BABES BOLYAI UNIVERSITY
FACULTY OF PHYSICS

PHD THESIS SUMMARY

Computational and analytical modelling of astrophysically important stochastic processes

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Motto

George Costanza: Here's the outlet.
Slippery Pete: The what?
George Costanza: The outlet. Where the electricity comes from.
Slippery Pete: Oh, you mean the holes.

"The Frogger - Seinfeld"

Contents

1 Introduction

All real systems with interacting components present signatures of stochasticity in their behaviour. In astrophysics especially, there is a series of processes that are yet unexplained based only on the application and/or extrapolation of deterministic laws.

The observables of these systems bear a signature of stochasticity. To model them mathematically a differential equation is needed, which in the most general formulation has a source term. To account for the randomness, the source term is a random process. The physical laws which govern each system fix the relation between available energy in the driving fluctuations, the answer of the system to the driving fluctuations and the exhibited observational properties of this answer.

For both analytical treatments and simulations, the statistical properties of the variables need to be constrained within a given interval. A major source of information regarding the constraints is the type of energy source in the system. Based on this, a brute and efficient classification is considered in this thesis between the various types of systems. The structure of the thesis follows this classification and consists of analysing processes which have a thermal source for their fluctuations and processes whose fluctuations are fed by a non-thermal sources, respectively.

Structure of the thesis

The thesis is structured in three main parts. The first part presents the **introductory notions** and emphasizes on those concepts that will be of great use when embedded into complex modelling of astrophysical and cosmological phenomena.

The second part deals with **thermal models**. This case is the most straightforward, in which the energy source supplying the driving is known and the reaction of the system to this driving is Markovian. It is somewhat safe to say that the physics is understood and that the most difficult thing to do is to numerically account for the multicomponent interaction (e.g. Brownian motion). To illustrate the importance of these systems we review some literature and present three original applications with implications in astrophysics and cosmology.

In the case of **nontrivial energy source (non-thermal)** the physics is still blurry and in some aspects is still a debated issue even at a conceptual level. We include here systems with a driver that has nontrivial statistical characteristics, systems which answer to this driver in a non-markovian way

and processes which exhibit observables with interesting statistical properties. It is known that accretion disks around supermassive black holes display all these properties and there is a consensus that what we think of as a ”heath bath” is represented by the magnetic field. We provide a review of phenomena and an investigation of observational data, which emphasize on these properties and the underlying physical significance. Some of the statistical properties of the recorded light curves exhibit particular characteristics. It is shown that if bosons are confined in an extreme potential well, a statistical state called Self Organized Criticality (SOC) will occur, with observables having much of the properties exhibited by the observational time series. Finally, we provide an analytical and numerical analysis of a magnetized accretion disk under the driving of a fractional Brownian Motion-type process.

2 Part I - Introductory notions

A stochastic process has a part of, or all of its dynamics governed by randomness. Because processes that occur naturally all have an intrinsic random component, a well developed mathematical framework must be used in order to be able to model and predict outcomes of such physical processes.

The associated mathematical framework has been historically developed by scientists in all fields, starting with the observations made by Brown and followed by the formulation and resolution of the random walk problem and the fluctuation-dissipation theorem put forward by Einstein. Currently, there is a broad scientific area where these notions may be used, since examples of stochastic processes are found in all fields of science. For example, stochastic processes are introduced to model the non-deterministic functioning of real neurons (Nishimori, 2001), conduction in inhomogeneous media (Freund & Poschel, 2000), the cosmological structure formation (Freund & Poschel, 2000), epitaxial growth of thin films, fluctuations in stock prices (Mahnke *et al.*, 2009), biological and ecological evolution (Derzsi & Néda, 2012), variability in the spectra of gas accreting around a supermassive black hole (Balbus & Hawley, 1998).

All these modelling ventures start from a handful of simple "toy systems", which are more or less exactly solvable analytically. We describe two of these basic models in detail, considering various improvements brought to these models, namely the **Random Walk** problem and the continuous counterpart of this process, the **Brownian Motion**.

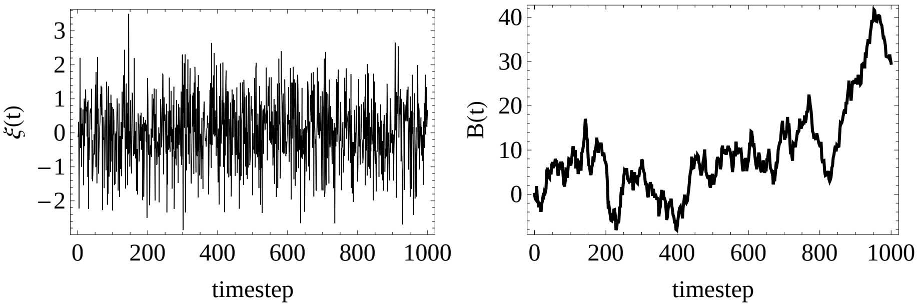

Figure 1: Left: One realization of the standard Gaussian process $\xi(t)$. The Power Spectral Distribution (PSD) of this curve is $P(f) = const.$. Right: One realization of a one dimensional Brownian motion $B(t)$. The PSD of this curve is $P(f) \sim f^{-2}$.

From a physicists point of view, one important characteristic of all stochastic processes is the degree of correlation between values of the same

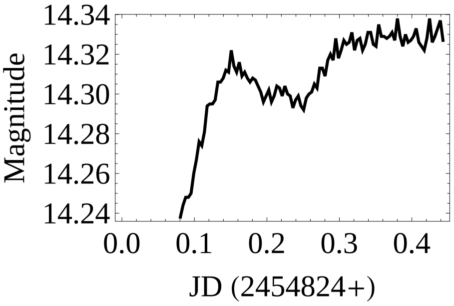

Figure 2: B (blue) band time series for the object Bl Lacertae S5 0716+714. A fit of the type $S(f) \sim f^{-\alpha}$ to the PSD of this curve returns $\alpha = 1.821 \pm 0.143$.

variable at different times. As each type of stochastic process is characterized by a specific autocorrelation function, power spectral distribution (PSD) analysis of observational time series is an important tool for identification of the underlying physical process responsible for producing the observed time series.

The appearance of what is called a well-behaved process can be seen in Fig. 1, for a realization of a Gaussian stochastic process (left) and for a Brownian stochastic process (right), respectively. But not all astrophysical phenomena exhibit well behaved information output. For example the light-intensity curve registered in the optical band of an accretion disk around a supermassive black hole is shown in Fig. 2.

3 Part II - Thermal models

There is a series of models and concepts that are of great importance to cosmology and astrophysics. One of the most interesting debates is about the origin of the Universe. While a lot of the items in such a discussion are still under scrutiny, it seems that there is one thing upon which everybody agrees on. This is the fact that for structure formation and evolution to occur, some physical fields must have fluctuated, tapping the energy necessary for these fluctuations from a heath bath.

It is our hope to bring a small contribution to clarify some of the problems associated to this topic. This part of the thesis is divided in three chapters.

The first chapter contains a simple model in which quark-gluon plasma and hadrons coexist as two distinct phases in a finite system near a first order phase transition. The transition is brought about by critical fluctuations of thermodynamic origin. Aside from theory, there is a way this can now be observed experimentally at the Large Hadron Collider (LHC) in CERN. We emphasize on the properties that may be measured at the LHC.

In the second chapter we discuss a momentarily "hidden part" of the Universe, indirectly confirmed by many observations and which is responsible for the directly observed expansion of the Universe, the dark matter. The reason it cannot be directly observed is its very weak interaction with what we call "ordinary matter" and there are many proposed particles which have the necessary physical properties. The possibility that dark matter is in the form of a Bose-Einstein Condensate of arbitrary finite temperature modelled by interaction with a thermal cloud is considered in detail. The consequences of the finite temperature are shown to have a major effect in the (theoretical) understanding of the evolution of the Universe.

The Universe was dynamical not only in what we call the cosmological past, but it is in perpetual evolution. In the third chapter we analyse the interaction, through their accretion disks, between stellar aggregates and black holes. The observational data suggest that a simple model for this process could be a stochastic harmonic oscillator tapping energy from a thermal reservoir. The interaction between the stars and the accretion disk is modelled as if the stellar aggregate would be a thermal heat bath and under the assumption that the disk undergoes thermal Brownian motion.

3.1 Critical fluctuations in quark gluon plasma

In central heavy ion collisions, due to the finite amount of involved matter the critical phenomena accompanying the phase transition is largely controlled by fluctuations. The dynamical development of the cooling and hadronizing quark-gluon plasma (QGP) is studied in a simple model assuming fluctuations of thermodynamical origin and a first order transition in a small finite system.

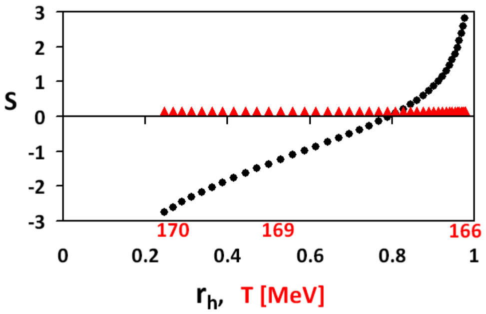

Figure 3: Calculated skewness as a function of the volume abundance of the hadronic matter (denoted as r_h, where 1 represents complete hadronization). The temperature scale is also indicated for clarity; the identifiers represent increments of 0.1 MeV in the temperature scale T. Results for a volume of the system of $500 fm^3$.

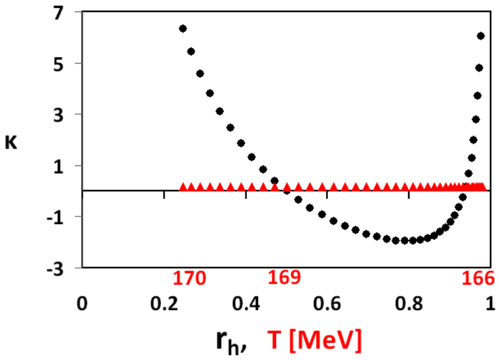

Figure 4: Calculated kurtosis as a function of the volume abundance of the hadronic matter (denoted as r_h, where 1 represents complete hadronization). The temperature scale is also indicated for clarity; the identifiers represent increments of 0.1 MeV in the temperature scale T. Results for a volume of the systems of $500 fm^3$.

The phase transition occurs when there is sufficient energy to form QGP in a sufficiently large volume. At lower energies this hadronization might happen well before the freeze out (FO) stage, and in such cases we have only rather indirect information about the phase transition.

This dynamical development is not directly observable, nevertheless, the FO moment can be observed and it should be similar for different locations in the system. At the same time, in a construction such as the Large Hadron Collider, the beam energy can be varied, which shifts the FO point versus the critical point, so a beam energy scan can provide us the series of information we are interested in. To this end, we study the energy density distribution of the two phases in the mixed phase domain in terms of the volume ratio. This analysis can be directly linked to experiments being performed at the LHC by analysing the skewness and kurtosis of this distribution (Figs. 3 and 4).

3.2 Dark matter as a Bose-Einstein condensate

The cosmological evolution of the finite temperature gravitationally self-bound Bose-Einstein dark matter condensates is studied. As a first step, by using the Hartree-Fock-Bogoliubov and Thomas-Fermi approximations, we obtain the equations of state of the arbitrary finite temperature dark matter condensate interacting with a thermal cloud. A finite temperature condensate can be described in terms of two fluids, the condensate and thermal excitations. Once the temperature of the system decreases, the number of the thermal excitations drops, and at zero temperature the system consists of the condensate only. Therefore the dynamics of a finite temperature Bose-Einstein-Condensate can be described as a system of two interacting fluids.

To see the effect of treating the dark matter as a BoseEinstein condensate we compare the currently accepted model of the Universe, i.e. which contains the standard pressureless ΛCDM dark matter with a Universe filled with dark energy, radiation, baryonic matter with negligible pressure and Bose-Einstein condensed dark matter. This is done through looking at the evolution of the scale factor in time, as shown in Fig. 5.

We have described the cosmological transition process in the framework of a two interacting fluid model. Due to the cooling of the Universe, the thermal cloud looses its particles and the thermal excitations enter into the condensate. Therefore, the dynamics of the dark matter is determined by the increase of the density, ρ_c, of the condensed component, which is initially increasing in time, and the corresponding decrease in the number of the thermal particles. However, after reaching a maximum value, the density of the condensate component will start to decrease. This happens at the moment

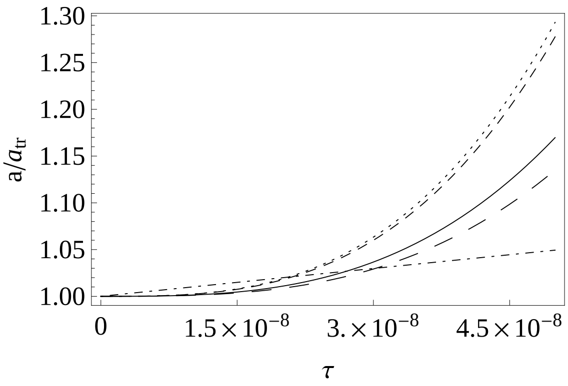

Figure 5: Time variation of the scale factor of the Universe filled with dark energy, radiation, baryonic matter with negligible pressure and Bose-Einstein condensed dark matter for $K_1 = 4 \times 10^{36}$ (solid curve), $K_1 = 3 \times 10^{36}$ (dotted curve), $K_1 = 2 \times 10^{36}$ (short-dashed curve), $K_1 = 0.6 \times 10^{36}$ (long-dashed curve), and of the standard ΛCDM model (dot-dashed curve). K_1 is a constant that dictates the energy transfer rate between the condensate and the thermal cloud.

when the time rate of the decrease in the condensate density due to the expansion of the Universe becomes higher than the density transfer rate from the thermal cloud. As a general result we have also found that the presence of the condensed dark matter will accelerate the expansion of the Universe as compared to the standard model. In the case of zero temperature condensed dark matter this behaviour has been already pointed out (Harko, 2011b; Chavanis, 2012; Harko, 2011a). However, a further increase in the speed of the expansion may be expected due to the finite temperature effects during the phase transition. Moreover, due to the presence of the two interacting fluid components, the cosmological dynamics of the finite temperature condensed dark matter is much more complicated than expected. Therefore the two-fluid finite temperature condensation transition may provide some clear cosmological signatures that could help in discriminating between Bose-Einstein Condensed dark matter models and the standard pressureless ΛCDM cosmological model.

3.3 Brownian motion in stellar aggregates-black holes dynamics

The interaction of black holes via their accretion disks with the stellar aggregate from their host galaxy is studied. This interaction is modelled as if the stellar aggregate would be a thermal heat bath, thus producing two independent forces on the disk: a friction force and a random force, respectively. The dynamics of the stochastically perturbed disks can be formulated in terms of a general relativistic Langevin equation. This equation is solved numerically to answer questions about the stochastic energy transfer between the aggregate and the disk. The observational signature of this interaction for accretion disks around supermassive black holes is compared to the observed light curves and their Power Spectral Distribution (PSD).

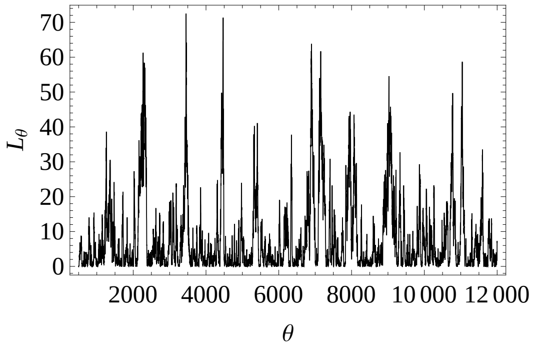

Figure 6: Dimensionless luminosity L_θ of the stochastically oscillating Schwarzschild disk as a function of timestep θ for a central object with mass $M = M^{10}M_\odot$ for a point found at seven gravitational radii away from the singularity.

Fluctuations of accretion disks around super-massive black holes may have important astrophysical implications. The emission of Galactic Black Hole Binaries (BHBs) and active galactic nuclei (AGN) displays a significant aperiodic variability on a broad range of time - scales. The Power Spectral Density of such variability is generally modelled with a power law, $P(f) \propto f^{-\alpha}$, where observations of BHBs and AGNs have shown that the spectral index is $\alpha \in (0.8, 2)$[1].

In the case of the present model of the disk oscillations (Fig. 6 for a Schwarszchild black hole and Fig. 7 for a Kerr black hole) under the effect of some stochastic uncorrelated force $\xi(t)$, the resulting Power Spectral Density is of the form $P(f) \propto f^{-2}$. This result is generally also valid for harmonically oscillating, undamped disks, and consequently it is independent

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[1] Such a dispersion of the power-law index reveals that there must exist some other mechanisms, different from purely thermal oscillations, which may be responsible for the observed value of α, like, for example, hydrodynamic fluctuations, magnetohydrodynamic turbulence, magnetic flares, density fluctuations in the corona, or variations of the accretion rate, caused by small amplitude variations in the viscosity. This is investigated in more details in the last part of the Thesis.

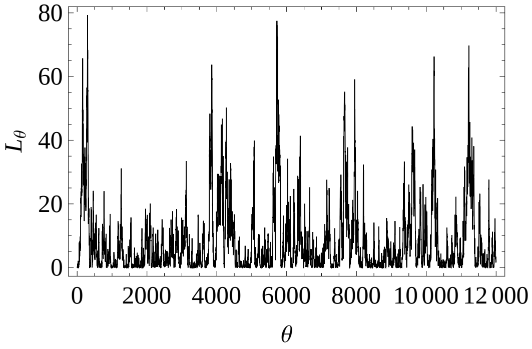

Figure 7: Variation of the luminosity L_θ as a function of timestep θ for a stochastically oscillating disk around a Kerr black hole with mass $M = M^{10} M_\odot$, rotation parameter $a = 0.9M$ and for a point found at seven gravitational radii away from the singularity.

of the general relativistic corrections to the equation of motion, since these only change the oscillation frequency of the compact object-disk system. The stochastic oscillation model can reproduce the aperiodic light curves.

4 Part III - Non-thermal models

One of the longstanding problems in astrophysics is the dynamics of magnetic fields. Reorganization of magnetic fields on all scales is believed to be an integral part of processes ranging from cosmological structure formation to flares in the Sun's atmosphere. While it is a simple matter to assume that the topology of magnetic fields varies, it is not as simple to find a mechanism that explains this process consistently for all the observed time- and length-scales.

One of the fields where the dynamics of magnetic fields is of major importance is accretion disk physics. Accretion disks are almost ubiquitous and their existence has been placed on firm grounds both theoretically an observationally (Krolik, 1999). The system of an accretion disk formed around an active super-massive black hole (Active Galactic Nucleus-AGN) exhibits very interesting observational characteristics which may be summarized as follows (e.g., Krolik (1999); Frank *et al.* (2002); Rosswoh & Bruggen (2007)):

- it occupies a volume of less than $1pc^3$ in the center of a galaxy but produces a luminosity 10^4 times larger than that of the host galaxy[2]. This incredible luminosity (between 10^{42} and 10^{48} erg/s) is recorded for almost all frequency bands of the electromagnetic spectrum;
- the continuum spectra of such an object is flat starting from the middle of the IR band and up until the hardest observable X-Rays (Fig. 8);
- it shows variability on all scales and in all electromagnetic bands. Perhaps the most interesting type of variability is the extremely short time scale variability, i.e. strong changes in the output luminosity occurring in a timescale of days or hours, termed Intra-Day Variability (IDV);
- the emergent radiation is linearly polarized in the range $0.5\% - 10\%$ and isolated cases show higher polarization;
- it shows strongly Doppler broadened emission lines. The exact mechanism leading to an apparent super-luminal motion of the plasma in the jets around AGNs is still debated.

The focus of this Part will generally be to analyse, model and reproduce the characteristics of the fast variability of a given subclass of AGNs, namely the Bl Lacertae objects. This part is divided in four chapters. The first

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[2] A parsec is an astronomically practical way to measure distances, $1pc \approx 3.08 \times 10^{16} m$.

two chapters discuss the theoretical motivation for considering the magnetic field the source of stochasticity and the observational evidence that this is so. The third chapter, starting from the idea of a nontrivial stochastic process, discusses some modelling techniques that we have employed to study observational data and to model such a system. The fourth chapter includes the analysis of an accretion disk subjected to a fractional Brownian Motion type disturbance in the density.

4.1 Magnetic field as a source of stochasticity

Reorganization of magnetic field lines occurs as a consequence of a process called magnetic reconnection. A classical approach to this problem (the Sweet-Parker model) is to infer that the small but finite electrical resistivity of cosmic plasma leads to the formation of a finite width current sheet. Field lines that were straight can break, change their topology and reconnect within this current sheet. However, classical Ohmic dissipation cannot assure a reconnection rate that corresponds with observed events.

Considering that the magnetic field has a weakly stochastic component is somewhat a natural extension of the Sweet Parker model. It can be shown that if the magnetic field is allowed to behave stochastically on some small scale, processes such as first order Fermi accelerations, flares (and especially solar flares), removal of magnetic flux from molecular clouds may be explained self-consistently. A quantitative comparison with observations, such as a Coronal Mass Ejection event (Ciaravella & Raymond, 2008), or reproducing the observed synchrotron power law behaviour of a microquasar (de Gouveia Dal Pino & Lazarian, 2005) looks very promising.

There is a (weak) debate of whether or not the source of the incredible energy release in an AGN is in fact due to accretion disks. In performing our analysis we assume that this is the case and review the mathematics behind calculating the intensity profile of the light curve. The Shakura-Sunyaev standard model (Shakura & Sunyaev, 1973) is reviewed and extensions of this model to include the effects of the magnetic field are discussed. One of the most promising viscosity mechanisms to assure fast accretion and in fact the only one that has not been dismissed when comparing to observations is the Magneto Rotational Instability (Balbus & Hawley, 1998). While it can be shown that a rotating disk is stable to hydrodynamic perturbations, perturbing even a weakly magnetized field leads to instabilities. In this case, the physics of mass transfer is not linear and the resulting radiated energy bears the signature of the underlying magnetic field.

One of the more subtle, but major consequences of the presence of a magnetic field is that the mass transfer is no longer homogeneous. Magnetic fields may lead to mass being transported at distant radii of the accretion disk along magnetic loops. Also, if there is a physical reason that might lead to a finite electric resistivity, the magnetic field will change topology and as a consequence will suddenly release large amounts of energy.

4.2 Analysis and interpretation of Active Galactic Nuclei observational data

Characteristic to all Active Galactic Nuclei (AGN) is a feature in their Spectral Energy Distribution (SED) called the Big Blue Bump (BBB, Figure 8) which occurs, as a function of their masses, in the optical or UV or soft X-Ray part of the electromagnetic spectrum. The flux recorded for each wavelength in this spectral region exhibits intra day variability (Figure 9). The underlying assumption is that the BBB is disk emission and the variability in the fluxes mirrors propagation times of disturbances in the disk between the characteristic radii for the respective wavebands.

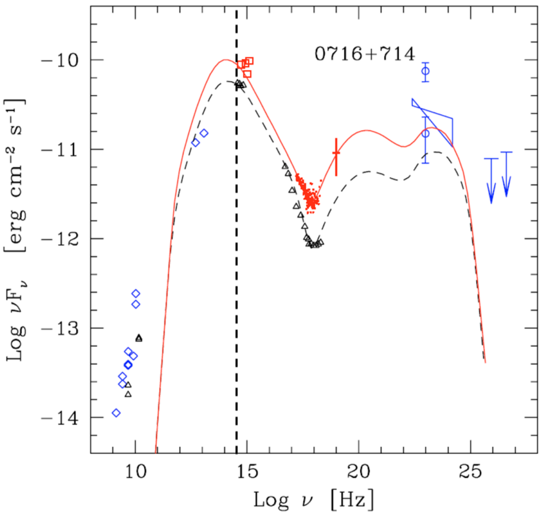

Figure 8: Spectral Energy Distribution of the object BL Lacertae S5 0716+714 (from Foschini *et al.* (2006)). Here F_λ is the specific flux, i.e. the rate at which energy arrives at a detector per unit area per unit wavelength λ.

There is one important note: similar objects with lower mass, such as X-Ray Binaries (XRBs) have their thermal emission in the X-Ray and are also better investigated than AGN. There are a number of characteristics

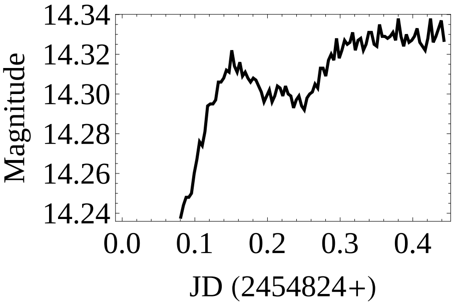

Figure 9: B (blue) band time series for the object Bl Lacertae S5 0716+714 at the wavelength indicated by the black dotted line in Fig. 8.

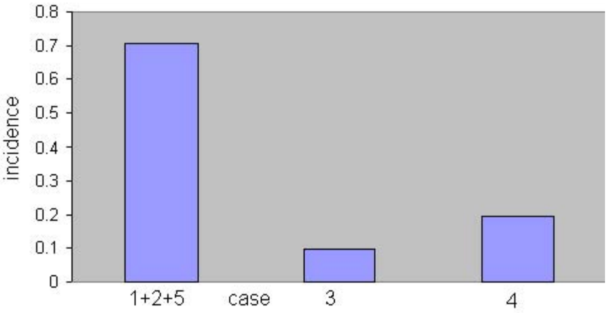

Figure 10: Histogram of the number of time-series acceptably fitted by hypotheses of the type $P(f) \sim f^{-\alpha}$ (the left column), in comparison with the cases which are not fitted by a power-law (the two right columns).

displayed by the X-Ray thermal emission that should be recovered by a model for optical variability for self consistency.

In order to get a taste for the "peculiarities" in accretion disk around AGN we present detailed analysis of sets of observational data. The focus is mainly the short time behaviour and the statistical properties of the light curve signal on these short timescales.

We find there are four aspects emerging from this discussion that are of importance:

- the existence and unknown nature of IDV (Fig. 8);
- the shape of the PSD dependency $P(f) \sim f^{-\alpha}$, during the IDV episodes (Fig. 10);
- the shape of the root-mean-square-deviation (rms) as a function of

the emitted flux during the IDV episodes (Uttley *et al.*, 2005). A linear function would show that the underlying process is log normally distributed, but observational data for IDV shows that this is rarely the case (Figs. 11, 12).

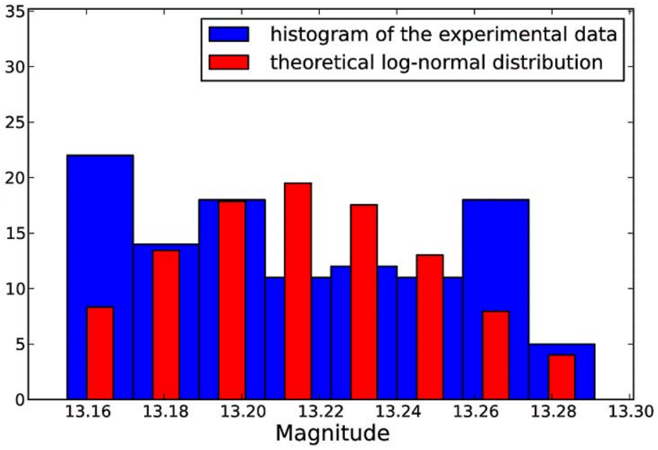

Figure 11: Histogram for the radiation magnitude of the Bl Lac object in the optical domain for IntraDay Variability. Data with a log normal distribution superimposed.

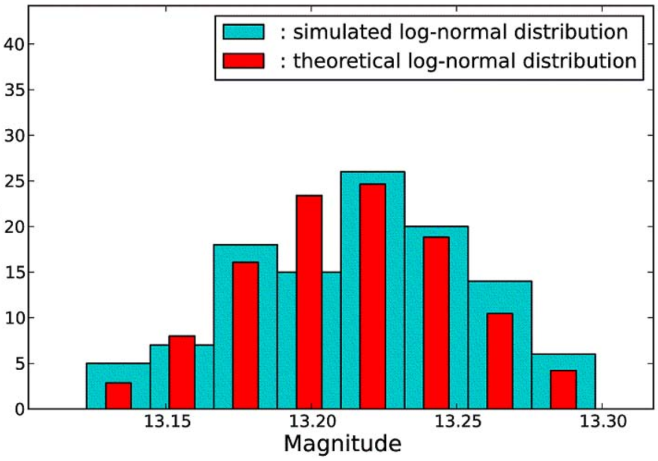

Figure 12: Simulated log normal distribution for a series with the same mean, variance and number of points as the observational data in Fig. 11.

- discrimination between models trying to explain IDV, based on the

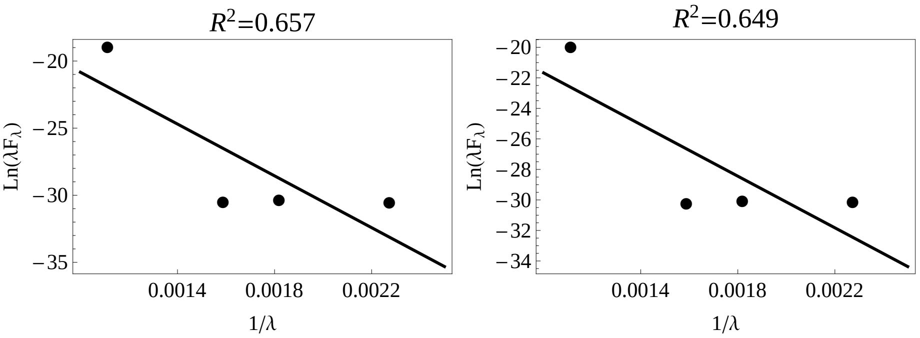

Figure 13: Plot of the logarithm of the medium value of λF_λ as a function of λ^{-1}, for the entire observational period (left) and for just one day (right). A fit of the type $\ln \lambda F_\lambda = a/\lambda + b$, expected for the MagnetoRotational Instability, was attempted, but it provides unsatisfactory results. Here F_λ is the specific flux, i.e. the rate at which energy arrives at a detector per unit area per unit wavelength λ.

observational analysis from the two above points. We excluded MRI as a source for IDV based on a set of observational data in the optical domain (Fig. 13).

All these issues are discussed in detail in the thesis.

4.3 Modelling tools for accretion disk dynamics

Apart from the numerical integration of Stochastic Differential Equations detailed in Part II (for the case of the stochastic oscillations of a General Relativistic Disk), we also analyse Cellular Automata type simulations. Cellular Automata (CA) proves to be very successful in modelling complex systems and its power resides in the fact that it reproduces global features of the behaviour of the system by using very simple local dynamics (Chopard & Droz, 1998; Wolfram, 2002).

In contrast, analytical treatment, through differential equations, may very accurately describe what happens locally, but it turns out not to be feasible in astrophysical conditions due to the very large number of boundary conditions involved in solving the associated differential equations (Isliker *et al.*, 1998).

One way to study accretion disks, due to various difficulties linked to direct observations, is through simulations which prove to be a very effective tool for tackling with this problem.

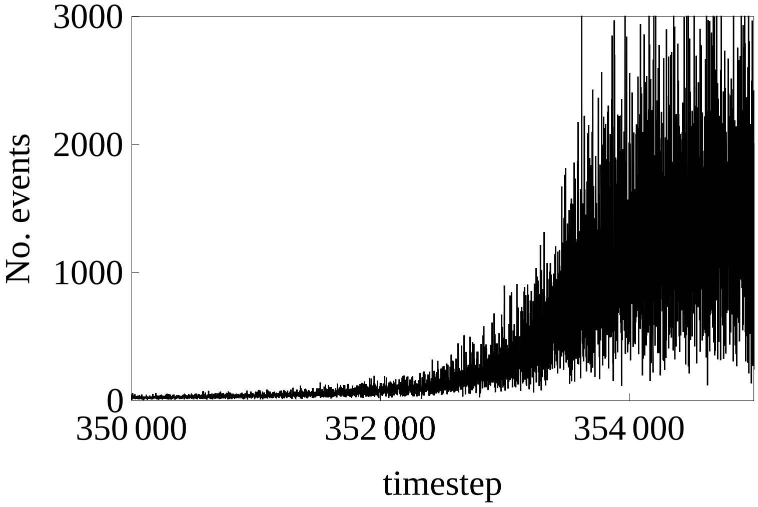

Figure 14: The distribution of event size, for a Quantum Chromo Dynamics (QCD)-axion cloud in superradiance around a BH. The cloud was initialized at 95% of its critical boson number.

CA simulations are especially useful because the picture of physical quantities varying smoothly over space and time cannot explain the observed rapid XRay variability in AGNs. The literature regarding Hydrodynamical and Magnetodhydrodynamical application of CA to astrophysical cases is reviewed.

We modelled the dynamics of unstable boson clouds by a simple cellular automaton and showed that it exhibits self-organized criticality (Fig. 14).

4.4 Magnetized disk perturbed by fractional Brownian Motion density

The purpose of this study is to investigate/map the effects that perturbations applied to an accretion disk might produce on the registered Light Curves (LC).

We work in the following framework: (1) start with the standard accretion disk (geometrically thin and optically thick); (2) the disk is perturbed at a radius R_{opt} thought to be the outer edge of the area that produces the optical emission. The perturbation will be, in turn, taken as deterministic or stochastic. The propagation of this perturbation plus associated emission will be followed through the next approximatively 20 gravitational radii. The support of this propagation is the disk itself. However, due to magnetic

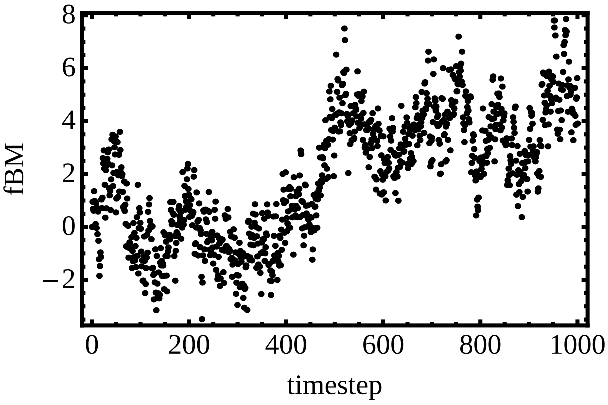

Figure 15: A sample path of fractional Brownian Motion (fBM). This is considered as a perturbation to the disk.

field pileup (a by-product of the accretion itself) the disk is in Self Organized Criticality. As already discussed, it has been shown that magnetic field in SOC produces a fractal structure, with known fractal index. This is confirmed both by analysis of solar magnetograms and 3D MHD simulations of accretion disk dynamics. When perturbed with a fractional Brownian Motion (fBM)-type form of the temporal variation of the density ($\dot{\rho}$ is a fractal, as shown in Fig. 15), the output luminosity of the disk (Fig. 16) for a fixed Hurst parameter gives a PSD slope $\alpha = 1.2706$, which is very promising.

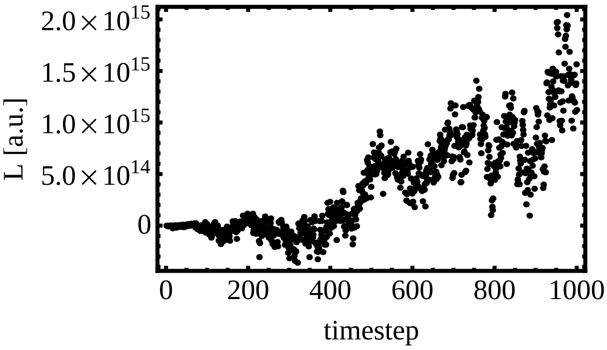

Figure 16: The luminosity variation resulting from the fBM input signal with $H = 0.2$.

Table 1: PSD of the output curve for a range of Hurst parameters of the input signal.

H	α	p_B
0.1	0.8478 [±0.04]	0.346
0.2	1.2706 [±0.04]	0.893
0.3	1.5483 [±0.04]	0.044
0.4	1.6188 [±0.041]	0.894
0.5	1.8012 [±0.038]	0.89
0.6	1.9923 [±0.045]	0.803
0.7	1.8470 [±0.038]	0.91
0.8	1.7944 [±0.039]	1
0.9	1.8090 [±0.04]	1

Table 2: PSD of the output curve for a range of masses of the central object and $H = 0.6$.

M/M⊙	α	p_B
10^7	1.8515 [±0.04]	0.339
10^8	1.9923 [±0.045]	0.803
10^9	1.9500 [±0.038]	0.391

The PSD of the output curve was calculated for a range of Hurst parameters of the input signal. The results are shown in Tables 1 and 2, where H is the Hurst parameter of the input fBM, α is the spectral slope resulting from the assumption that the PSD has the analytical form $P(f) \sim f^{-\alpha}$ and p_B is the Bayesian probability associated with this assumption (Vaughan, 2009). These results are in good agreement with observational data (See Poon *et al.* (2009); **Mocanu** & Marcu (2012) for a recent discussion of IDV in BL Lac S5 0716+714). By inspection of Table 2 it can be concluded that if one starts from the assumption that the light curve was produced in the framework discussed here, than there is a much higher probability that the central object has a mass of $10^8 M_\odot$ than $10^7 M_\odot$ or $10^9 M_\odot$. There is independent observational data that confirms that this object indeed has a mass of about $10^8 M_\odot$ (Fan *et al.*, 2011).

From the equations it is clear that as a consequence of the fBM perturbation, the magnetic field also becomes fractal. This is nicely embedded in our model. Complementary proof that the value of the magnetic field is a realization of a stochastic process is found in a series of works based on observational data analysis and/or successful interpretation in terms of the theory associated to stochastic processes (Kawaguchi *et al.*, 2000; Leung *et al.*, 2011; Ohsuga *et al.*, 2005; Takeuki *et al.*, 1995; Kawaguchi *et al.*, 1998; Carini *et al.*, 2011).

5 Conclusions and contributions

In the present thesis two different types of stochastic systems are analysed, differing from the point of view of the source of stochasticity. We were able to apply stochastic theory and numerical simulations to shed some light on a large number of astrophysical phenomena. This is not unexpected, as concepts arising from statistical physics are widely used in all fields of science, including ecology, sociology and transfer of information in the human brain. Large scale, i.e. astrophysical and even cosmological, evolution is known to be stochastic at least in some small percent. It is these phenomena that we have investigated, by using different analytical and numerical tools, in order to asses their effect on the processes observable nowadays.

Contributions and impact

The research described in this thesis has been published in six ISI papers, each containing original work. To describe our contributions to the field, we briefly enumerate the original work produced during the research period of the thesis[3]

- we proposed a new method that can be used as a tool to discriminate whether or not optical IntraDay Variability (IDV) comes from the disk (**Mocanu** & Sándor, 2012). This method is based on checking if the IDV data is log-normally distributed;

- we discussed the possibility that Dark Matter is in fact a Bose-Einstein condensate and studied its cosmological evolution together with the observational signature this framework would produce today (Harko & **Mocanu**, 2012a) (cited four times);

- we have shown that boson clouds around black holes eventually reach a statistical state known as Self Organized Criticality (**Mocanu** & Grumiller, 2012). As a result of this state, the emitted spectra in the gravitational wave domain is a crucial tool in determining the properties of the bosons. More specifically, a very interesting application is to see whether or not these bosons are axions, the most promising candidate for Dark Matter (cited four times);

- we studied the dynamical development of the cooling and hadronizing quark-gluon plasma (QGP) in a simple model assuming critical fluctu-

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[3] According to SAO/NASA Astrophysics Data System (ADS)

ations of thermodynamic origin and a first order transition in a small finite system (Csernai *et al.*, 2012) (cited two times);

- we analyzed the vertical stochastic oscillations of an accretion disk in a full General Relativity treatment by numerically solving the Stochastic Lagnevin equation associated to the system (Harko & **Mocanu**, 2012b). We found that in the case of an uncorrelated perturbation the spectral output of the disk is (thermal) Brownian motion (cited once);
- we carried a detailed PSD analysis of IDV observational data of a BL Lac object and based on this analysis concluded that the data was not produced by MagnetoRotational Instability, which is viewed as one of the most promising mechanisms to produce IDV (**Mocanu** & Marcu, 2012);
- we investigated the effect a fractional Brownian motion (non-trivially correlated) perturbation in density would produce on an accretion disk (work submitted to Advances in Space Research); our results are promising.

6 List of publications

List of ISI publications related to the thesis

1. **G. Mocanu**, A. Marcu, "Power spectral distribution of the BL Lacertae object S5 0716+714", Astronomische Nachrichten, Vol.333, Issue 2, p.166-173, 2012
2. T. Harko, **G. Mocanu**, "Stochastic oscillations of general relativistic discs", Monthly Notices of the Royal Astronomical Society, Volume 421, Issue 4, pp. 3102-3110. , 2012
3. T. Harko, **G. Mocanu**, "Cosmological evolution of finite temperature Bose-Einstein condensate dark matter", Physical Review D, vol. 85, Issue 8, id. 084012, 2012
4. **G. Mocanu**, D. Grumiller, "Self-organized criticality in boson clouds around black holes", Physical Review D, vol. 85, Issue 10, id. 105022, 2012
5. L.P. Csernai, **G. Mocanu**, Z. Néda, "Fluctuations in hadronizing quark gluon plasma", Physical Review C, vol. 85, Issue 6, id. 068201, 2012
6. **G. Mocanu**, B. Sandor, "Rms-flux relation in the optical fast variability data of BL Lacertae object S5 0716+714", Astrophysics and Space Science, Volume 342, Issue 1, pp.147-153, 2012

Submitted

- **G. Mocanu**, A. Pardi, N. Magyar, A. Marcu, "Appearance of an accretion disk perturbed by fractional Brownian Motion density", submitted to Advances in Space Research, 17 December 2012
- N. Verba, **G. Mocanu**, "Equivalent electric circuit for a harmonically perturbed accretion disk around supermassive black holes", submitted to Studia Physica, 30 January 2013
- C.S. Leung, **G. Mocanu**, T. Harko, "Generalized Langevin equation description of the stochastic oscillations of general relativistic disks", submitted as a conference paper to Journal of Astronomy and Astrophysics, January 2013

- T. Harko, C. Leung, **G. Mocanu**, "Generalized Langevin equation with coloured noise description of the stochastic oscillations of accretion disks", submitted to the Monthly Notes of the Royal Astronomical Society, February 2013

Conferences at which the work in the thesis was presented

- Conference "European Week of Astronomy and Astrophysics (EWASS)", July, 2012, Rome, poster presentation, "Rms-flux relation in the fast optical variability of BL. Lac. S5 0716+714", **G. Mocanu**, B. Sandor.
- Conference "Variability of BlazarsFrom Jansky to Fermi (VBJF)", December, 2012, "Generalized Langevin equation description of the stochastic oscillations of general relativistic disk", C.S. Leung, **G. Mocanu**, T. Harko

List of publications not related to the thesis

1. D.A. Pop, **G. Mocanu**, "Logical operator representation of velocity addition", Studia Physica, (57), 49-52, 2012
2. D.A. Pop, **G. Mocanu**, G. Arghir, "On the algorithmic behaviour of complex physical systems", Acta Technica Napocensis, Series Applied Mathematics and Mechanics, (55), 479, 2012
3. D.A. Pop, **G. Mocanu**, G. Arghir, "Object Oriented Programming (OOP) behavior of particle physics", Acta Technica Napocensis, Series Applied Mathematics and Mechanics, (55), 483, 2012
4. **G. Mocanu**, A. Marcu, "Simulation model for transverse loop oscillations: the effects induced by shock waves and opposition of the external medium", Romanian Astronomical Journal, (22), 23-42, 2012
5. **G. Mocanu**, D.A. Anastasiu, G. Arghir, "Algorithmic approach to basic thermodynamic notions", Acta Technica Napocensis, Series Applied Mathematics and Mechanics, 54, 1, 179-182, 2011

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