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ABSTRACT OF THE PhD THESIS

Developing middle school pupils' critical thinking through the study of Mathematics. Valences of problematisation and of learning by discovery

Programme of educational intervention for 6th grade

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ARGUMENT

In a world increasingly dominated by technology and artificial intelligence, everything that cannot be automated is becoming an increasingly valued and necessary skill, as OECD experts point out about critical thinking and creativity (Vincent-Lancrin et al., 2019).

Critical thinking and creativity are between the basic features of graduates, on the horizon of 2030, in the society of the future, as highlighted by all the conceptualizations regarding basic skills in the 21st century.

Either we are talking about the 16 competencies needed in the 21st century described by the World Economic Forum (2016), or we are talking about the Compass of 2030 learning developed by the OECD (2019b), or the empirical delimitation of the necessary competencies (Kaufman, 2013; Kirschner & Stoianov, 2020), to any reference of necessary competencies we refer, critical thinking is listed between them.

We constantly talk about the development of critical thinking in Romanian education throughout the post-1989 period, being elaborated specialized works since the 2000s (Dumitru, 2000).

However, the results of school pupils and students in international tests (PISA or TIMSS) and in the national ones are rather disappointing in terms of demonstrating mastery of this skill, but also of mathematical and scientific skills in general.

And at the last TIMSS test, in 2019, the report on the performance of Romanian pupils and students in this test, recently launched by specialists from the University of Bucharest, shows the very low ranking of Romanian pupils and students in both Mathematics and science (https: // unibuc. ro / testul-timss-2019-pentru-romania-de-la-results-la-recomendari-de-politici-educationale /). The pandemic period we went through highlighted, between other things, the need for scientific literacy skills, critical thinking to cope with the avalanche of fake news. This is why the 2022 PISA assessment prioritises mathematical, scientific literacy and critical thinking skills.

The results presented above made us investigate whether teaching Mathematics using problematization and discovery learning contributes to the development of critical thinking. From the structural point of view, the paper is developed along two sections comprising seven chapters addressing the issue of increasing school performance and developing school pupils' and students' critical thinking by teaching Mathematics to sixth graders.

CHAPTER I

CONSIDERATIONS ON CRITICAL THINKING AND APPROACHES IN ITS DEVELOPMENT

A critical thinker is described as "someone who is curious, open, flexible, and fair in nature, has a desire to be well informed, understands various points of view, and is willing to suspend judgment and consider other perspectives" (Lai, 2011, p. 5). According to Ennis (2011), “Critical thinking is reasonable, and reflective thinking is focused on deciding what to believe or do” (2011, p. 1). Lipman (1998) states that “Critical thinking is a skilled, responsible thinking that facilitates good judgment, because it is based on criteria, self-regenerates and is sensitive to context” (p. 38). For Paul and Elder (2006) “Critical thinking is the art of analysing and evaluating thinking in order to improve it” (p. 4), Willingham (2007) considers critical thinking as something that consists in seeing both sides of a problem, be open to new evidence that refutes certain ideas, reasoning disproportionately [...], drawing conclusions from available facts [and] problem solving ”(p. 8), whereas Dewey refers to critical thinking as“ reflective thinking” and a defines it as “an active, persistent and careful consideration of a belief or a supposed form of knowledge” (Fisher, 2009, p. 2).

„Critical thinking is the use of those cognitive skills or strategies that increase the likelihood of a desired outcome, [...] describes thinking that is intentional, motivated, and goal-oriented - a type of thinking involved in solving problems, formulating conclusions, calculating probabilities, and taking decisions, when the person uses skills that are well thought out and effective for the specific context and type of thinking task. ” (Halpern, 2002, p. 6)

Critical thinking skills are very important in learning Mathematics because these skills can improve the quality of learning Mathematics more deeply and significantly so it should be a systematic way to develop such skills by learning Mathematics in school (Cobb et al., 1992, apud. Firdaus, Ismail Kailani, Md. Nor Bin Bakar, Bakry, 2015).

Critical thinking and analytical skills are essential for most aspects of pupil and student life and beyond. The argument does not mean disagreement; it simply means supporting a point of view. You don't have to be an argumentative person to do this. However, the ability to be honest about one's own prejudices and preconceptions, flexible to consider alternatives and opinions, and willing to reconsider and revise views where honest reflection suggests that change is justified is essential to any analysis. Healthy scepticism must also be cultivated in statements beginning with "It is obvious that ...", arguments that are ungrounded and unbalanced, and arguments that have a particular political, professional, or anecdotal bias (as opposed to the evidence investigated). All these skills are supported by critical thinking, being essential to be not only developed, but also doubled by emotional, attitudinal and moral aspects to be translated into explicit behaviours. The sources of any research / literature that is considered must also be verified (Judge, Jones, McCreery, 2009).

Critical thinking generates new knowledge by capitalizing on a particular way of working with the existing knowledge. It seems to encompass a number of processes such as: understanding, analysis, synthesis, evaluation, etc., called "knowledge manipulation tools" (Moon, 2008). In other words, critical thinking involves accurate and skilful use of these knowledge manipulation tools.

Thus, even in Mathematics classes, for the development of algebraic thinking, Store (2017) uses constructivist theory as a theoretical framework for delimiting methodological approaches that lead to the development of critical thinking, by building new ideas and content, making mathematical connections, launching approaches questionnaires, work space creation and individual and group processing, connecting mathematical ideas from different student approach strategies, verbalizing reasoning, detachment and understanding of generalizations, reverbalization, application of generalizations, etc., thus appealing to more " literatures ", scientific, linguistic, social, numerical, entrepreneurial, etc. We notice how several constituent elements of critical thinking are developed interdependently. Similar investigative approaches, then generalizable, in an attempt to outline the basic explanatory theories (*grounded theory*), claimed to be missing in the explanatory context of the development of critical thinking through Mathematics, were made by other researchers, who tried to introduce various facilitators / conditioning factors for (formative approaches) of critical thinking development, factors ranging from creating socio-emotional contexts to steps aimed at maintaining cognitive consistency and rigor (Firdaus et al., 2015; Savva, 2016; Morgan, 2017; Store, 2017; Malara, Navarra, 2018).

When we talk about critical thinking, we are implicitly talking about a direction, a finality that we want to reach. We think critically to discover something specific, certain information that we had in mind from the beginning. This process is deliberate and active and usually results in judgments that can be reflective (assessing the quality of one's judgment or critical thinking processes). This reflective, transformative process has many connections with the theory of transformative learning (J. Mezirow, St. Brookfield), or with the theory of social criticism (J. Habermass), which, emphasizing the process of individual reflections or the process of reflections on the social context, highlights how processes critical thinking takes place either on their own experiences, reconsidering them, or on social stimuli (Butterworth, Thwaites, 2013; Lai, 2011).

Kaasboll (1998) improved critical thinking through pedagogical methods, such as: reducing the volume of course material, improving teacher training to encourage greater interaction with pupils and students, and using more effective project or inductive approaches, creating the framework of questionnaires, making connections, verbalizations of reasoning, generalizations, etc. (Store, 2018).

Also, modern approaches increasingly insist on didactic contexts that facilitate the development of critical thinking that take into account the social and emotional dimension, that ease the transfer of knowledge and reasoning in various contexts, as close to reality, that facilitates the joy of discovery, discussion collaborative and co-creation etc. (Stearns, 2020; Vincent-Lancrin, 2019; Store, 2018; Van Zoest et al.), also suggesting the transfer of the principles of gamification in the teaching of Mathematics in a way to stimulate the development of critical thinking (Yong et al., 2020).

The habit of asking (oneself): “can I solve the problem differently”, more “elegantly”, “in a shorter manner”, ”how can I use what I learned to solve it”, ”where did I get stuck”, “did I identify correctly all the data of the problem” etc., can be constantly encouraged, offering students dexterity in using comprehension questions, connection questions, strategy or reflection questions (Mevarech, Kramarksy, 2014; Ionescu, Bocoș, 2017).

Other studies, such as those of Noer (2013) and Sharadqah (2014) indicate that the critical thinking ability of eighth graders is not optimal, as only a few pupils (less than 15%) who are able to solve various academic tasks, become able to identify the given hypothesis, have the ability to formulate the main problems and the ability to determine the consequences of the decision taken.

Only 5% of students who are successful in solving different academic tasks given by the teacher have the ability to detect prejudices based on different points of view, the ability to express a concept / definition or theorem in solving the problem, the ability to evaluate the relevant argument in the solution to the problem (Umar, 2017).

At the same time, the results of Harti and Agoestanto's research showed that school pupils’ and students' ability to think critically in problem-based learning met the minimum learning criteria, especially in algebra. Pupils and students who belong to the group of children with high critical thinking skills have high globalization abilities, while those who belong to the group of children with medium critical thinking skills have generalization, transformation abilities that tend to be moderate. Pupils and students who belong to the group of children with low critical thinking skills have low generalization and transformation abilities (Harti, & Agoestanto, 2019).

Despite what one might believe, it seems that critical thinking is a greater predictor of life events than intelligence and as there is ample evidence that critical thinking can be taught, there is hope that teaching critical thinking skills could prevent the occurrence of negative life events. Thus, he advocates the acquisition of critical thinking as a way to create a better future for everyone (Butler, Pentoney, & Bong, 2017).

Critical thinking ability is the cognitive ability that always gets the centre of attention and is studied in research alongside the affective aspect, or other aspects such as the mathematical habits of the mind, which also begin to be studied by researchers because it is expected to stimulate students ’creativity and interest, as well as their positive attitude towards Mathematics (Umar, 2017).

Critical thinking is not only a necessary skill for students, but also for teachers, so there have been studies that have sought to demonstrate this.

In a study on physics students, in order to find out what effect teaching by methods involving scientific creativity has, the new contents were taught using the method of learning by discovery. The results showed that the learning model through the learning through discovery process had a significant effect on improving the capacity for critical thinking (Verawati, Sri, Ayub, & Prayogi, 2019).

CHAPTER II

DEVELOPMENT OF CRITICAL THINKING IN ROMANIAN MIDDLE SCHOOL MATHEMATICS TRAINING

From the perspective of concerns for the development of critical thinking, once the conceptualizations in the curricular reform had explicit formulations regarding the development of critical thinking and in the implementation of the reform and its implementation in the classroom, important developments took place. Teacher training has covered skills in the direction of developing critical thinking, books and specialized guides have appeared (Dumitru, 2000, Bocoș, 2013), most universities introducing this issue since the initial training of teachers. Unfortunately, more extensive changes were made at the beginning of schooling, and at the end of schooling, and the middle school cycle was often "left aside", becoming, over time, the "weak link", increasingly blamed in terms of performance, reflected in the school results of trainees, especially in those highlighted by the national assessment and in PISA tests (Kitchen, 2017; OECD, 2019).

After the curricular reorganization in 2009 (MECI, 2009), a new important stage in the reconsideration and restructuring and updating of the curriculum in the middle school took place in 2017 (MEC, 2017). Extensive curricular changes over a generation are needed not only due to contextual issues (e.g. unsatisfactory PISA test results), but also due to wider societal changes, new (proficiency) profiles that postmodern society demands, of some evolutions in science, technology, social, which presuppose wider curricular and educational reforms (Stan, 2001; Ungureanu, 1999; Ionescu, Bocoș, 2009, 2017).

Many of thorny aspects of the educational environment, of the teacher-student relationship, of the way of teaching and assessment, with effects on the stimulation of learning, were also highlighted by OECD experts (Kitchen et al., 2017).

As shown in the OECD report (Mevarech, Kramarski, 2014), in teaching Mathematics for today's and tomorrow's society we must increasingly shift the focus from solving routine problems to "solving complex, non-routine, unfamiliar problems, which it is based not on the reproduction of a memorized solving algorithm, but on the stimulation of logic, reasoning, deduction and intuition, on numerical sense and inferences, as innovative societies require creativity in Mathematics as well as in other fields. The approach to communication in Mathematics has changed as well, with students being encouraged to get involved in discussions, to share possible solutions and ideas, explaining their mathematical reasoning ”(Mevarech, Kramarski, 2014). This social, collaborative approach is also meant to stimulate metacognition, Mathematics being among the most important subjects of the curriculum for the development of metacognition, as the invoked study argues.

Similarly, the contribution of Mathematics to the development of critical thinking is self-explanatory by the direct link between the way in which aspects that are constituent elements of critical thinking are pursued in solving mathematical problems, as Butterworth, Thwaites (2013) revealed.

Therefore, the development of the ability to reason, to make predictions for controlled situations, the ability to select information and process data, to distinguish records, to make value judgments and to make correlations between the available data, to distinguish information of certainties, of reasoning, of making generalizations, of delimiting the balance of probability, etc. are skills specific to critical thinking (Butterworth, Thwaites, 2013), but which we aim to develop through mathematical activities, with a wide extension.

In recent years, at least in the last decade, PISA tests have become a phenomenon increasingly discussed in the Romanian public space in relation to the performance of the educational system. Basic skills are measured, namely literacy, numeracy, science, etc., all measured from the perspective of life skills, the items being formulated in such a way as to identify pupils' and students' abilities, to transfer in life situations what they learned in school.

The similarity of the items in the international PISA assessment tests with that of the critical thinking measurement items, such as Watson-Glaser, made us use the Watson-Glaser test (Goodwin, Glaser, 2002) in conducting the investigative study, as highlighted to the practical part of the thesis.

The results of Romanian students at the PISA tests should not be seen in the light of the fact that almost half of high school children do not know how to write, read, or count, as the test results show, but that they cannot actively use the knowledge they have, they cannot transfer what they have learned into life situations (Kitchen et al., 2017; OECD, 2019). This reality is all the more indicative of the importance to develop critical thinking with a much wider extension in school, to develop in students the ability to process information multidimensional, to make conceptual or property analogies (Magdaș, 2015), transferring what is learned in various contexts or to other disciplines, transdisciplinary.

Progressively designed to prepare students in an integrative, transdisciplinary manner, it has led to many reconsiderations to be implemented in the curriculum. Thus, the intermediate testing at the level of the sixth grade is designed for curricular, transdisciplinary areas (Bocoș, Avram, 2016).

Unfortunately, the necessary consistency is not applied in the design of the national assessment at the end of the eighth grade, which led teachers to grant less importance to intermediate assessment and its predictive capacity for improvement and integrated teaching, but to guide pupils' how to prepare for the national evaluation, which focuses more on disciplinary, reproductive aspects (Kitchen et al., 2017).

Of course, it is not only about cognitive performance, but also attitudinal and emotional performance, respectively, being worrying that students say they do not feel good at school, that they are unhappy or have a low ability to aspire, have low confidence in their own forces (OECD, 2019). These aspects reveal the quality of the didactic interactions, teachers' abilities to teach attractively, to create a pleasant climate in the classroom, to evaluate affirmatively and formatively, stimulating the school pupils' and students' self-confidence and self-esteem. This situation is often found in the case of Mathematics teachers, more oriented towards cognitive performance, respectively less towards stimulating the confidence to think "out of the box", creatively, to dare to solve problems in other ways than the taught algorithms, applying reasoning. problem solving to demonstrate a deep understanding of their explanatory and predictive capacity (Pasamentier, Krulik, 2009; Morgan, 2016; Wright, 2017; Malara, Navarra, 2018).

We consider that Mathematics taught through the use of active-participatory methods offers the satisfaction of discovery, the school pupils or students will not even be aware they have learned something at school, but that they discovered something they knew earlier. This more formative approach to teaching Mathematics, which also stimulates the satisfaction of discovery, the confidence to approach a problem solving differently, the ability to process what is learned and to transfer to different other contexts, to imagine solutions, etc., demonstrated by specialized research as more efficient (Cai, Leikin, 2020; Christiani, Siagian, Mukhtar, 2020; Steams, 2020; Chin et al., 2019; Fonseca, Arezes, 2017; Wilder, 2015), unfortunately contrasts with the current teaching practices.

"The purpose of developing thinking skills is to have a quality of thinking, as the quality of thinking is necessary not only in school but also outside of school" (McGregor, 2007). Thinking skills are related to pupils' and students' ability to understand the thinking process when studying the content of the subject (Swartz, 2001). Therefore, trainees must not only understand the content of Mathematics, but also the process of mathematical thinking (Cobb et al., 1992). Many educators argue that thinking skills can be learned and should be taught explicitly, and students should be informed about the types of thinking skills that are taught to them (Swartz, 2001; McGregor, 2007). Research shows that students' thinking skills can be developed if teachers create a classroom environment that supports thinking activities (Swartz and Parks, 1994; Mason, Burton, & Stacey, 2010). Teachers do not necessarily dominate and control learning activities, but should encourage trainees to play an active role and demonstrate good multilateral interaction between teacher and student (Firdaus, Ismail Kailani, Md. Nor Bin Bakar, Bakry, 2015).

Internationally, efforts to develop critical thinking skills in Mathematics have become the main agenda in the global Mathematics curriculum (NCTM, 2000; Mason, Burton, & Stacey, 2010; Innabi and Sheikh, 2006). Many researchers have shown that developing critical thinking skills can improve Maths achievement (NCTM, 2000; Silver & Kenney, 1995; Semerci, 2005; Jacob, 2012; Chukwuyenum, 2013). Similarly, critical thinking skills will encourage students to think independently and solve problems in school or in the context of daily life (NCTM, 2000; Jacob, 2012).

CHAPTER III

WAYS TO CAPITALIZE ON THE DIDACTIC METHODOLOGY IN ORDER TO DEVELOP THE CRITICAL THINKING OF MIDDLE SCHOOL PUPILS THROUGH THE STUDY OF MATHEMATICS

The diversity of learning situations requires a diversity of didactic approaches, i.e. of the ways in which the training methods can be applied and combined. We do not refer, from this perspective, only to the way in which one didactic strategy or another is resorted to, as a way of combining methods, didactic means and forms of organizing learning, but, rather, we are talking about the new trends outlined to delimit approaches teaching (*signature pedagogies*) as broader approaches that guide one approach or another of the teaching process (Vincent-Lancrin et al., 2019). Thus, according to OECD specialists, didactic approaches such as *design-thinking* teaching, dialogic teaching, metacognitive approach, project-based learning, research-based learning, etc., are just some of the didactic approaches designed to develop critical thinking and creativity, as empirical evidence has shown.

Similar approaches, which seek to stir the interest and involvement of pupils and students, to create learning situations as close as possible to life situations, which can be solved using even transdisciplinary knowledge, taking the form of projects or just complex problems, were presented. by various researchers, who highlighted these aspects regarding their valences and methodological suggestions (Abbott, 2017; Hackenberg, Creager & Eker, 2020). The solutions mentioned exploit both interdisciplinary approaches in the teaching of Mathematics and science, integrated approaches that highlight the way in which several basic competencies can be addressed, but they forward, on the other hand, suggestions to avoid disengaging pupils and students to help them understand the beauty and usefulness of Mathematics in life situations, to form a structured, logical, mathematical approach (*a mindset*) in understanding, analysing and anchoring in personalized contexts (Høgheim & Reber, 2017; Jack, B.M. & Huann-shyang Lin, 2017).

We will further present the advantages of using problem-solving methods and learning by discovery in order to develop school pupils' critical thinking as well as the advantages of using complementary methods of developing critical thinking: demonstration, algorithmization and solving exercises and problems.

By problematization the pupil is removed from the situation of receiving knowledge already systematized by the teacher and put in a position to find the solution of a problem alone. This method involves a total intellectual, emotional and volitional commitment on the part of the student. As Albu E. et al. put it (2004), "problematization does not seek to acquire as much knowledge as possible, but the formation of an individual style of work in conditions of mental tension, stimulating the spirit of investigation and courage in arguing and supporting personal opinions" (p. 86).

If the "optimal level of activation and stimulation" mentioned above is reached, the problematization stimulates the intrinsic motivation of the student for learning. Problem situations, through their atypical character, sometimes even contradictory, activate the student's curiosity and as a result support the student's effort to seek solutions (Ionescu, Bocoș, 2009, 2017). The joy of discovering the solution also contributes to strengthening the intellectual and behavioural attitude, specific to the research process in which the student has just become involved.

Due to the active involvement of the student in the learning process, learning by discovery leads to a deep and thorough knowledge and understanding and to a faster consolidation. The road to finding the solution requires the ability to transfer knowledge, perseverance and independent spirit. Due to the relatively high effort required by the student, the discovery supports intellectual development and even increases confidence in their own resources. The method offers the possibility of self-knowledge and self-control, stimulating interest in research and learning (Ardelean, Secelean, 2007, p. 111).

J. S. Bruner emphasizes the idea that the method of discovery contributes to the transformation of extrinsic motivation, based on rewards and punishments, into an intrinsic motivation, based on curiosity, desire for competence and satisfaction generated by discovering the solution itself (Albu, Silvaș, Filpișan, 2004, p. 88).

By using mathematical demonstration, pupils develop the ability to make thoughtful and reasoned decisions, not just based on intuition. In order to favour the development of critical thinking, it is important that even in mathematical demonstrations we persevere in asking pupils questions so that they can argue every relationship obtained, to coordinate them in making their own decisions to solve. Dewey emphasizes the rationality, the foundation and the evaluation of the rational approach, the trained rationality being the key of the critical thinking in his vision (Dewey, 1909, apud Fisher, 2009).

By using algorithms, pupils and students develop critical thinking both in the stage of discovering the steps of the algorithm to be applied, but especially in the stage of arguing the efficiency of the algorithm to the detriment of discovering their own methods of solving, when students develop the ability to take into account other perspectives. If we only apply algorithms, students are inhibited from the ability to make thoughtful decisions, to be curious, to ask questions, to gather relevant information and to analyse them, so it is important not only to apply algorithms but to we emphasize the argumentation of the stages of these algorithms and their optimization.

The method of exercises and problem solving contributes to the development of pupils' and students' critical thinking, because in order to solve a problem they need to combine previously learned rules and make their own solving strategy. However, this method does not develop their ability to make their own judgments, to take into account other perspectives.

The development of critical thinking and creativity are desiderata accepted by everyone as necessary to be built in school, conceptualized as such in the curricular documents.

However, it is more difficult for teachers to decode the way in which they can be trained concretely, through classroom teaching. This is the reason why OECD specialists have made available to teachers lesson plans, concrete examples of design, implementation and evaluation of the teaching approach, bringing together over 100 such examples from around the world (Vincent-Lancrin, et al., 2019).

Other concerns were linked either to determining the factors and contextual conditions for reducing anxiety about Mathematics (Arlsan, 2020; Awofala, 2019; Zsoldos-Marchiș, 2013), or developing and validating measuring instruments (Chin et al., 2019), or piloting and experimental modelling, to indicate ways to stimulate self-confidence in relation to Mathematics, motivation for Mathematics, for formulating and solving problems using mathematical thinking in various formats and life contexts (Bonner, 2013 ; Van Zoest et al., 2017; Voica, Singer, Stan, 2020).

A more focused body of research addresses the stimulation of critical thinking (*Fonseca, Arezes*, 2017), either by studying algebra or by studying geometry, using specific methodological approaches, such as collaborative activities, for example, for the development of metacognition (Smith, Mancy, 2018), autonomy (Marchiș, Balogh, 2010; Zsoldos-Marchiș, 2014), discovery learning (Batubara, 2019; Christiani, Pargaulan Siagian and Mukhtar, 2020), training differentiation (Hackenberg, Creager & Eker, 2020), or creativity (Bicer et al., 2020, Sanders, 2016; Bădescu, 2011), or for curricular conceptualization in order to develop disciplinary and transdisciplinary critical thinking, while evaluating in an integrative, transdisciplinary manner (McGuinnes et al., 2003; Bocoș, Avram, 2016) etc.

We will also present the results of an action research conducted in Turkey (Dolapcioglu, & Doğanay, 2020), on 5th graders where the way in which problem-solving and critical thinking skills can be developed has been identified.

According to the reports of Dolapcioglu & Doğanay (2020), the results of the research showed that practices based on authentic learning standards have led to improved features of critical thinking, such as comprehension, ability to compare, evaluate solutions, argue, offer new solutions, to reflect on the process of solving the problem.

Authentic learning was much easier when relevant problems were formulated for the pupils to solve from the proximity of the living environment, developing more easily the critical thinking skills. These results are also confirmed by my teaching practice (see page 61 where the example of a problem related to a trip to Timisoara is presented)

From another perspective, in his experimental research on teaching geometry by drawing several different geometric figures to investigate whether a statement can be shown to be valid or not, the Japanese researcher Komatsu (2017) highlights how such an approach facilitates the development of critical thinking, by using empirical examination both before and after the demonstration.

CHAPTER IV

GENERAL COORDINATES OF THE RESEARCH

In order to successfully integrate into a changing world, we all need the ability to select information and understand the correlation between them, to decide what is or is not important, to place ideas and knowledge in different contexts. us, to discover the essence of the things we encounter for the first time, so we must be able to give critical, creative and productive meaning to the information we face.

The topic of our research is part of a complex topical issue, with accentuated interdisciplinary valences.

Thus, through our research, we set out to reconfigure the way in which Mathematics is taught in middle school, by elaborating a methodological system called *CriticMath* and by creating a teaching auxiliary, contributing, we hope, to improving the school performance of students. We try to offer teachers support in carrying out activities that train students as autonomous people, able to set their own priorities and goals, with confidence in their own ability to reason, open to points of view. divergent world, flexible in considering alternatives and opinions. It is necessary to help them to be impartial in assessing a situation, honest in the face of their own prejudices, to eliminate their stereotypes of thinking, but especially with a willingness to reconsider and revise their views, if sincere reflection suggests that the change is justified.

Through the investigative approach we sought to train Mathematics teachers to support the development of critical thinking, and empirical evidence on assessing the impact of training shows that it is an approach worth multiplying and capitalizing on Romanian education, with effects on student performance and changes in the level of teaching practices and attitudes of teachers alike (Bădescu, Stan, 2019, 2020).

Since we set out to introduce a change, to show with empirical evidence that Maths classes can be done differently, with better formative effects, in order to develop critical thinking at the same time, the most appropriate investigative design we considered to be an experimental one, process-oriented (Bordens, Abbott, 2017).

The aim of the research is to identify the extent to which the use of problematization and learning by discovery in teaching Mathematics to sixth graders contributes to the development of their critical thinking, respectively leads to improving students' school performance.

In accordance with the stated purpose, the research aims to achieve the following objectives:

O1: establishing the causes that lead to learning difficulties of mathematical notions, as well as the criteria according to which students are guided in learning;

O2: inventorying the way in which teachers view the importance of developing pupils' critical thinking;

O3: finding out the opinion of Mathematics teachers in connection with the main aspects of teaching and learning Mathematics, especially regarding the didactic methodology used;

O4: designing and carrying out the training programme *Development of critical thinking by teaching Mathematics to the sixth grade*, Programme for teaching Mathematics in order to develop students' critical thinking;

O5: designing new contents and strategies for teaching-learning heuristics of the sixth grade subject, of a methodological system called *CriticMath*;

O6: analysing the impact of the training programme, in the short and long term, on the direct beneficiaries (Mathematics teachers) and indirect (their pupils).

The research was carried out for well-defined periods, during four school years, permanently pursuing the purpose of the research, the fulfilment of the proposed objectives and the verification of the hypotheses issued.

According to the research logic of the mixed methodology approach (Muijs, 2004; Creswell, 2018), we delimited questions corresponding to each stage of implementation of the study (pre-experimental stage, experimental stage and post-experimental stage), both with possible answers, in the form assumptions.

Starting from the problems identified following the analysis of the curriculum of the mathematical discipline of middle school, we formulated the following **research hypotheses:**

Hypothesis 1. Teaching Mathematics to sixth graders using an educational programme based on learning through discovery and problematization contributes to improving students' school performance in Mathematics.

Hypothesis 2. Teaching Mathematics to sixth graders using an educational programme based on learning through discovery and problematization contributes to the development of critical thinking of students.

Hypothesis 3. Teaching Mathematics to sixth graders using an educational programme based on learning through discovery and problematization contributes to the change of teachers' perceptions of the importance of critical thinking..

Independent research variable:

Teaching Mathematics to sixth graders through the method of problematization and learning through discovery.

Research dependent variables:

V.D. 1: the level of school performance in Mathematics.

V.D. 2: the degree of development of pupils' critical thinking.

V.D. 3: teachers' perception of the importance of critical thinking.

Thus, we set out to involve a total population of **668 pupils** studying in Caraş-Severin County and 83 teachers, who teach these pupils..

The research took place over four years, during the school years 2015-2016, 2016-2017, 2017-2018 and 2018-2019, respectively. During this period, the activities were carried out according to the curricula in force and in accordance with the plans of the teachers from the respective forms of pupils.

The content sample is made with:

- the contents taught during the experiment;

- the content of the training course *Development of students' critical thinking by teaching Mathematics to sixth graders*;

- the content of the tests applied during this period.

The contents taught during the experiment are in accordance with the curriculum in force for the Mathematics discipline, approved by the Order of the Minister of Education and Research no. 5097 of 9.09.2009.

The choice of these thematic units was based on the following considerations: the objectives pursued, the competencies we want to form, the recommendations from the curriculum, the representativeness of these chapters for the study of Mathematics, and as an example, we will present some situations.

The thematic and methodological content of the training course is based on a bibliography made up of works by the authors: Banea Horia, Berinde Vasile, Brânzei Dan, Cârjan Florin., Lupu Costică, Săvulescu Dumitru, Rus Ileana.

In this course, each thematic content was designed to be taught using the *CriticMath* methodological system. (Bădescu, 2016). This methodological system is designed by the author of the research and consists of the following:

- a real problem arises, for the solution of which the content to be taught will be used;

- a sketch of the described problem is drawn;

- that problem is transposed into mathematical relations;

- the content is problematized;

- the solving algorithms are discovered;

- the algorithms are applied in simple situations;

- the application of algorithms is problematized, those algorithms are modified to solve new problems;

- nonstandard problems are solved.

These contents of the advanced training course approached through the prism of the *CriticMath* methodological system were published in the brochure Discovering sixth grade Mathematics, Graph Publishing House, 2016.

Regarding the sample content of the tests, three tests were applied during the first semester, in the school year 2016-2017, in accordance with the school curriculum, tests aimed at establishing the school pupils' level of school performance. Also during this period, a test was applied that measured the level of development of students' critical thinking.

In order to carry out the test subjects, the measurement of the pupils' school performances, the works of the authors were consulted: Bălăucă Artur, Bădescu Ovidiu, Dăncilă Ioan, Gheorghe Turcitu, Zaharia Maria.

In order to achieve the topics of the Neutrino Contest for measuring the level of critical thinking of students, the works of the following authors were consulted: Linț Maranda, Bruck Jurgen, Havas Harald, Weber Ken but also the brochures with the subjects given at the LUMINA MATH National Mathematics Contest in 2014, 2015, 2016.

None of the methods used, no matter how complex and elaborate, would have been sufficient on their own to create the entire data table needed, so we resorted to a system of methods that, acting synergistically, contributed to the construction of a clear image of the current situation: *Method of psycho-pedagogical experiment, method of direct observation, study of curricular documents and other school documents, questionnaire-based survey, method of conversations, docimological tests, administered as written assessment tests*.

CHAPTER V

STAGES OF THE EXPERIMENTAL RESEARCH

The investigative approaches carried out by the author integrated three stages: the pre-experimental stage, the experimental stage and the post-experimental stage..

Pre-experimental stage

The purpose of the pre-experimental stage is to investigate and measure pupils' motivation to teach and learn Mathematics, to identify students' initial level of mathematical knowledge and the ability to apply this knowledge to solve Maths problems, to form the experimental group and the Control.

The division into the two samples, the experimental and the control was made so that the number of pupils in the two samples is the same and the averages from the pre-test are close, in our case the average of the control group was 5 , 27, and of the experimental one of 5.23.

At the Neutrino Contest, edition I, stage 2, where the pupils from the two samples participated, the average score of the two groups is close, of 55.50 in the experimental group and 53.16 in the control group.

The experimental stage

The formative stage of the research consisted in the elaboration and implementation of the support curriculum developed according to the CriticMath methodology, focused on improving school performance in Mathematics and the development of critical thinking of sixth grade students by teaching Mathematics.

The support curriculum was implemented among the experimental samples during the first semester of the 2016-2017 school year.

The implementation was done by the teachers who teach Mathematics to the classes of pupils from the experimental samples.

This stage took place between September 2016 and February 2017, the teachers of the experimental group taught using the ideas from the brochure “Discovering sixth grade Mathematics”, and during this period the tests took place:

- test 1, October 2016: test on the contents of the sixth grade curriculum: The set of natural numbers. The straight line

- test 2, December 2016: test on the contents of the sixth grade curriculum: The set of positive rational numbers. Angles

- test 3, January 2016: test on the contents of the sixth grade curriculum: The set of natural numbers. The set of positive rational numbers. The straight line. Angles. Congruence of triangles.

The subjects of these tests are adapted to the requirements of the school curriculum and the test results are used to measure students' school performance.

To measure the level of development of critical thinking of pupils we used the results of the Neutrino contest, of 9.12.2016.

The purpose of the experimental phase of the research was to study whether teaching Mathematics using problematization and learning by discovery contributes to the improvement of school performance among pupils' in the experimental group and to development of their critical thinking.

At this stage also a questionnaire was applied in order to investigate the change in attitude towards the importance of critical thinking of teachers participating in the training course *Development of critical thinking by teaching Mathematics to sixth graders*. This questionnaire was filled in both at the beginning of the training course and at the end of the course, to see the changes. The questionnaire can be consulted in *Annex 7* and by applying this questionnaire we pursued the following objectives:

- establishing the level of importance of critical thinking;
- the manner in which problematization and discovery learning are used

All teachers in the teaching experiment, both those in the experimental group and those in the control group, were given a questionnaire in May 2016 and the same questionnaire on September 14, 2016, when the teachers in the experimental group completed the training course *Development of critical thinking by teaching Mathematics to sixth graders.*

The 42 teachers who attended the training course *Development of critical thinking by teaching Mathematics to sixth graders* were applied a questionnaire in order to discover their attitude towards the importance of developing critical thinking, a questionnaire applied both before the start of the training course and after its completion. As a result, at the beginning of this course, only 4 teachers said that critical thinking is important, 31 teachers said it was not important, and the remaining 7 teachers did not know how to give an answer. At the end of this training, the weight was totally changed: we have 32 teachers who consider it important, 8 who consider it not important, and the number of those who could not give an answer is 2.

Also in this stage of the research, 3 tests were applied to measure school performance in Mathematics, both to the pupils in the experimental group and to those in the control group, whose results we present below. Their presentation will be made schematically, and we will return with the interpretation of the SPSS results in the next chapter.

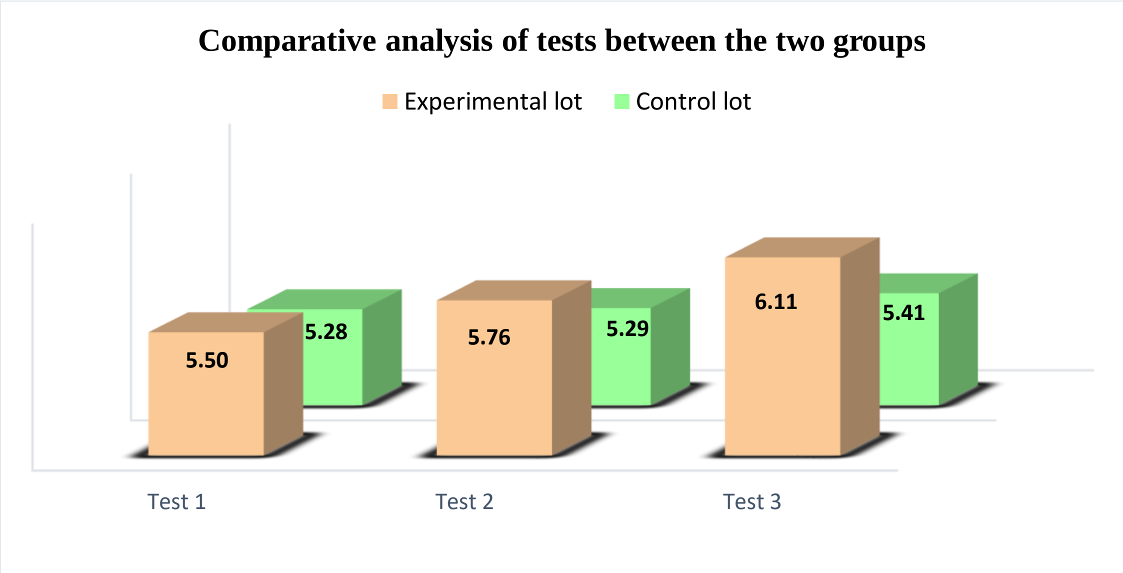

Figure V.1. Comparative analysis of the average grades of the 3 tests for measuring the school performance of the students from the two groups

The average grade in test 1 for measuring school performance is 5.50 in the experimental group and 5.28 in the control group, in test 2 it is 5.76 in the experimental group and 5.29 in the control group, and in test 3 for measuring school performance it is 6.11 in the experimental group and 5.41 in the control group.

Also in the experimental stage we applied the second test to measure the level of critical thinking of pupils in the two samples, intending to detect whether teaching based on problematization and learning by discovery to the pupils in the experimental group leads to the development of critical thinking of the pupils in this group.

In this competition for measuring the level of critical thinking of students (second edition, stage 1), the average score of the pupils in the experimental group is 69.69, by 17.26 higher than that of the control group.

Post-experimental stage

In the post-experimental stage, the post-test was administered both to the pupils in the experimental group and to those in the control group, three months after the end of the intervention (*Annex 16*).

Also in this stage we want to see if the performance of the pupils from the experimental group is significantly superior to that of the pupils from the control group, this being reflected in the results obtained at the National Evaluation exam at Mathematics, test that took place at the end of the 8th grade, namely 2 years and 3 months from the end of our intervention.

In order to verify the stability over time of pupils' critical thinking skills, we compare the results of the experimental group with those of the control group at the second stage of the Neutrino Contest, held in May 2017. We shall see that the change produced is not only long-term and does not decrease over time, but also that the intervention has created a number of critical thinking skills that pupils continue to use to improve their performance, although the intervention has ended.

The purpose of the post-experimental stage of the research was to verify both the stability of pupils' school performance and the maintenance of critical thinking.

Three months after the end of the intervention, in order to verify the stability of the skills developed in the pupils from the experimental group, a post-test was applied and the results show that the average grade of the pupils in the experimental group is 6.50 and that of the pupils in the control group is 5.49.

At a distance of 2 years and 3 months from the end of the intervention, at the national evaluation exam in June 2019, the average grade of the pupils from the experimental group was 7.35 compared to 6.50 which is the average grade of the same group in the post-test.

At the Neutrino competition, a competition for measuring the level of critical thinking of pupils (2nd edition, stage 2), the average score of the pupils in the experimental group is 68.09 compared to the average score of the pupils in the control group, which is 52.11.

CHAPTER VI

RESEARCH RESULTS: DATA ANALYSIS, PRESENTATION AND INTERPRETATION

The verification of the research hypotheses was performed with the SPSS statistical programme with which the pair *t t*ests were run, in order to be able to ascertain whether the obvious differences between the results that the pupils obtained in the evaluation tests are statistically significant. Then bifactorial mixed Variance Analysis (ANOVA) was used, with *Time* factor (initial test, test 1, test 2, test 3) and *Group* factor (Experimental, Control) (Mauchy, 1940) with Greenhouse-Geisser corrections (Greenhouse & Geisser, 1959) and *t* tests for independent samples to compare whether there are significant differences between the performance of the experimental group and the control group during and at the end of the intervention.

The descriptive analyses of the research and the analysis of the data collected through the questionnaire applied to the teachers involved in both the experimental study and the control group, were verified using the Word Excel programme. The results were analysed by identifying the proportions of teachers' answers to the questions in the questionnaire and were represented by diagrams..

The testing of Hypothesis 1, i.e. *Teaching Mathematics to sixth graders using an educational programme based on learning by discovery and problematization contributes to improving pupils' school performance in Mathematics* was performed by applying the *t-pair* test.

Next, in Table VI.2, we remark that the average of the grades obtained by the pupils participating in the experimental study is higher in test 1 (M = 5.49, DS = 2.07), than the average of the grades obtained by the same students in the initial test (M = 5.23, DS = 2.09), statistically significant difference for t (333) = - 8.82, p <.001.

The comparison between test 2 (M = 5.75, DS = 2.10) and test 1 (M = 5.49, DS = 2.07) indicates a statistically significant increase for $t_{(333)}$=-9.87, p<.001.

Also, the increase of the students' averages in the tests, due to the proposed intervention, is also valid between test 2 (M = 5.75, DS = 2.10) and test 3 (M = 6.10, DS = 2.06), for $t_{(333)}$=-16.78, p<.001.

The analysis of the data for the control group does not show a statistically significant increase between the initial test (M = 5.26, DS = 2.07) and test 1 (M = 5.28, DS = 2.04), nor between test 1 and test 2, where the means are identical. (M = 5.28, DS = 2.04 / 2.05), but shows a statistically significant increase between test 2 (M = 5.28, DS = 2.05) and test 3 (M = 5.40, DS = 2.07), for $t_{(333)}$=-5.27, p<.001.

Table VI.2. Average grades at the evaluation tests of the pupils participating in the study (N=334)

Group			Average grade	No. of participants	Standard deviations	Standard error
Control	Pair 1	Initial_test	5.26	334	2.07	.11
		Test_1	5.28	334	2.04	.11
	Pair 2	Test_1	5.28	334	2.04	.11
		Test_2	5.28	334	2.05	.11
	Pair 3	Test_2	5.28	334	2.05	.11
		Test_3	5.40	334	2.07	.11
Experimental	Pair 1	Initial_test	5.23	334	2.09	.11
		Test_1	5.49	334	2.07	.11
	Pair 2	Test_1	5.49	334	2.07	.11
		Test_2	5.75	334	2.10	.11
	Pair 3	Test_2	5.75	334	2.10	.11
		Test_3	6.10	334	2.06	.11

Table VI.3. Differences between the results of the assessment tests of the participating pupils

Group			Differences between pairs					t	df	P
						95% trust interval of difference between means				
			Mean	Standard deviation	Standard error	Low level	High level			
control	Pair 1	Initial_test - Test_1	-.01	.60	.03	-.08	.04	-.49	333	.618
	Pair 2	Test_1 - Test_2	-.00	.44	.02	-.04	.04	-.07	333	.941
	Pair 3	Test_2 - Test_3	-.12	.41	.02	-.16	-.07	-5.27	333	.000
experimental	Pair 1	Initial_test - Test_1	-.26	.55	.03	-.32	-.20	-8.82	333	.000
	Pair 2	Test_1 - Test_2	-.25	.47	.02	-.31	-.20	-9.87	333	.000
	Pair 3	Test_2 - Test_3	-.35	.38	.02	-.39	-.31	-16.78	333	.000

The data obtained from the statistical analyses continue to verify the results with the ANOVA analysis presented in Table VI.4 (Within Effects) and Table VI.2 (Between Effects). For the Within effects, the assumption of sphericity was not observed, as shown by Mauchy's test (.67, p <.001) - as a result, we report these effects containing Greenhouse-

Geisser corrections. As can be seen, we obtained a significant main effect of the time variable, ($F_{(2.39,\ 1572.55)}$=198.8, p<.001, η^2=.20), which indicates that there are differences between test times. The time * group interaction is also significant, ($F_{(2.39,\ 1572.55)}$= 129.4, p<.001, η^2=.13), indicating that there are differences between the two groups at different moments in time.

Table VI.4. Maths performance results, Within effects.

	Sphericity correction	Sum of squares	df	Mean of squares	F	p	η^2
Time	Greenhouse-Geisser	86.78	2.39	36.25	198.8	.001	0.20
Time ✻GROUP	Greenhouse-Geisser	56.50	2.39	23.60	129.4	.001	0.13
Residual	Greenhouse-Geisser	286.76	1572.55	0.18			

Regarding the Between effects related to the results indicating the mathematical performance of the pupils, the main effect of the group factor is also significant, (F (1,659) = 5.31, p = .021, η^2 = .008), so the two groups have significantly different performance (table VI.5).

Table VI.5. Maths performance results, Between effects.

	Sum of squares	df	Mean of squares	F	p	η^2
GROUP	90.27	1	90.27	5.31	.021	.008
Residual	11166.01	657	17.00			.008

We used *t* tests for independent samples in the following sections to see if the control and experimental groups are already different along the way, at the time of the initial test and test 1, but also at the end of the intervention, at the time of test 3. The results can be seen in Table VI.6.

Table VI.6. Math performance results, independent samples t tests

	t	df	p
Five_months_T1	-1.52	657.0	.127
Seven_months_T2	-2.90	657.0	.004
Posttest_8 months_T3	-4.77	657.0	.001

We can remark a significant difference between the two groups already during the test, at the time of test 1, after 7 months of intervention ($t_{(657)}$=-2.90, p=.004), which confirms our Hypothesis 1.

The validation in time of the intervention programme implies the analysis of the differences in time at the level of school performances and after a period of 3 months from the end of the intervention but also at two and a half years after its completion.

The analysis of the differences in time at the level of school performance and after a period of 3 months from the end of the intervention involves an analysis that performed the pair t test and the results show a statistically significant increase, t (333) = - 12.86, p <.001 (table VI.8), so that the test results at 3 months after the end of the intervention are higher (M = 6.49, DS = 1.98) than those at the end of the intervention (M = 6.10, DS = 2.06) (Table VI.7) . The same statistically significant increase is also observed for the control group, t (333) = - 3.65, p <.001, where the results from test 3 are lower (M = 5.40, DS = 2.07) than in the test from 3 months after completion of the intervention (M = 5.49, DS = 2.16).

Table VI.7. The averages of the evaluation tests of the pupils participating in the study after the completion of the intervention

Group			Mean	No. of participants	Standard deviation	Standard error
control	Pair 1	Test_3	5.40	334	2.07	.11
		Test_after_3_months	5.49	334	2.16	.11
experimental	Pair 1	Test_3	6.10	334	2.06	.11
		Test_ after _3_months	6.49	334	1.98	.10

Table VI.8. Differences between the results of the assessment tests of the participating students three months after the intervention

Group			Differences between pairs					t	df	P
			mean	Standard deviation	Er. a standard	95% trust interval of difference between means				
						Low level	High level			
control	Pair 1	Test_3 - Test_after_3_m	-,08	,42	,02	-,13	-,03	-3,65	333	,000
experimental	Pair 1	Test_3 - Test_after_3_m	-,38	,54	,03	-,44	-,32	-12,86	333	,000

A significant difference between groups was also obtained at the time of the test after 3 months ($t_{(657)}$=-4.77, p<.001), indicating that, at the end of the test, the pupils in the experimental group performed better than those in the control group. In other words, we can note that this difference between the two groups seems to increase gradually, as indicated by the threshold p of statistical significance: if at time T1 the two groups were not significantly different (p = .127 - it seems that 5 months are not enough), at time T2 the difference is already significant (p = .004), and at the end, after a total of 8 months, the difference will be extremely significant (p <.001).

Following this analysis, it can be mentioned that the level of school performance is maintained even after a period of 3 months from the end of the intervention.

Ensuring that the effects of the intervention were maintained even two and a half years after its completion for the experimental group required the comparison of the results from test 3 (M = 6.20, DS = 1.98) which are lower than those from the national evaluation (M = 7.34, DS = 1.74) (Table VI.9), statistically significant results for $t_{(326)}$=-26.68, p<.001 (table VI.10). For the control group, it can also be seen that the results from the national evaluation are higher (M = 6.62, DS = 1.74) than those from test 3 (M = 5.50, DS = 2.01), statistically significant for t (324) = - 31.82, p <.001. This result highlights that the change produced is not only long-term in the sense that it does not decrease over time, but also that the intervention has created a number of skills that students continue to use to improve their performance, with all that the intervention ended.

Table VI.9. Average grades at the evaluation tests of students participating in the study at the end of the intervention programme and at a difference of more than two years

Group			Average grade	No. of participants	Standard deviation	Standard error
control	Pair 1	Test_3	5.50	325	2.01	.11
		National_evaluation_2019	6.62	325	1.74	.09
experimental	Pair 1	Test_3	6.20	327	1.98	.10
		National_evaluation_2019	7.34	327	1.74	.09

Table VI.10. Differences between the results of the pupils participating in the study at the National Evaluation

Group			Differences between pairs					t	df	P
			Mean	Standard deviation	Standard error	95% trust interval of difference between means				
						Low level	High level			
control	Pair 1	Test_3 - Nat._evaluation_2019	-1.11	.63	.03	-1.18	-1.05	-31.82	324	.000
experimental	Pair 1	Test_3- Nat._evaluation_2019	-1.14	.77	.04	-1.23	-1.06	-26.68	326	.000

The pupils in the experimental group and those in the control group performed significantly differently, as indicated by the t test for independent samples. ($t_{(650)}$=-5.30, $p<.001$) - this difference can also be seen in Figure VI.3. According to our expectations, hypothesis 3 is confirmed, indicating that the average grades of the experimental group are higher than those of the control group at moment T5. As a result, we conclude that the skills acquired as a result of the educational intervention have long-term effects and are applied in a different testing context (national exam).

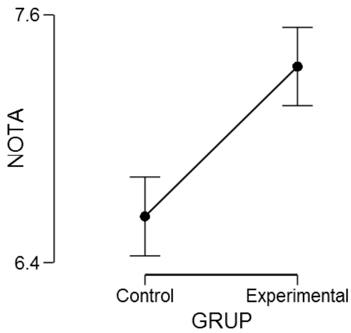

Figure VI.3. The grades obtained by the students from the two groups at Mathematics at the national exam.

Hypothesis 2 referred to the development of pupils' critical thinking by teaching sixth grade Mathematics using an educational programme based on learning by discovery and problematization, so the comparison between the averages of students participating in two competitions over 6 months was used.

In the experimental group it can be observed that the score of the second contest (M = 69.61, DS = 24.46) is higher than that of the first contest (M = 56.86, DS = 24.01) (tables VI.12), for $t_{(270)}=-13.55$, $p<.001$ (table VI.13), and for the control group the difference is not statistically significant, which comes to strengthen the hypothesis already supported by the data.

Table VI.12. The scores for the critical thinking contest of the pupils participating in the study

Group			Average grade	No. of participants	Standard deviation	Standard error
control	Pair 1	Contest_score_V_27.05.2016	54.86	225	23.77	1.58
		Contest_score_VI_9.12.2016	54.77	225	23.74	1.58
experimental	Pair 1	Contest_score_V_27.05.2016	56.86	271	24.01	1.45
		Contest_score_VI_9.12.2016	69.61	271	24.46	1.48

Table VI.13. The differences between the scores from the critical thinking competitions of the students participating in the study

Group			Differences between pairs					t	df	P
			Mean	Standard deviation	Standard error	95% trust interval of difference between means				
						Low level	High level			
control	Pair 1	Contest_score_V 27.05.2016 – Contest_score_VI 9.12.2016	.08	10.57	.70	-1.30	1.47	.12	224	.900
experimental	Pair 1	Contest_score_V 27.05.2016 – Contest_score_VI 9.12.2016	-12.74	15.48	.94	-14.60	-10.89	-13.55	270	.000

The results of ANOVA for the tests that measure critical thinking are presented in tables VI.14 and in tables VI.15, for the two groups, at the moments of the Neutrino contest of 20.05.2016 and of 9.12.2016. For the analysis of critical thinking, only the data from those pupils who participated in both moments of the test were used. As a result, data from 257 pupils contributed to the experimental group and 225 pupils to the control group. A significant main effect of the time variable was obtained, ($F_{(1, 482)}$=75.48 $p<.001$, η^2=.13), which indicates that there are differences between the score at the contest of 20.05.2016 and the score at the contest of 9.12.2016. The time * group interaction is also significant, ($F_{(1, 482)}$=18.20, $p<.001$, η^2=.03), indicating that there are differences between the two groups at different times. Regarding the Between effects, the two groups do not have significantly different performances, if the effects of the Time factor are not taken into account.

Table VI.14. Results for critical thinking, Within effects.

	Sum of Squares	Df	Mean Square	F	p	η²
Time	4216	1	4215.81	75.48	.001	.13
Time✻Group	1017	1	1016.73	18.20	.001	.03
Residual	26420	482	55.86			

Table VI.15. Results for critical thinking, Between effects.

Between effects	Sum of Squares	Df	Mean Square	F	p	η^2
Group	1087	1	1087	1.06	.302	.002
Residual	481583	482	1018			

The results indicate that the two groups really have significantly different performances at the time of the contest on 20.05.2016 ($t(482) = 2.00$, $p = 045$). In other words, Hypothesis 2 appears to be confirmed, as the pupils in the experimental group developed superior critical thinking skills as a result of educational intervention.

The data analysis continued with the verification of the maintenance in time of the critical thinking capacity developed by the pupils following the participation in the previously presented programme. For the experimental group, the results show statistically significant differences as follows: the score at the third contest is higher (M = 68.05, DS = 24.06) than the score at the first contest (M = 57.05, DS = 24.05) (tables VI.16) for $t_{(266)}=-11.39$, $p<.001$ (table VI.17), and the score at the third competition (M = 68.05, DS = 24.06) is lower than the score at the second competition (M = 70.16, DS = 24.24), a significant result for $t_{(265)}=3.42$, $p=.001$. for the control group the results are not statistically significant.

Table VI.16. Scores, after three months after the intervention, of the pupils participating in critical thinking competitions

Group			Average grade	No. of participants	Standard deviation	Standard error
control	Pair 1	Contest_score_V_27.05.2016	54.60	226	23.97	1.59
		Contest_score_VI_19.05.2017	54.35	226	24.02	1.59
	Pair 2	Contest_score_VI_9.12.2016	53.41	234	24.28	1.58
		Contest_score_VI_19.05.2017	53.09	234	24.54	1.60
experimental	Pair 1	Contest_score_V_27.05.2016	57.05	267	24.05	1.47
		Contest_score_VI_19.05.2017	68.05	267	24.06	1.47
	Pair 2	Contest_score_VI_9.12.2016	70.16	266	24.24	1.48
		Contest_score_VI_19.05.2017	68.12	266	24.08	1.47

Table VI.17. The differences between the scores at the critical thinking contest three months after the intervention

Group			Differences between pairs: Mean	Standard deviation	Standard error	95% trust interval of difference between means: Low level	High level	t	df	P
control	Pair 1	Score_V 20.05.2016 – Score _VI 19.05.2017	.24	11.32	.75	-1.24	1.72	.32	225	.747
	Pair 2	Score _VI 9.12.2016 – Score _VI 19.05.2017	.32	9.81	.64	-.94	1.58	.49	233	.618
experimental	Pair 1	Score _V 20.05.2016 – Score _VI 19.05.2017	-10.99	15.75	.96	-12.89	-9.09	-11.39	266	.000
	Pair 2	Score _VI 9.12.2016 - Score _VI 19.05.2017	2.04	9.76	.59	.87	3.22	3.42	265	.001

Changing the behaviour specific to critical thinking in the experimental group is not only long-term and does not decrease over time, but even increases over time, possibly because the intervention has created a number of skills that pupils continue to use to improve critical thinking. Thus hypothesis 2 is supported by statistical data.

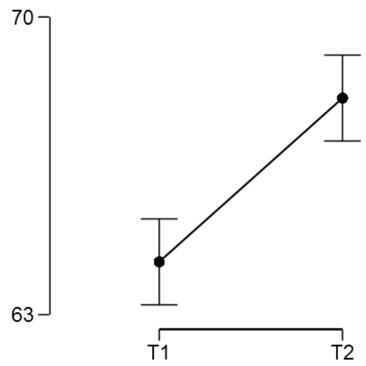

Figure VI.4. The evolution of the level of critical thinking over time

The upward change in performance at the time of T2 - Posttest indicates that the critical thinking skills of the experimental group continue to develop over time, after the educational intervention has ended. The Oy axis indicates the average results of the tests applied at the two moments in time (measured on a scale from 0 to 100).

As for the improvement of teachers' perception regarding the importance of critical thinking as a behaviour in the context of teaching Mathematics, the statistical opinion of the interviewed teachers is presented below.

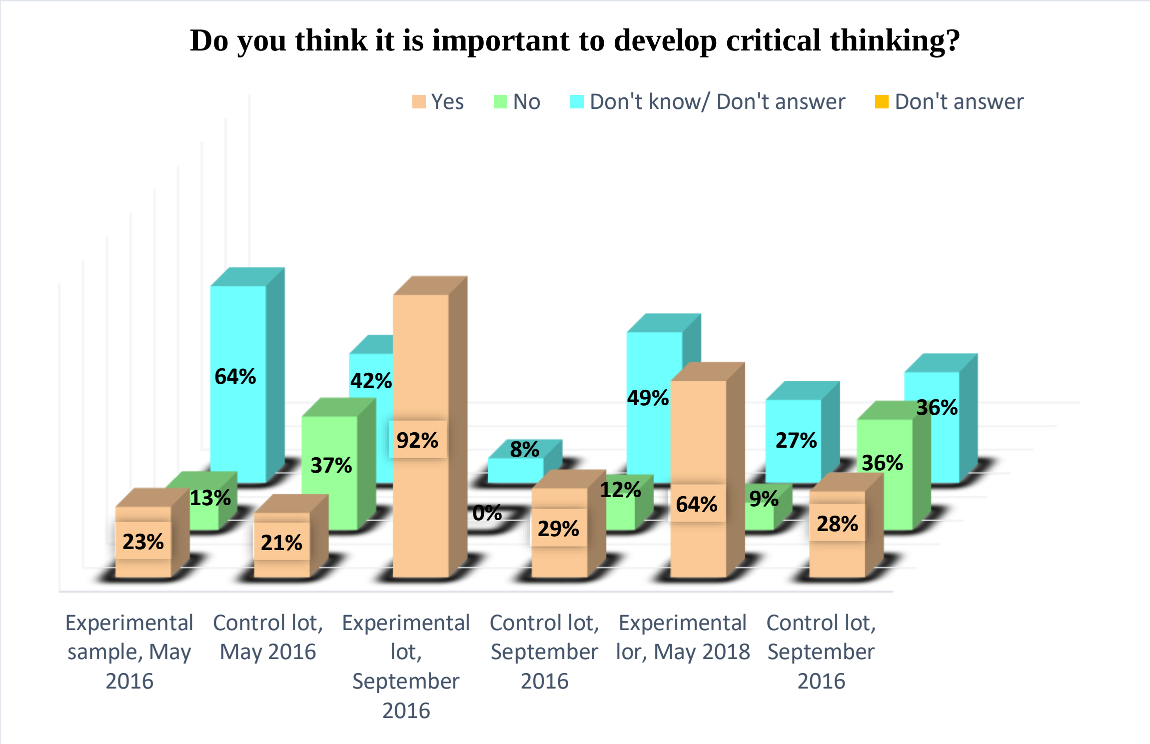

Figure VI.12. The opinion on the importance of critical thinking

The above data show that 79% of the respondents of the experimental group, in the initial stage considered that it is not necessary to develop critical thinking or prefer not to respond. The situation changes radically after the formative intervention, 92% of them considering that it is necessary to develop critical thinking (Sept. 2016). The quasi-identical percentage at the initial stage in the control group of 79% who consider it unnecessary or unresponsive is equally worrying. About the same remains in 2018, 72%. Such data have the potential to explain the performance of students at PISA, unfortunately.

The teachers from the experimental group who participated in the training course started teaching using problematization and discovery learning. As a result, the percentage of teachers who develop critical thinking using problematization and learning through discovery increases in the experimental group, reaching 48% in September 2018.

CHAPTER VII

RESEARCH CONCLUSIONS

We will analyse the research results from the perspective of the three main directions:

1. Teaching Mathematics to sixth graders with the help of an educational programme based on learning by discovery and problematization contributes to the improvement of students' school performance in Mathematics.

We can summarize the results in terms of mathematical performance of the pupils in the two groups as a result of educational intervention, so:

- the pupils from the experimental group started to have better results than those from the control group already during the intervention after 7 months from the initial test; these differences are accented over time, as shown by the results from 8 months after the initial test.
- the superior performance of the pupils in the experimental group persists with the end of the educational intervention, indicating that the learning produced has medium-term effects.

The skills acquired by the pupils in the experimental group following the educational intervention have long-term effects and are applied in a different testing context, as the results of the national exam show us..

The better results of the pupils in the experimental forms compared to those of the students in the control forms allow us to conclude that teaching Mathematics through problematization and learning by discovery leads to increased school performance of students, but also produce favourable effects in terms of knowledge and acquired skills, in line with similar results obtained by other studies in the field (Bocoș, 1998, Vlaicu, 2013).

2. Teaching Mathematics to sixth graders starting from the premises of the educational programme based on learning by discovery and problematization contributes to the development of critical thinking of pupils.

The systematic use of problematization and discovery influences the very level of development of pupils' critical thinking. If at the beginning of the experiment the pupils approached the solution of Mathematics problems based on the random combination of the data from the hypothesis, towards the end of the experiment the pupils in the experimental group constructed the reasoning of the solution, aiming to obtain the conclusion by the simplest method.

Gradually, the pupils in the experimental group become aware that a reasoned decision has more chances of success in solving a problem and transforms their own learning from an accumulation of isolated mathematical knowledge, into a learning that emphasizes understanding new acquisitions, correlating with the old ones, on predicting, explaining and solving concrete problems. Only in these conditions can we say that the learning is conscious, that is, the students understood, integrated the new acquisitions in their own cognitive structure and that they internalized the respective informational content.

Evaluated in contests organized over a long period of time to confirm the validity of the intervention programme, the level of critical thinking increases significantly only in the experimental group, which comes to support the role of implementing such a programme in working with children. Thus, we conclude that the pupils in the experimental group develop critical thinking skills superior to those built the control group, an ability that continues, it seems, to develop over time, after completing the intervention.

The results obtained allow us to conclude that teaching Mathematics through problematization and learning by discovery leads to the development of critical thinking of pupils, in line with similar results obtained by other studies in the field (Fischer, 2009, Halpern, 2014, Cai, Leikin, 2020).

3. Teaching Mathematics to sixth graders using an educational programme based on learning by discovery and problematization contributes to changing teachers' perceptions of the importance of critical thinking.

The completion by the teachers of a training course in which they learn to apply in the teaching activity the problematization and learning through discovery leads to a major change in their opinion regarding the importance of developing students' critical thinking. So we need to learn Maths in a way that makes sense and relevance, rather than through isolated subjects.

Teachers’ favourable opinion on the importance of developing pupils' critical thinking also increased during this experiment, from 23% to 64%.

A class that promotes critical and creative thinking provides opportunities for:

- thinking at a higher level in authentic and meaningful contexts;
- complex problem solving;
- reasoned answers;
- decision making.

Following the development of this educational programme based on problematization and learning by discovery, the initiator of this programme contributes to:

- establishing the analogy of PISA problems with the Watson-Glaser critical thinking measurement tool;

- designing and teaching the training course *Development of critical thinking by teaching Mathematics to sixth graders*. This course whose impact on the way in which Mathematics teachers have changed their attitude and didactic behaviour in order to develop critical thinking in students, is presented in detail in Bădescu, Stan (2020);
- designing the *CriticMath* methodological system, *published* (Bădescu, 2016);
- designing, based on the Watson-Glaser critical thinking measurement tool, the pupils' critical thinking measurement items. These items were used in the topics of the Neutrino contest that we initiated and held annually.

Research limits

The first of the limitations of this research is the duration of the intervention, which can influence the results. A longer duration of the intervention, of at least 1 year, would have been more eloquent in this respect.

Another limitation of the research was the design of the progress tests. These Progress Tests (tests 1-3) contained, in addition to the standard problems, also a problem in which pupils had to find the solution. This could be an advantage for the pupils in the control group, because solving the standard exercises and problems was already an automation for them, but the non-standard problem was an advantage for those in the experimental group.

The non-participation of all pupils in the critical thinking tests, moreover, the different percentage of 77% in the control group and of 85% in the experimental one, could have influenced the results.

The choice of teachers participating in the experimental group and the control group was made according to the group in which their own students were included.

Premises of future research

Participating in our teaching experiment provided teachers with ways to improve the learning process. Their enthusiasm during the training course as well as throughout the experiment makes us say that not only the pupils were won, but also their teachers. The different attitude and involvement of teachers has produced change in terms of improving school performance and developing students' critical thinking..

As a result of this didactic experiment, we propose some suggestions to teachers for increasing school performance and developing critical thinking of their own students.:

1. to be motivated and motivating when teaching Mathematics;
2. to carefully plan the teaching-learning strategies they will use;
3. to use, when possible, problematization and learning by discovery as teaching methods;

4. to encourage pupils' progress, not just the grade obtained;
5. to encourage pupil-teacher and pupil-pupil dialogue;
6. to present the contents starting from a practical applicability, from a challenge they have to solve and to show pupils how what he teaches them will help solve the problem;
7. to focus on pupils' collaboration, not competition;
8. to collaborate with other colleagues teaching the same subject and other subjects;
9. to accept pupils' personal opinions;
10. to help pupils, by asking them questions, to realize if their reasoning is correct or wrong.

Next, we intend to continue the project through which most of the contents of middle school Mathematics will be approached from the point of view of problematization and learning by discovery.

We want to extend this project to high school Maths as well. In this regard, we have already created the site www.ovidiubadescu.ro where in section https://ovidiubadescu.ro/clasa/ all the contents of the high school subject are already approached in this way.

The next step would be a database of video lessons posted on the Internet on this content, taught through problem solving and learning by discovery. This database would be very useful now in times of pandemic and beyond because teachers can be inspired in the preparation of lessons, and school pupils and students will still have a way in which that content will be taught.. We elaborated thus *Maths for all (https://www.youtube.com/channel/UC7zZBJu0VOZyswXkkVwFAPA)*, a youtube channel that contains video lessons with middle school and high school Maths lessons.

Organizing training courses at the national level in which teachers have the chance to discover the importance of critical thinking and how critical thinking can be developed would be beneficial if we wish to have a high-quality education.

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