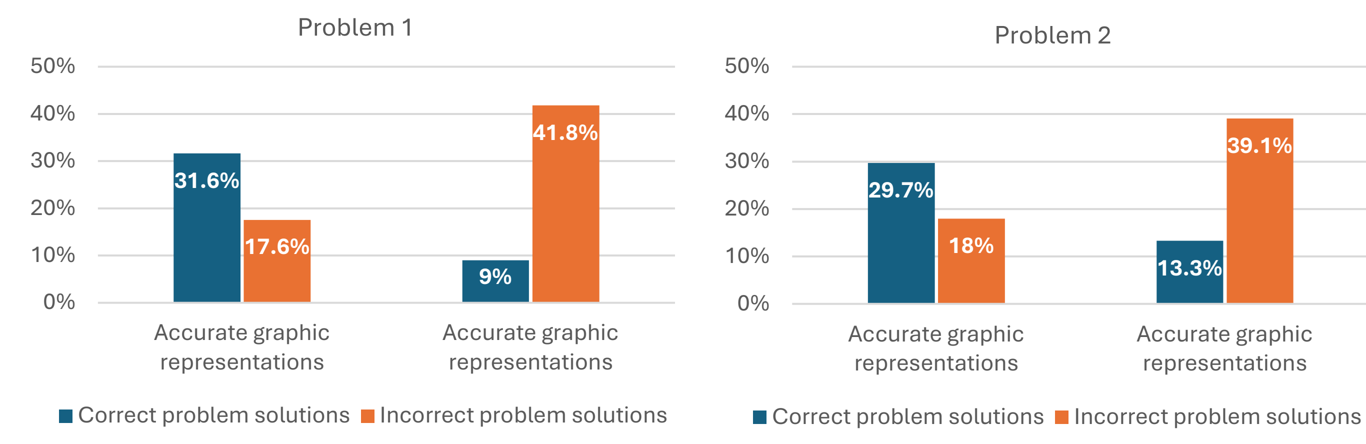
The correlation coefficients calculated for the relationship between *reading comprehension skills* and the *accuracy of student-created graphic representations of each of problems 1* (rpb = 0.27, n = 256, p < 0.001) and *2* (rpb = 0.2 9, n = 256, p < 0.001) in the posttest indicated a *positive, medium-strength, statistically significant correlation* between the two variables, **confirming hypothesis 2**; the high reading comprehension skills also correlating with the large number of correct problem solutions in pretest (subchapter IV.2.6.2.).

IV.2.6.3. Research question 3: What is the relationship between the main types of first-grade student-created graphic representations of compare-combine word problems and the correctness of problem solutions in posttest?

Hypothesis 3 presumes a positive correlation between the accuracy of first-grade students' created graphic representations of the compare-combine word problem content in posttest and the correctness of problem solutions.

The relationships between the accuracy of student-created graphic representations of the problem content in posttest and the correctness of problem solutions were analysed using the Chi^2 test (Figure No. 32. IV.).

Figure No. 32.IV. Distribution of correct problem solutions in posttest among the accuracy of student-created graphic representations in posttest



The distribution of correct problem solutions across each category of student-created graphic representations shows, for both posttest problems, a higher frequency of correct problem solutions among accurate graphic representations and a higher frequency of incorrect problem solutions among inaccurate graphic representations (Figure No. 32.IV.).

The results obtained from the data analysis of each problem (Table No. 26.IV.) indicate Chi^2 test values higher than the critical value: $\chi^2(1)$ (57.59) > $\chi^2_{critical}$ (3.84) for problem 1 and $\chi^2(1)$ (35.52) > $\chi^2_{critical}$ (3.84) for problem 2, indicating the correlation between the two variables.

Table No. 26.IV. Chi^2 test coefficients for the relationship between the accuracy of graphical representations and the correctness of solutions to problems 1 and 2 in posttest

	Chi^2	**χ^2 critic**	**df**	**p**
Accuracy of graphic representations – correctness of problem 1 solutions (posttest)	57.59	3.84	1	< .001
Accuracy of graphic representations – correctness of problem 2 solutions (posttest)	35.52	3.84	1	< .001

The Chi^2 test values (Table No. 26.IV.) associated with the Pearson contingency coefficient values and the V. Cramér coefficient values (= 0.47 for the relationship between the variables in problem 1 and 0.37 for the variables in problem 2) highlight the *existence of a statistically significant association of medium strength, with p < 0.001, between the accuracy of the student-created graphic representations of the problem content and the correctness of the solutions provided by students for each of the two problems in posttest,* thus **confirming hypothesis 3**.

Similar results were found after conducting the Chi^2 test on the association between the frequencies of correct problem solutions and the frequency of accurate student-created graphic representations across the entire sample in posttest. The Chi^2 test value for the two variables was higher than the critical value, with $\chi^2(4)$ (70.97) > $\chi^2_{critical}$ (9.49), and the Pearson contingency coefficient, C = 0.58, along with V. Cramér = 0.37, indicates a *statistically significant, medium-strength association, with p < 0.001, between the frequency of correct graphical representations and correct problem solutions*, providing further **support for hypothesis 3.**

IV.2.6.4. Research question 4: To what extent do the accurate student-created graphic representations of compare-combine word problems increase the success rate of solving these problems?

Given the medium-strength correlation between the accurate graphic representation of problem content and the correctness of problem solutions, it was hypothesised that students accurately creating graphic representations of compare-combine word problems would increase their success rate in solving these problems (hypothesis 4).

The effect of accurate student-created graphic representations of the problem content in posttest on the success rate of solving those problems was examined using a logistic regression analysis for each of the two problems in posttest.

The logistic regression model for problem 1 was statistically significant, $\chi^2(1)$ = 60.25, $p < 0.001$, explaining 28.3% (R^2 Nagelkerke) of the variation in correct problem solutions, with an accuracy rate of 73.4%. The odds ratio indicated that, *when the graphic representation of problem 1 was accurate, students were 8.37 times more likely to solve the problem correctly than not*.

The logistic regression model for problem 2 was also statistically significant, $\chi^2(1)$ = 36.34, $p < 0.001$, explaining 17.7% (R^2 Nagelkerke) of the variation in correct problem solutions, with a correct classification rate of 68.75% of the observations. Due to the *accurate graphic representation of the content in problem 2, the likelihood of students solving the problem correctly is 4.86 times higher than the likelihood of not solving it correctly.*

The analysis of the odds ratio for correct solutions to the two problems in posttest, given the accurate first-grade student-created graphic representation of the problem content, indicated that *the accurate graphic representation of problems 1 and 2 in posttest increased the chances* *of correctly solving procedure by 8.37 and 4.86 times, respectively,* **confirming hypothesis 4**. Based on the data presented above, we can conclude that the accuracy of the graphic representation of a compare-combine problem's content is a predictor of its solving success.

IV.2.6.5. Research question 5: Is there an improvement in first-grade students' solving performance in posttest compared to their solving performance in pretest?

The increased likelihood of correctly solving compare-combine word problems when students create an accurate graphic representation of the problem's content (subchapter IV.2.6.4.) supports the assumption that *first-grade students' solving performance in posttest will improve compared to the pretest (hypothesis 5).* The comparative analysis of the correctness of the problem solutions in pretest and posttest was based on the similarities in the solving strategies for each of the four problems in the content sample (subchapter IV.2.3.). Therefore, a qualitative pretest-posttest comparison was carried out for solutions of problems where solving strategies involve a subtraction and an addition operation (problem 1 in the pretest and problem 2 in the posttest) and for problems requiring two additions (problem 2 in the pretest and problem 1 in the posttest). Differences in students' solving performances between pretest and posttest were analysed using McNemar's Chi^2 test for paired samples.

The analysis of the difference between students' performance in solving problems involving a subtraction and an addition operation indicated a calculated value of McNemar's Chi^2 test lower than the critical value $\chi^2(1)$ (3.44) < $\chi^2_{critical}$ (3.84) and p = 0.064. The p-value > 0.05 indicates that *there is no statistically significant difference between the solving performance of first-grade students who improved and those who regressed in posttest*.

The analysis of students' solving performance on problems involving two addition operations indicated that the calculated Chi^2 value for the McNemar test was greater than the critical value $\chi^2(1)$ (5.01 > 3.84) and p = 0.025. The p value < 0.05 highlights *a statistically significant difference, indicating an improvement in students' solving performance in posttest compared to solving performances of the same students in pretest* (Table No. 47.IV.).

The differences between the solving performance recorded by students in pretest and posttest **partially confirm hypothesis 5**, with the number of students who improved their solving performance in posttest being higher than the number of students who recorded a decline in solving performance in posttest, only for solving strategies of problems involving two addition operations, the difference between the solving performances recorded by students in problems involving a subtraction and an addition is not statistically significant.

IV.2.6.6. Research question 6: To what extent does the explanation information in compare-combine word problems statement lead to an increased number of accurate student-created graphic representations of word problems?

Stern and Lehrndorfer (1992) emphasised the importance of explanation information in understanding the quantitative comparison between sets of elements described in the problem statement. Therefore, it was examined whether *the presence of explanation* *information in the compare-combine problem statements would increase the number of accurate graphic representations of the problem content created by first-grade students* (hypothesis 6). To test this hypothesis, the student-created graphic representations of both problems in posttest were compared: problem 1 contains only solving information, while problem 2 incorporates some explanatory information in its statement.

The difference in accuracy between student-created graphic representations of the two problems (assessed as accurate or inaccurate) was analysed using McNemar's Chi2 test for paired samples. The analysis of the difference in the accuracy of the graphic representations of the two problems in posttest indicates a Chi2 value, calculated for the McNemar test, much lower than the critical value $\chi^2(1)$ (0.12) < $\chi^2_{critical}$ (3.84) and p = 0.734. The p-value, being greater than 0.05, indicates that the difference in accuracy between the graphic representations of problems 1 and 2 in posttest is not statistically significant. The results obtained **reject hypothesis 6**, suggesting that *the presence of explanation information in problem 2 does not lead to an increase in the number of accurate student-created graphic representations of the problem content.* To properly test this hypothesis, a different research design approach is needed, along with a rewording of the problem statements, so that the effect of explanation information on the accuracy of the graphic representation of the compared sets can be effectively highlighted.

IV.3. Conclusions

IV.3.1. Conclusions regarding personal contributions – theoretical and practic-applicative

In mathematical activities, the development of problem-solving skills aims to transfer abstract mathematical knowledge to the concrete operational level, which is specific to different contexts in everyday life.

Romanian literature and specialised methodologies offer various descriptions and classifications of mathematical problems, based on the number and type of operations needed to solve them or the specific reasoning involved in devising the solution strategy. They focus mainly on outlining the steps teachers should follow to organise and conduct lessons for solving mathematical problems. Despite Romanian students frequently recording poor results in international assessments (PISA, TIMSS), the specialised literature in our country does not adequately address the causes behind these performances. Therefore, it is essential to conduct a descriptive analysis of the primary errors made when solving various types of problems and to identify the underlying causes.

The scientific literature on mathematical problem solving by students provides detailed analyses of errors in reasoning used to solve problems. This paper draws on some of these research findings to describe and explain the mistakes that primary school students make when solving math problems involving a single addition or subtraction operation, as the existing literature does not offer sufficient data on these difficulties. The results from analysing the current research data show how first-grade students understand and represent a set of elements in comparison with another reference set, as well as the main errors in reasoning when solving these problems.

As a result of analysing the latest research, gaps in knowledge were identified regarding the role of first-grade student-created drawings of simple word problems in developing a solving strategy. This paper proposes a novel approach to the relationship between the two variables. We believe that our research findings significantly contribute to the enrichment of scientific knowledge and educational practice, emphasising the importance of first-grade student-created drawings in assessing their understanding of problems and identifying errors that occur at different stages of solving procedure.

IV.3.2. Research conclusions

Qualitative and quantitative analyses of the solving strategies and first-grade student-created graphic representations of compare-combine word problems allowed us to conclude the relationships between the research variables.

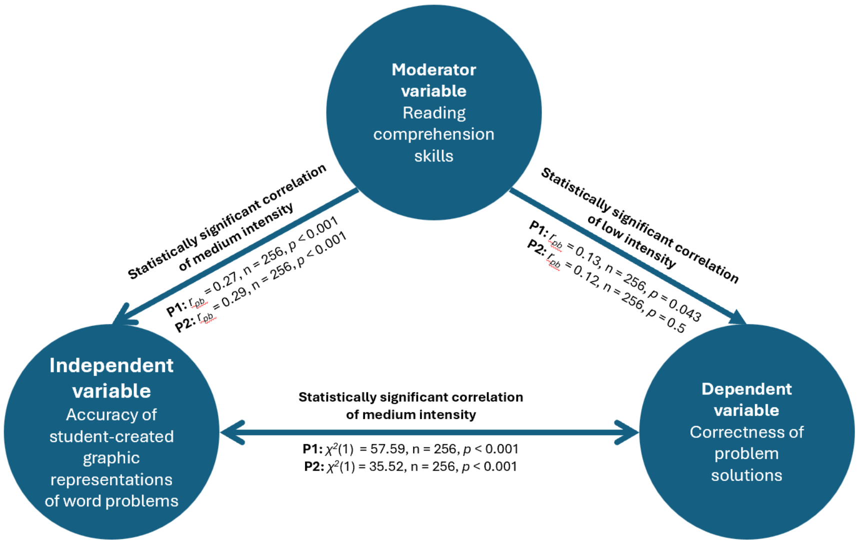
Question I. What are the main types of first-grade student-created graphic representations of compare-combine word problems?

The analysis of first-grade student-created graphic representations of compare-combine word problems outlined that graphic representations of solving information are the main category of these representations. Less than half of the graphic representations of solving information for both problems in posttest were correct (subchapter IV.2.4., Table No. 19.IV. and Figure No. 25.IV.). These findings **reject hypothesis I**, which assumed that *most first-grade students would produce accurate graphic representations of solving information,* as evidenced by the fact that less than half of the participants created accurate graphic representations. For both posttest problems, the number of inaccurate graphic representations slightly exceeds 50% of the total graphic representations of solving information (subchapter IV.2.4., Figure No. 25.IV.).

Question II. To what extent will the first-grade students' created graphic representations of compare-combine word problems positively influence problem-solving strategies and solutions?

The positive correlations established between the three variables of the research: the correctness of compare-combine problem solutions determined by first-grade students, their reading comprehension abilities, and the students' created graphic representations of compare-combine word problems, highlights the role of graphic representations of solving information as indicators of understanding the problem content, more precisely, as indicators of understanding the relationships between known and unknown data in problem statement.

Figure No. 33.IV. The correlations existing among the three research variables



At the same time, the increased likelihood of success in solving problems when the problem content is accurately represented makes the accuracy of graphic representations a predictor of successful problem solving (subchapter IV.2.6.5.), a finding also confirmed by Boonen et al. (2014). A comparative analysis of the solving performance recorded by students in pretest and posttest indicated a higher number of correct solutions in posttest for problems whose solutions involved two addition operations, with the student-created graphic representations of the problem content clarifying the solving process.

The analysis of students' problem-solving performance based on their reading comprehension skills showed that performance improved significantly among students with optimal developed reading comprehension, compared to those with good, sufficient, or insufficient reading comprehension skills.

Further analysis of the role of explanation information in the word problems that made up the content sample showed *that the presence of explanatory information does not enhance the quality of students' created graphic representations and, implicitly, their understanding of the problem content*.

Based on the arguments presented above regarding the relationships between problem comprehension indicators, we conclude *that student-created accurate graphic representation of compare-combine problem content has the potential to positively influence the development of solving strategies and the determination of correct problem solutions in the case of problems whose unknown (the compared value) is determined by addition*, **partially confirming hypothesis II of the research**.

IV.3.3. General conclusions

Investigating the implications of first-grade student-created graphic representations of compare-combine word problems on understanding the problem content and building the solving strategy highlighted the role of student-created graphic representations as an indicator of problem understanding. Considering the accurate graphic representation of the problem, the predictor of solving success (subchapter IV.2.6.4.), inaccurate graphic representations detail the errors that occur in understanding the relationships between the data in the problem statement, providing a clear picture of what students "do not understand" about the problem.

The use of the Talking Drawings strategy to assess understanding of texts that form mathematical problem statements revealed a similar relationship between the quality of student-created drawings for solving information and their understanding of the statement, similar to the relationship highlighted by Cappello & Walker (2021), Mc. Connell (1993), and Paquette et al. (2007) between the informative texts read and the content of children-created graphic representations based on those texts.

In mathematical activities, the use of visual language specific to graphic representations helps identify gaps in understanding a problem's content. Moreover, enriching maths classes with teaching sequences that incorporate children's drawings offers a comprehensive view of teaching educational content and an opportunity to tap into students' imagination and creativity in a subject often seen as dull, too abstract, and quite challenging.

IV.3.4. Research limits

The obstacles and difficulties that could not be anticipated during the pilot research or the development of this investigative approach's project were later identified, particularly in data collection and analysis, and manifested in the research's limitations.

1. Collection of valid data. The initial sample consisted of 320 first-grade students. Following the analysis of the collected data, 20% of the participants were eliminated: those who were absent from at least one testing stage, those who did not fully complete the work tasks, and participants from classes with an unreasonably high number of correct problem solutions to all problems or where similar solving strategies were identified in terms of structure and organisation, indicating a possible failure to follow test administration procedures.

2. Overestimation of students' reading comprehension skills. The score distribution used to assess students' reading comprehension skills revealed a primary school teacher's tendency to overestimate their abilities. This can be attributed either to subjectivity or the lack of standardised evaluation criteria clearly outlined in the curriculum documents. The large number of participants and the limited time available within the specific deadlines of the doctoral programme made it impossible to test each participant individually using standardised assessments.

3. The numerical values of problem 1 statement in pretest. In the particular case of problem 1 in pretest, the use of incorrect reasoning leads to the accidental obtaining of the correct numeric result, an aspect that was not anticipated in the stage of developing the content sample.

4. Insufficient written explanations of the problem solutions. Given the limitations caused by the underdeveloped writing skills of first-grade students and the variations in teachers' instructional styles, students were not explicitly asked to explain their results from calculations. This made it difficult to assess some of the solving strategies they used.

IV.3.5. Future research perspectives

Exploring research limitations facilitated identifying several future research directions:

1. The differences between students' reading skills and their understanding of the problem statement, expressed in the correctness of problem solutions and the accuracy of graphic representations they created of the problem content, require detailed investigations into differences between students' reading comprehension skills as assessed by primary school teachers and as evaluated by standardised tests. Additionally, it involves examining the causes behind the tendency of primary education teachers to overestimate the reading comprehension levels of first-grade students.

2. Incorrect ISe-type solving strategies were identified only in problems where the statement describes two distinct sets of elements, subordinate to a higher-level set that names the category of those elements. It is important to explore how the organisation of elements in the sets, as described in the problem statements, affects the correctness of the solving strategies used by first-grade students.

3. The difference between the number of correct problem solutions determined by students to problems whose solving strategy involves performing a subtraction and an addition and the number of correct solutions determined to problems whose solving strategy involves performing two addition operations, makes it necessary to investigate the causes that determine this solving behavior of first grade students.

The data obtained from investigating how first-grade students create graphic representations of compare-combine word problems, and how these representations affect their understanding of the problems and the development of their solving strategies, serve as a starting point for exploring how young schoolchildren understand different types of mathematical word problems.

Glossary

Ability to visualise (referring to word problems): ability to represent and reflect on images through visual means, what is not explicitly presented in mathematical problems, by making inferences and deductions, capitalising on specific relationships between data.

Accurate/ inaccurate graphic representation (of solving information) (cf. Boonen et al., 2014): specific type of graphic representation created by a problem solver, in the form of a drawing, illustration, scheme, or diagram, which organises the solving information into a coherent structure, allowing the solver to develop the solving strategy of a word problem.

Explanation information: information in the problem statement that details and explains the solving information or the situational information.

Graphic representation: a form of external representation of informational or conceptual content expressed through drawings, illustrations, graphs, schemes, or diagrams. Graphic representations help to understand phenomena, concepts, and products; demonstrate relationships between different elements; and aid in solving problems (for example, the graphical method used to solve a specific type of mathematical word problem).

Illustrative representation: graphic representation of situational information from the word problem statement. Although rich in detail, these representations are descriptive in nature, often being irrelevant for solving strategy.

Information (general) (cf. Baciu et al., 2022): Set of new elements related to previous knowledge of a subject, contained either in the structure of a message or in the meaning of a symbol (text, image/sequence of images or sounds).

Mathematical model: the equation, system of equations, and/or mathematical expressions that encode the relationships between the known and unknown data of the problem.

Mental images: internal reflections of various aspects of external reality; cognitive products evoked in the absence of the object of perception, similar, to a large extent, to spatial representations.

Numeric result: a mathematical product (such as a number, theorem, property, etc.) that is obtained by answering a requirement with a mathematical statement (like an exercise, problem, or question) after solving it, or that is discovered through logical inference or mathematical reasoning starting from existing data.

Problem condition(s): the relationship(s) established between the known and unknown data in the problem.

Problem content: information in the problem that presents known and unknown data, along with the relationships established between them.

Problem solution: the answer provided to the problem task obtained through explanation, evaluation, and/or interpretation of the numerical result within the specific context of the problem. The problem solution is explicitly stated in an enunciative sentence at the end of the solving plan, following the identification of the numerical result.

Problem statement: The text that presents the known and unknown data in the problem, along with the relationships established between them, what is given and what is required. The statement can present the problem's data either using abstract mathematical language or using common language, by evoking a particular situation or context familiar to the students.

Situational information: information from the problem statement that particularises the solving information, presenting it in a specific life context, through descriptions and details that refer to students' real-life experiences. Situational information facilitates the understanding of solving information, without importance for solving strategy or the problem model.

Spatial representation: an internal or external representation of informational or conceptual content in three-dimensional form that depicts the topological relationships between the components of a configuration (size, proportions, depth, distances), on which the subject can perform mental transformations.

Solving information: information from the word problem content that describes the essential data for solving strategy: the known data, the unknowns and the relationships that are established between them.

Solving plan: The set of interrogative or affirmative sentences outlines the successive stages of reasoning required to solve a mathematical word problem. Proper development of the solving plan helps in creating the solving strategy.

Solving process: the set of systemic steps (problem analysis and preparing the solving plan, writing and performing the mathematical operations corresponding to the solving strategy), completed by the solver to determine the problem solution.

Solving strategy: the specific method of finding a solution to a problem by systematically using the relationships between known and unknown data, in a logical and correct order, along with the solver's understanding of the context where this data appears.

Visual reasoning: complex metacognitive ability that involves reflecting on one's visual representations and organising these visual representations into operational structures used in the solving process of mathematical word problems.

Visual representation: internal or external representation of informational or conceptual content as a model that is mainly perceived visually.

Visualization (referring to word problems): the process of recalling, creating, using, and interpreting visual representations (internal or external) to comprehend information and develop new ideas and meanings, through which a global understanding of a mathematical word problem is constructed.

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