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EXTENDED ABSTRACT

The Effect of a Play-Based Intervention Program on the Ability of Length Measurement and Self-Efficacy in Geometry among Five-year-old Children

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Abstract

The present research involved a study of knowledge in 5-year-old children with respect to length measurement, spatial sense and sense of self-efficacy in geometry, while addressing the impact of a play-based intervention program on the acquisition of the above skills.

The aims of the study were to develop comprehensive tools, validated and reliable, to assess children's knowledge and to plan instruction that is suited to length measurement, spatial sense and identification of a sense of self-efficacy in geometry. Using the developed tools, the study examined whether an educational play-based intervention program based on meaningful tasks advances abilities in the way of length measurement, spatial sense and a sense of self-efficacy in geometry.

The mixed methods paradigm was chosen for carrying out the research. In the quantitative part of the study a clinical interview was developed and validated, including a unique and detailed index mapping knowledge on the part of 5-year-old children with respect to the three attributes studied. Following approval of the research it was used to examine the effectiveness of the intervention program, based on direct, explicit teaching.

After about five weeks, on conclusion of the quantitative part of the research, a qualitative study was carried out, aimed at examining implementation and transformation of the direct, contextualized instruction during geometric game playing. The qualitative paradigm included analysis of the children's geometric discourse according to the commognitive approach, as well as analysis of the transcribed interview that was held with the kindergarten teacher in the kindergarten.

The principal findings of the doctoral study point to significant differences in length measurement abilities and spatial sense between the children who participated in the intervention program and those who did not. As regards the sense of self-efficacy in geometry, overall no difference was found between the two groups.

The findings emerging from discourse analysis show that the children in the experimental group, unlide the children in the control group, always acted according to the correct, precise measurement sequence, and in full mutual exchange. They used diverse strategies, manifested in a process of proof and verification.

The research supports and reinforces neo-Piagetian theories and theories regarding the development of quantified tools among pre-school children. It points to the fact that

cognitive development is the result of a growth in the available capacity of the memory, allowing the child to cope with increasingly complex information and perform increasingly complex tasks.

The research makes a significant contribution to our understanding of the geometric discourse of young children by means of the four features of the commognitive approach. It adds an invaluable viewpoint regarding the identification of different components in thinking and learning processes undergone by the children who participated in the intervention program.

It is imperative to stress that this is a preliminary study that aims to assess the theoretical and empirical feasibility and value of the intervention program and the research tools.

Keywords: pre-schoolers, geometry, length measurement, spatial sense, clinical interview, sense of self-efficacy in geometry, play-based intervention program

Introduction

Background

The present research involved a study of knowledge in 5-year-old children with respect to length measurement, spatial sense and sense of self-efficacy in geometry.

The choice of pre-school age was based on the consensus that has been reached in recent years regarding the importance of advancing mathematical knowledge in children already at pre-school age, as reflected in the Standard set by the NCTM (2000) the curriculum determined by the Israel Ministry of Education (2010) and elsewhere in the world (Clements & Sarama, 2021, Dunphy et al., 2014), and research carried out in the field (Clements & Stephan, 2004; Rittle-Johnson et al., 2019; Sarama et al., 2022; Verdine et al., 2017; Zacharos & Kassara, 2012). Broadly speaking, length measurement and spatial sense are not only important in their own right but also support mathematical perceptions and skills (Arcavi, 2003; Gómezescobar et al., 2023; Verdine et al., 2014). They are significant predictors of the children's success in mathematical performance in later life (Aunio & Räsänen, 2016; Markowitz, 2018; Smith et al., 2015) – this being the reason for the importance attached to their engagement with the subject from early childhood.

The core of the present research is an intervention program involving play-based development that was developed by the researcher for pre-school children. The program was based on the mathematics curriculum for kindergarten children in Israel (Ministry of Education, 2010) and took into account the maturity of the child, namely, his learning potential (Vygotsky, 1978), his capacity for personal development (Piaget, 1952) and his level of development of geometrical thinking (Van Hiele, 1999).

The program rooted the process of instruction and the learning environment in which the experiment took place on direct, contextualized teaching, building up the geometric concepts layer by sequential layer (Aharoni, 2015), while supporting and advancing the sense of self-efficacy in geometry (Bandura et al., 1997). In order to assess the effectiveness of learning in the framework of the intervention program, a cognitive clinical interview was developed (Ginsburg, 2012), including a special, detailed index mapping and assessing knowledge on the part of the 5-year-old children in the three attributes studied.

Gap in Knowledge

Research in length measurement has dwelt in particular on young children's knowledge and strategies, as well as on the development of abilities related to length measurement. However, they have focused largely on knowledge without addressing the meaningful aspect of performance (e.g. Lozada & Carro, 2016; Matsuo & Nakawa, 2019; Szilágyi et al., 2013; Van den Heuvel-Panhuizen & Elia, 2011). Consequently, the children have no means of applying the abilities they have acquired in other processes that involve the solving of related problems. In light of the existing knowledge in the field and mapping of the stages that children go through in acquiring length measurement abilities (Hiebert, 1981). the need arose for research of the present kind to examine the use of strategies and measurement tools among pre-school children using a range of meaningful activities necessitating multistage performance, enabling the children to understand the relevance of the tasks assigned them (Clark et al., 2023; Clements et al., 2023; Laursen & Rasmussen, 2019; Savery & Duffy, 1995; Van Hiele, 1999), while advancing length measurement using units of measurement.

The present research narrows a number of gaps in existing knowledge, as follows:

- Proposal of an intervention program involving development based on direct, explicit instruction. The program includes play-based activities, necessitating multistage performance accompanied by explanations.
- Creation of authentic measurement tools suited to pre-school age in order to assess length measurement abilities, spatial sense and sense of self-efficacy in geometry.
- Use of qualitative methodology based on analysis of mathematical discourse as per the commognitive approach (Lavie & Sfard, 2019), enabling understanding and interpretation of cognitive footprints (Jonassen et al., 1999) deriving from learning in the framework of the intervention program.

Research Aim

The main aim of the present research is to examine whether play-based learning deriving from meaningful tasks promotes length measurement ability, spatial sense capability and sense of self-efficacy in geometry when solving assignments relating to length measurement and spatial sense.

The Research Contribution to Knowledge

The findings of the present research may make a significant contribution not only to instruction in length measurement and spatial sense, including encouragement of a

sense of self-efficacy, but also to research in mathematical education for pre-schoolers in Israel and elsewhere.

On the theoretical level: the research findings contribute to enriching the corpus of research knowledge on length measurement with young children using units of measurement, and to an understanding of the geometric discourse of children according to the commognitive approach, as reflected during planned game playing.

On the methodological level: creation of a unique clinical interview in the Hebrew language, aimed at examining children's knowledge in terms of their abilities in length measurement, spatial sense and sense of self-efficacy in geometry, contributing to the development of future research studies.

On the practical level: the research contributes to the construction of mathematical intervention programs in kindergarten that are suited to other cultures and countries.

CHAPTER I. THEORETICAL BACKGROUND

The literature review covers a wide range of sources addressing the theories that served for the purposes of this research, while acting as the basis for the key concepts associated with the field. A review was also carried out of previous research studies in six principal subjects that constitute the bedrock of this research and form the conceptual framework of the thesis: the kindergarten as an educational environment; cognitive and social development in pre-schoolers; teaching-learning with pre-schoolers; the mathematics curriculum for pre-schoolers; geometry for pre-schoolers, with the focus on spatial sense and length measurement; and instructional intervention programs for pre-schoolers.

1.1 The Kindergarten as an Educational Environment

Pre-school age (3 to 6) is a particularly vulnerable period, providing a window of opportunity for emotional, cognitive and social development (Holmes & Farnfield, 2022). Kindergarten in Israel is a complex educational institution, one that is constantly developing and changing, with a population of children from all ethnic backgrounds and walks of life existing together in a multicultural society (Snapir et al., 2012). The kindergarten teacher fills a key position, having the capacity to impact processes of optimal development in each child. Among other things, she is responsible for the kindergarten's work plan and its implementation.

1.2 Cognitive and Social Development in Pre-schoolers

This section focuses on the cognitive and social development stages of kindergarten children.

1.2.1 Piaget's Cognitive Theory and Neo-Piagetian Theories

Piaget's theory describes development as a general process that is not field-dependent (Piaget, 1952). This constructivist approach (Twomey Fosnot, 1996; von Glasersfeld, 1995) perceives learning as a process of building knowledge by self-exploration that takes place through interaction between the child and the environment.

Piaget assumed that thinking develops with age, and as the child grows, his/her concepts multiply and become increasing complex. The child is more logical in thinking and is able to solve increasingly difficult problems (Piaget, 1952). However, neo-Piagetian theories (Hallowell, 2020), which take into account interpersonal and cultural differences between children, provide different explanations for specific variations in development and unique thinking mechanisms on the part of children in the pre-operational stage.

1.2.2 Cognitive-Social Theory: Learning According to Vygotsky

An additional approach describing cognitive development in children is the social cultural approach determined by Vygotsky, which characterizes the learning process as one in which the child appropriates cultural tools as his/her own through negotiation with the environment (Vygotsky, 1978). According to this approach, participation by children as active partners in social actions involves interchange, in the course of which they internalize concepts and acquire skills as a result of negotiation, thus creating the meaning that occurs in a given event.

A knowledge and understanding of how children of pre-school age learn can help educators choose suitable teachding methods.

1.3 Learning and Instruction in the Kindergarten

Learning with many children is characterized by the fact that they are learning all the time and everywhere (Levin, 1995). They learn from both incidental and guided experiences, each experience contributing to the development process.

Harpaz (2020, 2012) claims that in order to advance and encourage learning, the aspiration should be in the direction of meaningful learning. This is learning in which the learner constructs anew his insights and creates a foundation for more enriched insights in the future. The major challenge faced by educators and the educational

system in general is to create an educational environment that increases the probability that children will acquire an awareness of **"involvement in the process and understanding of the product"**. Such an environment must create and support a connection with the social-emotional world of children (CASEL, 2020), while emphasizing appropriate teaching-learning methods such as experiential learning (Panigua & Istance, 2018), collaborative learning (Rasmussen et al., 2020) and play-based learning (Tyilo, 2021). This advances active experimentation, encouraging motivation and involvement, which in turn promote the building of high-order knowledge and thinking.

1.4 Kindergarten Mathematics Curriculum

The subjects included in the mathematics curriculum for kindergartens in Israel are arranged according to three sections: the concept of the number; spatial sense and geometry; and quantitative concepts in everyday life (Ministry of Education, 2010b). The curriculum makes reference to the important place that mathematics occupies in daily living, and to the many actions performed by children that are connected to mathematics. The present research study focuses on the section on spatial sense and geometry, involving size relationships and measurements.

1.5 Geometry at Pre-school Age

Geometry is important for familiarization with the surrounding world and with orientation within it. Children develop and act in an environment that contains objects and shapes, and familiarization with the environment helps them to orient themselves in space (Markowitz, 2018).

Van Hiele et al. (1957, 1986) developed a theory with respect to the five development levels of geometric thinking. Children of pre-school age lie in one of the first two levels: recognition and analysis. Advancement in the level of thinking depends to a great extent on education and learning. High importance is therefore attached to the kindergarten teacher's knowledge of the level of thinking in the child for this can guide her into choosing learning opportunities that are suited to the child.

The present research focuses on spatial sense and length measurement in pre-schoolers.

1.5.1 Spatial Sense at Pre-school Age

In the present research spatial sense is defined as the ability to identify, perceive and process information on the shape and location of stimuli in space (Uttal et al., 2013). Spatial sense consists of two key components: visualization and orientation (Stanic &

Owens, 1990). The present research also rests on the four central components of spatial skills as defined by Okamoto et al. (2015): visualization and representation; navigation; mental rotation and transformation of static and dynamic shapes and objects; and identification, disassembly and assembly of geometric shapes.

Development and consolidation of these components, which are instrumental in building spatial intelligence, allow us to interpret the visual information received in order to orient ourselves in the world and understand it. We need this intelligence both for routine everyday tasks and for more complex ones, such as length measurement.

1.5.2 Length Measurement at an Early Age

As a preliminary definition, it may be said that length is the attribute of an object that can be assigned a numerical value by quantification between the two end points of the object. Distance refers to the empty space between the two points (Clements & Sarama, 2021). Measurement can also be carried out relatively by comparing sizes.

The stages of measurement (Nir Gal et al., 1996), on which the present research focused with a view to developing the acquisition of length measurement skills are: direct comparison, comparison with the help of a mediator, and measurement using units of measurement.

The following concepts must be established in learning length measurement in order to understand how children conceive of space when carrying out measurement involving physical division (Clements, 1999; Lehrer et al., 2003): the units of measurement must be placed contiguously such that no gaps are created; the units of measurement must be placed at the starting point of the object to be measured; the units of measurement must by chosen such that they will be suited to the object to be measured; if the same object is measured using different units of measurement, two different results are obtained, both of which will be correct, depending on the unit used.

Length measurement is a subject that can be introduced in the early stages of learning in kindergarten (Ministry of Education, 2010b). Young children find it difficult to understand the concept of measurement, but do succeed in performing measurements once they are given the opportunity. Regrettably, reports are rife on reductions that are taking place in the teaching of geometry (Clements et al., 2022), a fact that leads to poor proficiency in this ability. One of the reasons for the difficulty encountered could be the traditional manner in which the subject has been taught, using conventional measuring instruments, without allowing the child to be creative and to understand the

need for measurement (Kamii & Clark, 1997; Kamii, 2006). In order for length to be "seen", conceptualization of the term is necessary.

The kindergarten teacher has a key role to play in developing these skills. Curricula (Ministry of Education, 2010b; NCTM, 1989) suggest that the subject of measurement include concrete experimentation, in which the children use measurement procedures in order to create interaction with their environment and actively explore the real world. Children must be able to have command over the choice of size and type of units of measurement that are the most suitable for a given situation (NCTM, 2000).

1.6 Sense of Self-efficacy

The concept of self-efficacy was defined by Bandura (1977) as judgment by an individual with respect to his ability to successfully maintain certain behaviours that will lead him to a desired outcome. Self-efficacy is not a general attribute but one that is dependent on the characteristics of the specific context (Schunk, 1991). It is a dynamic cognitive processs that people undergo when considering their performance capabilities, based on a perception of the connection between their skills and the requirements of the position. Self-efficacy has three dimensions – magnitude, generality and strength – and four sources: personal experience (success or failure); observation of others; verbal persuasion; and emotional awakening (Bandura, 1977).

1.6.1 Sense of Self-efficacy at Pre-school Age

Very few research studies have been devoted to self-efficacy in pre-schoolers, for the possible reason that children at this age find it difficult to distinguish between what is real and what they would like to be real (Stipek et al., 1984). Some research has found that young children may incorrectly associate effort with competency (Ruble et al., Stipek & Iver, 1989) and when asked about their self-efficacy they tend to rate it as across-the-board high (Tirosh et al., 2013; Wilson & Trainin, 2007).

The present research focused on pre-schoolers. One of the aims of the research was to investigate the beliefs the children have regarding their self-efficacy as related to specific geometric tasks, both familiar and unfamiliar, and to compare these beliefs with actual performance.

1.7 Previous Learning-based Intervention Programs in Mathematics for Pre-schoolers in Israel and Worldwide

This section presents the extensive mathematical work carried out in the framework of various intervention programs, highlighting the importance of such programs for advancing children's abilities using diverse methods and activities.

The research literature claims that cultivation of mathematical literacy calls for structured pedagogic activities that are in keeping with the thinking and learning characteristics of pre-school children. Interventions that applied explicit teaching, with continuous instruction according to a specific order – generally from easy to difficult – and with clear advancement in the subject, lead to improved results in mathematical learning (Clements & Sarama, 2021; Clements et al., 2018). Moreover, an effective intervention program for advancing mathematical abilities in pre-schoolers must be play-based, using tangible objects (Jarrad et al., 2021). The incorporation of games in the teaching of geometry in kindergarten allows learning through enjoyment, intensifying the children's motivation and activity in a real environment that strengthens an understanding of spatial relations and concepts relating to size, while developing mathematical language.

1.8 Play-Based Intervention Program for Advancing Length Measurement Ability and Spatial Sense Developed for the Present Research

The core of the present research is a development-oriented, play-based intervention program that is aimed at advancing abilities in length measurement, spatial sense and sense of self-efficacy in geometry among 5-year-old children.

Development of the intervention program was based on three main axes (Fig. 1): the formal axis of mathematical learning at pre-school age; the axis based on theories relating to child development, focusing on learning at pre-school age in general, and learning of mathematical concepts at pre-school age in particular; and the axis asking the central question – How does one advance learning at pre-school age? These present the paths to meaningful learning while focusing on the principles of learning that incorporate value, involvement and relevance.

The program based teaching and the learning environment in which the experiment was conducted on direct, contextualized instruction, building the geometric concepts layer by sequential layer (Aharoni, 2015), the intention being to also support and advance the sense of self-efficacy in geometry (Bandura, 1997).

Work took place in the kindergarten once a week for about 25 minutes for each group of children. It was carried out in an environment that allowed the children maximum concentration.

The researcher ran all the activities in the intervention program for about six months.

Intervention Program Structure

The intervention program was based on 21 games that develop spatial sense and length measurement at different levels of sophistication. The first game group included seven games that develop spatial awareness and organization in space, for example:

- Disassembly of a given polygon into different polygons, such as disassembly of a rectangle into two triangles or two different squares and assembly of a polygon from given polygons.
- Covering an image using "Pattern Blocks".

The second game group includes 14 games that combine experiential and meaningful learning and develop length measurement skills using a variety of measuring instruments, for example:

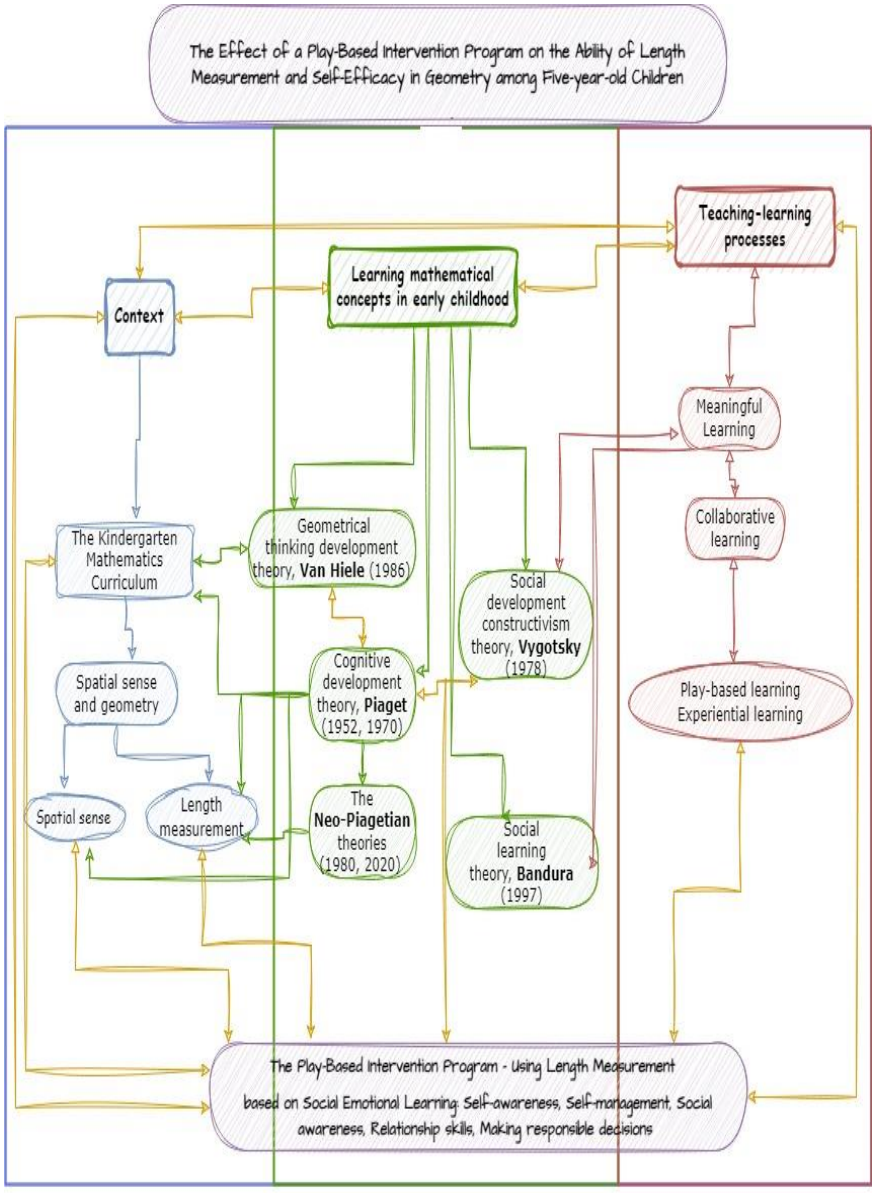
- What is the length of the ribbon you need to make a crown for your head?
- Measuring and preparing a tablecloth for a birthday celebration or a planned party.

The uniqueness of the intervention program developed by the researcher is manifested in the following:

- The program offers only games and meaningful activities in which measurement and spatial sense are the means for achieving a meaningful goal for the child.
- The games and activities include use of a wide range of materials allowing measurement amid implementation of a variety of strategies (direct comparison; use of a mediator; use of natural, arbitrary measuring units; estimation).
- During the course of learning the children were asked to explain their measuring procedures and verbalize arrangement of the various objects in space. In addition to their explanations, they were asked questions that encouraged discussion and reflection.
- The framework enabling implementation in groups of up to four children or a pair of children encouraged observation of partners who were performing and explaining their performance. Such observation advanced the sense of self-efficacy in geometry.

Figure 4

Conceptual framework of the research



The following chapter begins with an overview of the research design followed by a detailed description of the research tools, participants and ethical considerations.

CHAPTER II. RESEARCH DESIGN and METHODOLOGY

2.1 Overview of the Research Design According to the Studies

The following table summarizes the research design and methodology, the process and its different studies.

Table 1

A detailed research design and methodology

Study	**Aim**	**Participants**	**Research Question**	**Research Hypothesis**	**Tools**	**Analysis**
1 Creation of a valid, reliable protocol to map the knowledge of five-year-olds in length measurement, spatial sense and sense of self-efficacy in geometry	Design of the research tool: Creating a valid and reliable protocol that includes a clinical interview and an index that maps the knowledge of five-year-old children in: 1. Measuring length 2. Spatial sense 3. Sense of self-efficacy in geometry.	29 children (5-6 years old) in three different kindergartens from the experimental and the control group. **Convenience Sample**	What are the research tools that allow assessing the abilities of five-year-old children when performing tasks of: A. Length measurement B. Spatial sense C. A sense of self-efficacy in geometry when solving tasks related to length measurement and spatial sense?		Development of a research protocol and tool specifically for the purposes of the current research.	Construct validity – developing a theory-based tool. Content validity set by five experts in the field**.** Internal reliability by Cronbach's alpha.
2 Pre- and post-evaluation of the intervention program effectiveness in terms of the ability to measure length, ability in spatial sense, and sense	Execution phase: Pre- and post-testing the effect of the intervention program among five-year-old children: 1. Ability to measure length	60 kindergarten children (5-6 years) in all, of which 30 in the experimental group and 30 children in the control group (not the same children from study 1). **Convenience**	1. Will direct play-based learning in length measurement and spatial sense improve the following indicators: A. Length measurement; B. Spatial sense; C. Sense of self-efficacy in geometry	1. The children who will take part in the intervention program will demonstrate a greater improvement in their length measurement abilities compared	Clinical interview and documentation index examines: 1. Length measurement. 2. Spatial sense 3. Self-efficacy in geometry in the tasks of measuring length and spatial sense	ANOVA analysis of repeated measurements in which the independent variables will be the intervention program and time, and the dependent variable will be the questionnaire.

of self-efficacy in geometry (Quantitative study)	2. Ability in spatial sense 3. Sense of self-efficacy in geometry.	**Sample** The selection of the children into groups was based on matching that took into account the following variables: belonging to the group, age, gender, socioeconomic status, and the kindergarten teacher's seniority.	when solving tasks related to length measurement and spatial sense. 2. Will there be a difference between kindergarten children who were exposed to direct play-based learning in measuring length and spatial sense (the experimental group), and those who studied geometric subjects only as part of the government curriculum (the control group) in the following indicators: A. Length measurement capabilities; B. Capabilities of spatial sense; C. Sense of self-efficacy in geometry.	to the children in the control group. 2. The participants in the program will demonstrate a greater improvement in spatial sense compared to the control group. 3. The participants will demonstrate a higher increase in the sense of self-efficacy in geometry compared to the children in the control group. 4. Age, gender and group affiliation will predict the improvement in the ability to measure length, the ability to spatially perceive and the sense of self-efficacy in geometry.		The interaction effect in which the difference between before the intervention and after the intervention will be measured.
3 Implementation and	To examine the implementation and transformation of	12 children from the experimental group and 12 children	1. How do children implement and demonstrate		Participant observation	Mathematical discourse analysis

transformation of direct instruction when performing authentic activities (Qualitative study)	the learning that took place in the intervention program.	from the control group. The children from the experimental group and the control group were selected using a stratified sample based on the score in the clinical interview of measuring length and spatial sense after the implementation of the intervention program. The kindergarten teacher of the experimental group.	transformation of the learned geometric concepts five weeks (T_1) after completion of the intervention program? 2. How do children implement and demonstrate transformation of the learned geometric concepts, as reported by the kindergarten teacher, five weeks (T_1) and ten months (T_2) after completion of the intervention program?		Semi-structured interview recorded on video.	based on the Commognitive Approach. Content analysis of the interview.

2.2 Research Population

The average age of the children who participated in the research was 5 years and two months (5;2) (SD=0.37). They studied in regular state kindergartens. The kindergartens that were chosen for the study are located in a large neighbourhood in a city in southern Israel. The socioeconomic status of most of the population in the neighborhood is average to high average.

2.2.1 Characteristics of the Participants in Study 1

In this study use was made of convenience sampling that included 29 children. The children who participated in this part of the research were different from the children who participated in the following stage, studying in different kindergartens from the children who comprised the experimental and control groups.

2.2.2 Characteristics of the Participants in Study 2

In this study, use was made of convenience sampling to choose the two groups for the research: experimental and control.

The experimental group included 30 children from one kindergarten only. The children chosen for the control group were matched to the children in the experimental group with respect to all mediating variables: age, gender and socioeconomic status. The control group included 30 children from two kindergartens.

2.2.3 Characteristics of the Participants in Study 3

The participants in this study were as follows: 12 children in the experimental group and 12 children in the control group, selected according to stratified purposeful mixed methods sampling, based on the score in the clinical interview on length measurement and spatial sense in the post-test.

A total of four children were chosen from each group, exhibiting the following abilities: above average, average, below average.

An additional participant at this stage was the kindergarten teacher of the children in the experimental group.

2.3 Research Tools

2.3.1 Research Tools for Study 1

The instrument for this study was developed by the researcher for the research with the aim of examining abilities in the way of length measuremenet and spatial sense in 5-year-old children, as well as to examine the children's sense of self-efficacy in these

skills. The instrument included a unique, detailed index mapping and assessing the children's knowledge in the three areas under study.

2.3.2 Research Tools for Study 2

The research instrument developed in Study 1 was used by the researcher to examine the effectiveness of the intervention program.

2.3.3 Research Tools for Study 3

At this stage of the research use was made of two research tools:

- Participant observation, carried out with the aim of identifying processes that had taken place in the course of the intervention program and assessing their nature.
- Two semi-structured interviews conducted with the kindergarten teacher with the aim of investigating her thoughts, emotions and beliefs with respect to the experiment carried out in her kindergarten.

2.4 The Researcher's Role

The researcher in this study was a participant in each of the stages of the research from the moment the study was formulated, including construction of the tools and implementation of the intervention up to the stages of evaluation and analysis.

In order to overcome the limitation of the researcher's involvement, the following steps were taken:

- The researcher had no prior acquaintance with the research participants.
- Construction and verification of the content of the clinical interview and the content of the intervention program. The researcher consulted with five specialists in mathematics education, early childhood education and early childhood mathematics education.
- Analysis of the received data in the tool-building phase was conducted with other specialists in the field in order to neutralize the involvement of the researcher. In cases where it was not possible to ascertain the children's knowledge unequivocally, additional objective and uninvolved judges were sought to review the researcher’s analysis.
- Analysis of the final data following intervention, together with two external judges, was done as part of a joint discussion, referring to the compatibility of the analyses with the research goals and examination of the interpretations from a critical and objective point of view.

2.5 Ethical Considerations

In this study, ethical issues were taken into account, mainly because of the young age of the participants and the involvement of the researcher in the various stages of the study. The researcher maintained clear ethical guidelines following national and international legislation with regard to research involving children.

The participants in one part of the research were pre-school children. As such, basic ethical rules had to be observed, as required in research with minors (ages 0-18):

- All research conducted in Israel in a school with children must be approved by the Office of the Chief Scientist of the Ministry of Education (Ministry of Education, 015; AERA, 2011).
- Any research with minors requires receipt of approval from their parents.
- When carrying out an intervention program in an educational institution that includes children (interviews / questionnaires / observations) the following must be taken into account: (1) The children cannot be forced to participate in the program; therefore activities must be planned for those who choose not to participate; (2) The detailed program and lesson plans must be submitted to the Chief Scientist's office for approval. The approval process for an intervention program is longer than the approval process for research without an intervention program (Ministry of Education, 2015).

Another participant in the study was the kindergarten teacher in whose kindergarten the experiment was conducted. A relationship of trust and partnership must be created between the researcher and the participants, allowing the participants to voice their opinions without judgment or criticism, in order to minimize ethical damage (Shkedi & Weinberg, 2021).

CHAPTER III. RESEARCH FINDINGS

This chapter presents the research findings according to the research questions and hypotheses.

3.1 Findings Relating to the Creation of a Valid and Reliable Protocol, Including Clinical Interview and Index Mapping the Five-year-old Children's Knowledge in Length Measurement, Spatial Sense, and Sense of Self-Efficacy in Geometry (Study 1)

Verification of the construct validity, in accordance with the theories on which the theoretical variables are based, and the content validity, based on the evaluation of specialists in the field, attests to the fact that the research instrument encompasses the entire content of length measurement and spatial sense in 5-year-olds. This was proved essential for the purposes of validity and reliability of the items in the clinical interview. Items that were found to be in only minor correlation with other items in the interview were eliminated, while other items were added, such that Cronbach's alpha reliability coefficient for length measurement was 0.75, for spatial sense was 0.81, and for sense of self-efficacy was 0.64. This allowed the instrument to be used for Study 2 as well.

3.2 Findings Relating to the Effectiveness of the Intervention Program in Promoting Length Measurement, Spatial Sense Abilities, and a Sense of Self-Efficacy in Geometry (Study 2)

This section presents the analysis of the findings relating to the experimental and control groups on this subject.

3.2.1 The Differences Between the Experimental and Control Groups at the Time of the First Measurement

Statistical analyses were carried out prior to the first measurement time (before intervention) in order to rule out the existence of differences between the groups prior to the start of the intervention program. No significant differences were found between the experimental and control groups in the three study variables in the first measurement.

3.2.2. The Differences Between the Experimental and the Control Group Abilities in Length Measurement, Spatial Sense and Self-Efficacy in Geometry

Following are the findings with reference to the research hypotheses:

1. *The children who will take part in the intervention program will demonstrate a greater improvement in their length measurement abilities compared to the children in the control group.*

Table 2 presents the average pre- and post-intervention longitudinal measurement scores in each of the study groups.

Table *2*

T-tests for samples depend on the improvement in each of the study groups

	Before intervention		After intervention		t value
	M	SD	M	SD	
Experimental group	1.96	0.46	2.61	0.40	10.69***
Control group	1.92	0.53	2.00	0.46	0.79

**p<.01, ***p<.001

According to Table 1, it appears that while in the experimental group the improvement between measurement times is significant, in the control group there is no significant difference between the times.

Furthermore, additional analyses were carried out in order to examine the source of the differences between the experimental and control groups in length measurement by examining each of the sub-scales of the instrument separately (through direct comparison, use of a mediator, and use of a unit of measurement). The findings are presented in Table 3.

Table 3

Independent t-tests to examine improvement in the sub-scales of length measurement according to group

	Experimental N=30		Control N=30		t value	Significance
	M	SD	M	SD		
Direct comparison	1.33	0.38	0.71	0.55	5.07***	<.001
Use of a mediator	0.91	0.44	0.22	0.74	4.35***	<.001
Use of a unit of measurement	0.77	0.66	0.29	0.66	2.85**	.006

**p<.01, ***p<.001.

A look at Table 3 shows that, according to the research hypothesis, the improvement among children in the experimental group was greater than the improvement among children in the control group in the three sub-scales of length measurement.

Conclusion: The first research hypothesis was confirmed.

2. *The children who will take part in the intervention program will demonstrate a greater improvement in spatial sense compared to the control group.*

Table 4 presents the average scores of spatial sense before and after intervention in each of the study groups.

Table 4

T-tests for samples according to spatial sense

Division into groups	Before intervention		After Intervention		t value
	M	SD	M	SD	
Experimental group	1.99	30.58	3.00	0.51	12.71***
Control group	1.81	0.66	1.98	0.62	1.78

***p<.001

Conclusion: The second research hypothesis was confirmed.

3. The children who will take part in the intervention program will demonstrate a higher increase in the sense of self-efficacy in geometry compared to the children in the control group.

In order to test this hypothesis, an analysis of variance (anova) was carried out for repeat measurements. Table 5 shows the significance of the effects in the variance analysis model.

Table 5

Significance of the effects in the variance analysis model for the sense of self-efficacy in geometry

Effect	DF	F	η^2_p
Time		5.11*	0.08
Group	(1,58)	2.28	0.04
Interaction		0.32	0.85

*p<.001

As shown in Table 23, the sense of self-efficacy in geometry did not show a significant interaction effect between the measurement time and the group.

Conclusion: The third research hypothesis was not confirmed.

Additional Findings for the Third Hypothesis

An additional analysis was performed to examine the children's sense of self-efficacy in geometry. A new variable was created for the purpose of the analysis, expressing the relationship between self-efficacy and performance. The variable was obtained by the difference between the child's own estimated self-efficacy and his actual performance (the estimated self-efficacy before performance of the task less the actual score obtained on the task). This index is intended to examine the extent to which the level of self-efficacy perceived by children reflects reality. In order to check differences between the groups a t-test was carried out for independent samples in which the independent variable was the group (experimental or control) and the dependent variable was the level of correspondence between the sense of self-efficacy and actual knowledge on conclusion of the intervention program.

A significant difference was found between the groups (experimental and control) in the sense of self-efficacy vs. performance. In other words, the children in the experimental group were able to estimate their competence in terms of their knowledge following intervention more accurately than the children in the control group. In the control group, the children continued to estimate their self-efficacy as high despite the fact that their knowledge, as measured, proved otherwise.

Conclusion: It was found that following intervention, the sense of self-efficacy of children in the experimental group reflected more accurately their geometric knowledge of length measurement and spatial sense than that of the children in the control group.

4. *Age, gender and group affiliation will predict improvement in the research variables, namely: length measurement ability, spatial sense and sense of self-efficacy in geometry*.

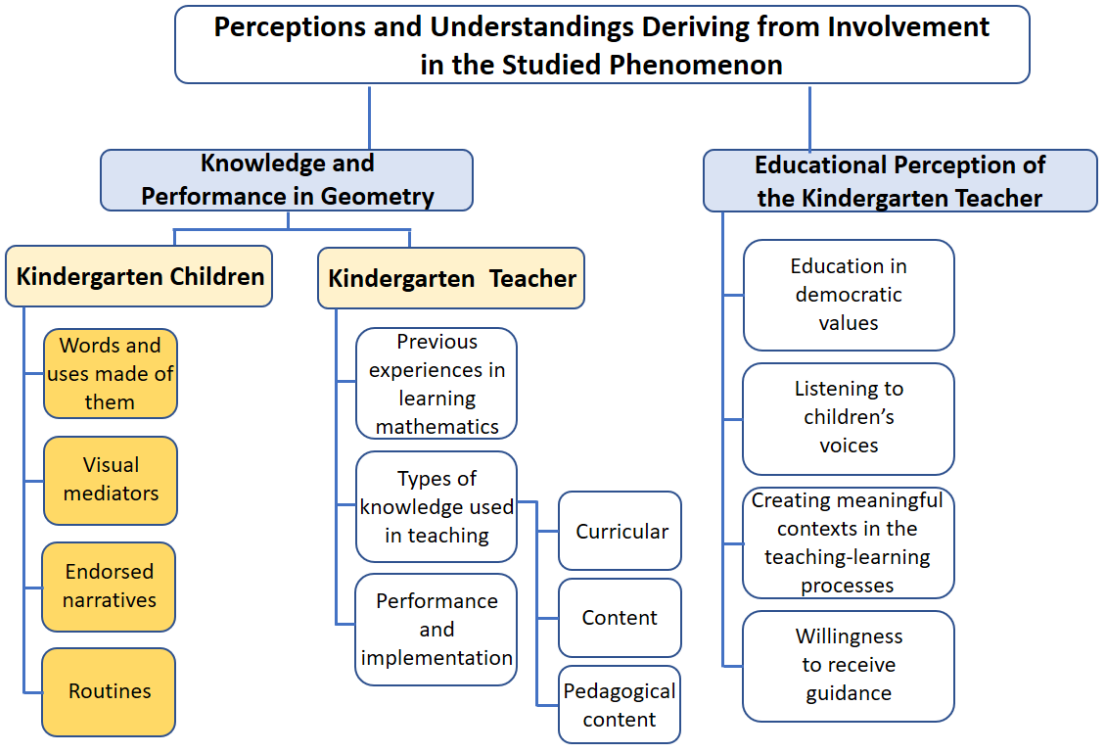
Three multiple regression tests were carried out, as shown in Table 24. The only definite predictor was group affiliation in both length measurement ability, $t(59)=4.91$, $p<.001$, and spatial sense, $t(59)=6.32$, $p<.001$. This prediction indicates that in the experimental group, the improvement was significantly greater than in the control group for both variables.

3.3 Findings relating to Examination of Implementation and Transformation of Learning Conducted in the Course of the Play-based Intervention Program (Study 3)

As regards the research questions, the themes that emerged from qualitative analysis of the transcribed discourse (during game playing by the children) and the interviews (with the kindergarten teacher of the experimental group) are: knowledge and performance in geometry (with the kindergarten children and with the kindergarten teacher in the experimental kindergarten) and the kindergarten teachers's educational perception, as presented in Figure 2.

Figure 5

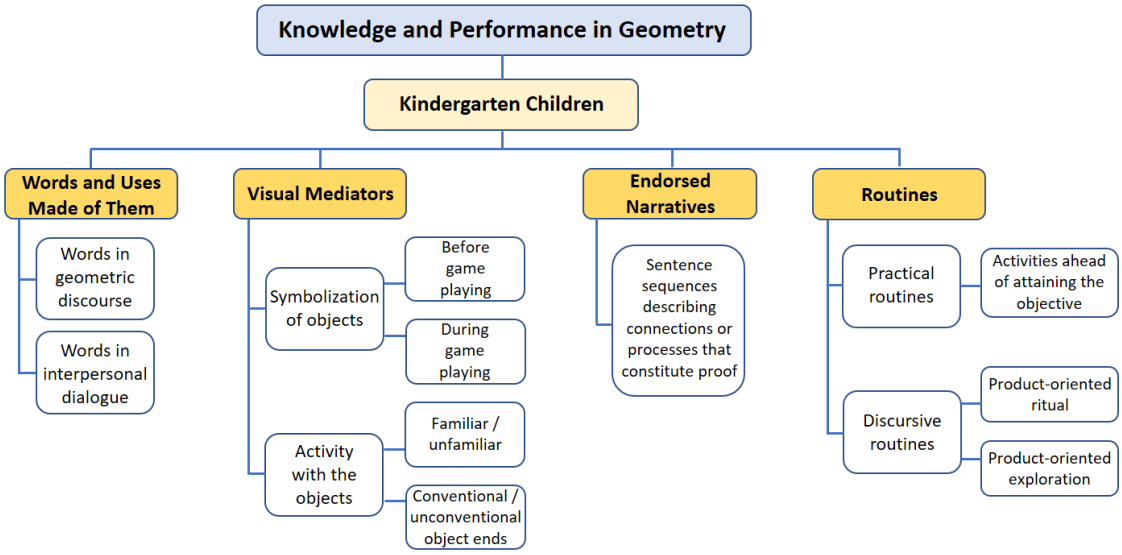
Perceptions and understandings deriving from involvement in the studied phenomenon – classification of the findings from the transcripts and interviews (qualitative research)



The central theme emerging from analysis of the children's discourse during geometric game playing reflected knowledge and performance in geometry among the kindergarten children (both experimental and control groups), as shown in Figure 3.

Figure 6

Knowledge and performance in geometry among the kindergarten children (experimental and control groups)



The differences between the learning processes in the two groups are evident in the mathematical discourse that accompanied geometric game playing, including words and the way they were used, together with the use of visual mediators and endorsed narratives (Sfard, 2008, 2012). In comparing the two groups – the children who participated in the intervention program and those who studied according to the state curriculum – particularly conspicuous was the use made of the components of discourse. The findings show that among the children who participated in the intervention program use was made of accepted mathematical terms for length measurement, length estimation, use of different mediators, non-avoidance of mediators with unconventional ends, and multiple exploration routines leading to results, while basing the answer on endorsed narratives, attesting to processes constituting proof. This is evidence of the use of skills, use of knowledge and meta-cognitive experiences (Lavie et al., 2019).

In contrast, among the children who studied according to the state curriculum it was found that use was made of accepted mathematical words for length measurement only at times. They also avoided choosing objects with unconventional ends (Tabach & Nachlieli, 2016).

It was also found that the children who did not take part in the intervention program used ritual routines with high frequency, this being the first and most necessary part of

learning. They did not demonstrate situations involving proof and use of endorsed narratives since these start being formed only once there is a command of knowledge.

In light of the above, it appears that the children who studied according to the state curriculum did not bring with them knowledge that boosted confidence in dealing with exploration routines and proof during length measurement activities. In other words, additional learning is required, taking into account all the layers comprising the concept of length measurement, in order to build up knowledge layer by layer, including consolidation of each layer separately.

The findings emerging from content analysis of the two semi-structured interviews held with the teacher in the experimental kindergarten present a broader picture, rooted in the educational field in which the research was conducted, and attesting to the abilities of the children from her point of view.

The qualitative findings add an important dimension, attesting to the fact that the children performed and implemented length measurement abilities in the framework of the educational activities carried out in the kindergarten in many and varied contexts. They also express the personal voice of the kindergarten teacher, illustrating how, as part of the routine educational activities she initiates in the kindergarten, there are manifestations of geometric discourse around the subject of measurements. She adds that these are supported by a knowledge and understanding of the essence of the activity, as expressed in the dialogue between the children and the procedures they carry out.

CHAPTER IV. CONCLUSIONS AND RECOMMENDATIONS

This chapter makes reference to the main conclusions deriving from the research findings, to the research limitations, and to the theoretical, methodological and practical implications.

4.1 Main Conclusions of the Doctoral Research

Validation of the Research Instrument – the Clinical Interview

As part of the present research a new research instrument was developed, including a unique, detailed index mapping and assessing the knowledge of 5-year-old children in the three areas studied: length measurement, spatial sense and sense of self-efficacy in geometry. The validity of the research instrument is based on the following criteria:

construct validity, content validity, and internal reliability of the three sub-instruments, which was found to be high.

The findings from the processes of validation and reliability of the research instrument led to the conclusion that it is possible to use the instrument as is, in all its parts, although only the length measurement ability and spatial sense can be examined separately.

This clinical interview was created for pre-school children in Israel, but it can be used in any country that so desires, being adaptable to meet the needs of cultural sensitivity.

Contribution of the intervention program to abilities in length measurement, spatial sense and sense of self-efficacy in geometry among 5-year-old children

The research findings show that the intervention program – based on direct and explicit instruction in a range of activities calling for multistage performance – has a significant effect on the abilities of the children who participated in it.

The conclusion deriving from the research findings is that the intervention program resulted in an improvement in the following indices:

Length measurement abilities: the program built precise layers that teach and consolidate the principles of measurement according to a development sequence, with strict attention paid to teaching the stages characterizing the development of length measurement. The play-based experiential activities in the intervention program advanced and consolidated one stage each time, in accordance with the developmental sequence of measurement, from measurement by means of direct comparison, through measurement with the help of a mediator, to measurement using units of measurement. In the present research not only did the children's abilities improve, they also demonstrated impressive measurement capabilities using units of measurement. Learning in the intervention program allowed the children to focus on the goal facing them during performance of the task, but also to optimally rearrange all that they had learned in the past and would be learning in the future – thanks also to an educational environment that enhanced the children's probabilities of developing an awareness regarding **"intervention in the process and understanding of the product".**

Spatial sense: the intervention program focused on development of spatial sense, including two key components – visualization and orientation. The program advanced and consolidated the children's spatial skills. The research findings show that the

children enriched their storehouse of visual structures amid play-based collaborative learning and active mediation.

Sense of self-efficacy in geometry: no difference was found in the sense of self-efficacy in geometry between the children who participated in the intervention program and those who did not. An examination of the difference between the sense of self-efficacy before performance of the task and during performance of the task in children who took part in the intervention program shows that **a relationship exists between the sense of self-efficacy in geometry and performance.**

Relationship Between Emotion and Learning

Observing and listening to the children's discourse suggested the existence of a close relationship between emotion and learning. The emotions that were expressed during geometric game playing attest to the quality of the learning that the children experienced while participating in the intervention program, this learning being related to the goals they set themselves, their beliefs, their expectations and their predispositions.

4.2 Theoretical, Methodological and Practical Implications

Theoretical Implications

The research supports and reinforces all neo-Piagetian theories (Case, 1985; Fischer, 1980; Hallowell, 2020) and development of quantitative tools for children of pre-school age. The results of the research provide additional evidence in support of the fact that young children can indeed – through partially guided instruction based on a range of relevant activities and provision of appropriate scaffolding (Baroody et al., 2019) – advance the acquisition of skills based on the freeing of space in the working memory. The children acquired diverse skills in length measurement in different contexts with increasing complexity, and these skills were included in other measurement situations, advancing the children to successful measurement using units of measurement (Fischer, 1980). The research fills a gap in knowledge in all matters related to length measurement abilities among young children using units of measurement.

The present study makes a significant contribution to understanding the geometric discourse of young children by means of the four attributes of the commognitive approach (Sfard, 2008). Discourse analysis exposed a detailed profile of rich geometric

engagement in task-oriented ritual discourse routines and product-oriented exploration discourse routines. In-depth observation with purposeful attention to the children's statements allowed the researcher to turn latent knowledge into overt knowledge.

In conclusion, significant insights deriving from the discussion on the research findings – with respect to assessment of the sense of self-efficacy among young children during the performance of geometric activities – reinforce the existing literature and confirm that young children are not able to report consistently on their level of self-efficacy. They tend to rate their ability as high across the board.

An examination of the relationship between the sense of self-efficacy and actual performance enabled a more precise assessment of the sense of self-efficacy on the part of the children who participated in the present research, as manifested authentically and concretely during performance of the activities.

Methodological Implications

A new research instrument was developed in the framework of the present study, including a detailed index mapping and assessing knowledge on the part of the 5-year-old children in the three abilities studied: length measurement, spatial sense and sense of self-efficacy in geometry. The research instrument was subjected to processes of validation and reliability. Through these processes it was found that the clinical interview was validated and reliable, and that it can be adapted and used for different cultures and countries.

The advantage of the instrument lay in the fact that it could be used as is in all its parts. It is simple to implement and can serve as a working tool for researchers in the field. It can also be used as a tool for kindergarten teachers by means of which they can identify children's incipient knowledge in these fields.

It is recommended to make the instrument available for training educators of pre-school children, making it part of their toolkit.

Practical Implications

For the purposes of the present research a special intervention program was developed, including a total of 21 activities involving direct play-based instruction with a view to advancing abilities in length measurement and spatial sense, while reinforcing the sense of self-efficacy in geometry. The program, as presented in the current research, can serve as an operative, implementable and accessible work tool for educators of pre-schoolers, as well as for first and second grade teachers in primary schools. It can also

constitute a milestone in meaningful instruction in other countries and other languages following adaptation to the local culture.

Exposure of the processes required for learning length measurement and developing spatial sense as emerging from this research could serve as a practical recommendation on the national and state level. It is recommended to add to the curriculum the sequence of activities that are responsible for building and consolidating geometric knowledge layer by layer.

4.3 Recommendation for Future Research

Following analysis of the findings and derivation of conclusions, as presented above, the following recommendations are made for further research:

- It would be beneficial to carry out a research study that would validate use of the clinical interview with children of different ages, speaking different languages and belonging to different cultures.
- The present experiment is the first to be performed in implementation of an intervention program. The program for kindergarten children is of extended duration and intensive. It is recommended to conduct further studies in order to create a balance between the possible number of activities to be included – arriving, through fine tuning, at the optimal number that is desirable for building an intervention program suitable for implementation in multiple kindergartens.

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