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PHD THESIS

– RESUME –

ESTIMATING VALUE AT RISK FOR FINANCIAL RISKS

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Introduction

The financial environment is formed from set of entities that interact constantly. These entities are companies, investors, financial markets and regulators. Companies are traders that offer for consumption various products and / or services in order to maximize their profits. Investors are natural or legal persons who have financial resources and invest capital in order to obtain positive returns and thus maximize their earnings. Financial markets are the places where all transactions take place, the "meeting place" of the financial environment participants, while the supervisory authorities are the ones who establish the regulations and policies on the basis of which all the activity on the financial markets is carried out.

Every participant at the financial environment aims to obtain and maximize profits, in case of traders, and to obtain maximum returns in the case of investors. However, the financial environment is characterized by a certain degree of uncertainty and, often in financial markets, the gain of one entity is associated with the loss of another. Due to this uncertainty, there are certain risks that every participant should consider, such as: credit risk, market risk, liquidity risk, exchange rate risk and other types of risk. The greatest impact is given by the extreme risks associated with financial crises, which have a high degree of uncertainty.

In some cases, however, a higher risk is also a possibility to obtain higher gains. Every financial asset on the market has a certain degree of risk, and the decision to invest can be made based on return to risk ratio, on the one hand, but also taking into account the risk aversion shown by every participant in the financial market, on the other hand. A conservative investor will generally be satisfied with lower returns as long as the risks are kept to a minimum, while an aggressive investor will seek higher returns by implicitly taking on a higher degree of risk.

In general, risks are produced by unexpected events, classified by Malevergne and Sornette (2006) into three categories: the first category refers to random events that can be quantified and modeled by known probability laws; the second category includes stochastic events that can be partially modeled in terms of known probability laws, and the third category includes all purely random events, which are surprises and / or have been considered impossible until the moment they actually happened. Peng et al. (2011) classify financial risks in credit risk - the probability that a counterparty will not return the amounts owed to the creditor, investment risk - given by the probability that the investment will not generate returns (this may include market risk and interest rate risk), business risk - which includes all the factors that can affect the smooth running of a trader, and operational risk - risks generated by human error or other errors in the daily processes carried out at the level of an economic entity.

Due to the evolution of financial markets and the increasing complexity of traded instruments and products, the risks associated with certain events have also acquired a high variety and complexity. Given the two aspects, it is very important for an entity operating in the financial environment to quantify its risks in an adequate manner. It is not enough to just be aware of them, but also to select a mathematical model through which they can be properly evaluated.

Classical methods do not provide satisfactory results in modeling risks with a high degree of complexity, especially in periods of financial instability. In view of this, methods that take into account both the characteristics of the risks and the nature of the data are needed. The financial returns series do not have a normal distribution, often showing features such as excessive kurtosis, fat tails and heteroskedasticity or left skewness. Therefore, specific methods are needed that take these characteristics into account so that the results have a sufficiently high degree of accuracy.

Among the statistical measures for financial risks, volatility, value at risk (VaR) and expected loss (ES) can be found in literature. In the current paper, VaR measure is mainly analyzed, as it is relatively easy to understand in comparison to ES and it has a higher complexity than the volatility. Also, in order to obtain consistent estimators for value at risk, a smaller volume of data is required than in the case of expected loss, the latter being estimated on the values found in the tail of the distribution. Every measure has its own advantages and disadvantages, but there are sufficient ways in which the advantages can be emphasized and the disadvantages removed or at least diminished. The VaR measure was attractive due to its simplicity and due to the fact that there are many ways in which its disadvantages can be treated. Alexander (2008) highlights the main elements that make this measure attractive: (i) VaR corresponds to an amount of money that can be lost with a certain probability, (ii) it measures the risk associated with each factor and their sensitivity, (iii) it can be compared across several markets, for various exposure levels, (iv) it is a universal measure that can be applied to any type of risk, in various fields, (v) it can be measured at any level, starting from a stock or a portfolio of financial assets up to a value associated with an entire company that includes all of its risks, and (vi) when VaR is aggregated or decomposed, it takes into account the dependencies between components.

In literature there is a relatively large number of methodologies through which value at risk can be estimated. They are found in the nonparametric area, in the purely parametric area as well as in the semi-parametric sphere. Every VaR methodology has its own advantages and disadvantages when it comes to anticipating future risks. The field of research on VaR measure remains attractive due to the multiple possibilities - some really complex - to estimate it and, at the same time, to the simplicity of its understanding and interpretation. Abad et al. (2014) present a compendium of existing methodologies, a study that can be updated. Recent papers such as Babat et al. (2017) propose portfolio optimization techniques based on VaR, or Mohammadi and Nazemi (2020) which propose a VaR approach that considers replacing portfolio selection models with linear programming problems as well as the use of neural networks.

The approach of this paper focuses on modeling financial risks using various methods of estimating VaR. Although there is a lot of research on this topic that aims to model volatility and measure the accuracy of VaR estimation methods, the studies focus only on certain aspects of such methods. Also, due to the changes that continuously influence the financial environment, the study of the proposed topics remains relevant. Thus, a first objective of this paper is to highlight the measures of quantifying financial risk together with the presentation of their advantages and limitations, and to present a brief description of the of risk estimation methods proposed in the literature, with emphasis on the hypotheses which the development of every group of methods was based.

This literature review facilitated the selection of methods proposed for use in the empirical studies, developed in the following chapters. The methods proposed in the empirical analyses take into account the specific characteristics of financial returns: heteroskedasticity - successfully captured by ARCH / GARCH family models on both univariate and multivariate levels, fat tails - which can be modeled with extreme value theory techniques and quantile regression, asymmetric information - taken into account by APARCH-type specifications at the univariate level or by the asymmetric conditional correlation at the multivariate level, and the correlation between the returns of the financial assets of a portfolio - incorporated in the multivariate conditional correlation.

The main objective of this paper is to identify approaches in estimating VaR that have a high degree of accuracy, when used for various data sets. The main purpose was to contribute to the existing literature by combining techniques such as filtered historical simulation, asymmetric models for volatility and elements of extreme value theory, each modeling specific aspects of the financial returns series. The empirical results obtained proved to be satisfactory in terms of accuracy, when compared with those resulting from many other models found in the literature. The accuracy degree of the used approaches was assessed using the following methods: failure rate, Lopez's quadratic loss function (1999), the unconditional coverage test introduced by Kupiec (1995), the conditional coverage test proposed by Christoffersen (1998), the dynamic quantile test developed by Engle and Manganelli (2004) and the loss function of Gonzalez-Rivera et al. (2004).

The added value of this thesis consists in some novelty elements briefly described in the following paragraphs and also in the sections dedicated to the research objectives and conclusions of each chapter. A first contribution is given by the use of filtered historical simulation (Barone-Adesi et al., 1999) combined with an APARCH model (Ding et al., 1993) used to estimate value at risk for a set of 14 international stock market indices. The results of this approach proved its efficiency as well as its high degree of accuracy in forecasting value at risk.

The next contribution to the literature is the use of filtered historical simulation in combination with volatility models from multivariate ARCH-GARCH families. This methodology was applied on a portfolio of 11 financial assets from Bucharest Stock Exchange. The models that take into account the informational asymmetry at multivariate level, and also having a high degree of accuracy, were noted as appropriate.

The third contribution element is the use of a methodology that combines the generalized Pareto distribution, the APARCH model and the rolling window method. This approach was applied to a data set consisting of 20 stock indices collected from international markets of all types (developed, emerging and frontier). The results indicate a high degree of accuracy, as well as the fact that the use of the rolling window method contributes to an increase in accuracy levels.

Another contribution of this paper is given by the VaR estimation for high frequency data – intraday type. Most researches proposing the use of VaR measure for market risk management are performed on daily data due to unavailability or difficulties in obtaining high frequency data.

The research of these methods has been and continues to be attractive due to the fact that the results, although highly accurate, can be further improved. Another element that makes this research attractive is the practical applicability of the techniques presented in the following chapters. They can be used in both academic and professional environment for statistical modeling of risks and to anticipate potential losses.

The paper is structured in five chapters, each addressing a specific topic, preceded by the introduction, and it ends with a section dedicated to the conclusions and future directions of research. This structure was selected in order to highlight the large number of approaches that can be used in VaR estimation and also to deepen specific topics such as VaR obtained by filtered historical simulation, multivariate VaR, value at risk obtained by techniques from extreme value theory and VaR related to intraday data.

The first chapter analyzes the main risk measures, focusing on the value at risk. In the first section, financial risk is defined according to the literature and the main ways in which it can be quantified are presented: volatility, value at risk and expected shortfall. Also in this section, the properties which a coherent risk measure must have - according to Artzner et al. (1999), and the section ends with the description of the advantages and disadvantages of risk measures. The second section contains comprehensive information on statistical methods and models used in literature to estimate value at risk. Non-parametric methods, purely parametric methods and semi-parametric methods are analyzed in detail in the paper. The last section is dedicated to value risk back testing methodologies.

In the second chapter, the emphasis was on determining the value at risk in the univariate case. The proposed methodology consists in using the four stages of the filtered historical simulation (FHS) together with an APARCH type model for the volatility series. The APARCH model is one of the most suitable for financial returns due to the fact that it takes into account asymmetric information and because it incorporates as sub-models families such as GARCH, GJR-GARCH, but also others. The first section of this chapter presents the research objectives and highlights the elements of originality. The second section includes the methodologies used in order to estimate VaR (two pure parametric, two from the field of extreme value theory and two based on filtered historical simulation) as well as the description of back testing methods. The following section presents the data sample, which contains 14 international stock market indices, and shows the results obtained from applying the proposed methods, among which the FHS-APARCH is highlighted. The chapter ends with the conclusions' section.

The third chapter is dedicated to the study of value at risk in a multivariate framework. The first section of this chapter includes the objectives of studying the multivariate value at risk, while highlighting the chapter's specific elements of originality. The second section presents the results of several similar studies as well as a brief description of the ten approaches used in the empirical study (the Riskmetrics approach, a method that uses higher order moments, four models from multivariate GARCH families and four models based on filtered historical simulation). A method of method of risk decomposition (Mina and Xiao, 2001) for every individual portfolio component is also presented. The empirical section describes the data sample - a portfolio of 11 assets from BSE - and presents the empirical results of the back testing methods as well as the risk decomposition technique into individual risk components. The chapter ends with a section dedicated to the conclusions and final remarks.

Chapter 4 focuses on estimating value at risk using methodologies from extreme value theory (EVT). The first section contains the research objectives of this chapter (VaR study by methods within EVT framework). The second section contains a short review of studies that used value at risk in combination with elements of extreme value theory. The 12 VaR estimation methods are briefly presented, one being the Riskmetrics approach, six use EVT, and five use quantile regression techniques. The latter were selected for comparative purposes. The empirical section describes the data sample - a set of 20 international stock indices - and presents the back testing results, highlighting the GPD-APARCH-skew model estimated with the rolling window method, and then the conclusions are presented in the last section.

The fifth chapter includes case studies regarding the application of VaR methodologies on high frequency data (intraday), collected from the Romanian financial market. The first section contains a brief presentation of studies containing methodologies specific to high frequency data - mainly - methods based on realized volatility. Each of the following three sections include a case study conducted using different specific methods and data sets. The first case study was conducted on a set of 12 companies from Bucharest Stock Exchange and it includes six VaR methods: a historical method, a parametric method (classical Riskmetrics approach), a Monte Carlo version of the parametric method, a Riskmetrics approach that uses realized volatility and its Monte Carlo version, as well as the approach proposed by Francois-Heude and Van Wynendaele (2001) which takes into account the liquidity based on the BID-ASK spread. By taking liquidity into account, more accurate results have been obtained. The second case study considers the same data set, but other risk estimation methods were used, one specific to intraday data (based on an ARFIMA model), and the other applicable after aggregating the data at daily frequency (filtered historical simulation with an APARCH model). Both generated satisfactory results. In the third case study, four VaR models (Riskmetrics with realized volatility and its Monte Carlo version, an ARFIMA model and filtered historical simulation) were tested on the intraday series of the BET index. The filtered historical simulation was highlighted.

This paper ends with a section dedicated to the general conclusions and future research directions, followed by the bibliographic references section and the appendixes section, respectively.

Chapter 1 – Resume

Due to the fact that the financial environment is characterized by a certain level of uncertainty, it implies certain risks. It is necessary for a financial environment participant to quantify the risk adequately in order to ensure its profitability or to minimize its losses.

According to Danielsson (2011), a measure of risk is a mathematical method by which risk is quantified. According to the same author, the most commonly used measures for quantifying financial risks are volatility, Expected Shortfall (ES) and Value at risk (VaR).

Volatility is one of the most accessible measures of risk due to its simplicity. It can be easily expressed by the standard deviation of financial returns:

$$\sigma = \sqrt{E[(R_t - \mu)^2]}$$

where $\mu = E(R_t)$ and E() denotes the average value or the expected value.

According to Habart-Corlosquet et al. (2013), value at risk (VaR) measures the largest loss an entity expects, given a certain time horizon, under normal market conditions, at a certain level of confidence. This measure quantifies the level of risks specific to a company or a portfolio, in a certain time horizon. To define this measure mathematically, we will consider a random variable L, representing a financial loss as well as a confidence level $\alpha \epsilon (0;1)$. Then, the VaR_α corresponding to a certain underlying asset, at the confidence level α, is given by the lowest number *l* for which the probability that the loss L exceeds the value *l*, is not higher than 1 - α. This version of defining VaR is similar to the one found in Habart-Corlosquet et al. (2013):

$$VaR_\alpha = inf\{l \, \epsilon \, \mathbb{R}: Pr(L > l) \leq 1 - \alpha\}$$

Expected Shortfall will be defined in a similar approach to the one found in Danielsson (2011): if L is a variable that represents a potential loss of a financial asset or a portfolio of financial assets, and $VaR_\alpha(L)$ is the value at risk of the variable L for the confidence threshold α, then the expected loss of L is given by the following conditional average:

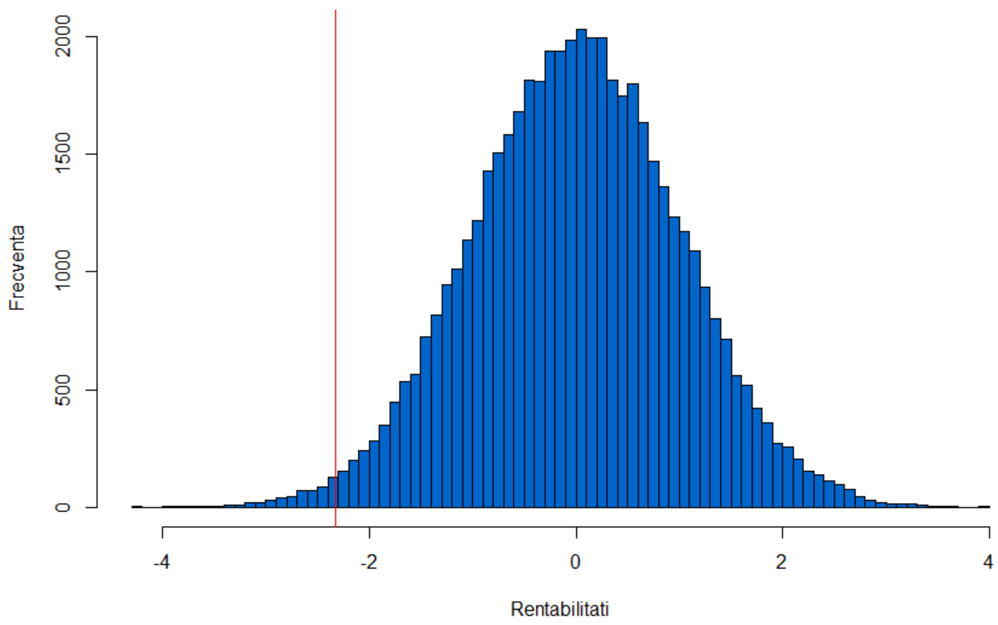
$$ES_\alpha(L) = E[L|L \geq VaR_\alpha(L)]$$

This measure quantifies the expected value of the loss conditional on exceeding VaR, practically, the value recorded by the variable L in cases where the value $VaR_\alpha(L)$ is exceeded.

Value at Risk is a measure that enjoys a high degree of popularity in both business and academic environment, due to its simplicity and applicability and to the many methods by which it can be estimated.

A detailed classification of value at risk estimation methodologies can be found in Abad et al. (2014). They can be divided into non-parametric, parametric and semi-parametric methods.

Figure 1.1 – Value at risk (1%) via historical simulation for an empirical distribution



Source: own generation in R environment

The first category includes historical simulation and estimation methods based on non-parametric density. One of the simplest ways to determine the value at risk is historical simulation. This consists in the direct extraction of VaR from the distribution of returns, as exemplified in Figure 1.1.

The second category of methods is a comprehensive one encompassing ARCH-GARCH type models, realized volatility models, density functions and time-varying conditional higher order moments. Due to the large number of parametric methods, only the APARCH model introduced by Ding et al. (1993) is presented in this resume. In its simplified form, APARCH (1,1), has the following equation:

$$\sigma_t^{\delta} = \omega + \alpha_1(|\varepsilon_{t-1}| - \gamma_1\, \varepsilon_{t-1})^{\delta} + \beta_1\sigma_{t-1}^{\delta}$$

Where:

- σ_t^{δ} , σ_{t-1}^{δ} represents the measure of volatility at time t and t-1 respectively;
- $\omega, \alpha_1, \beta_1, \gamma_1, \delta$ are the parameters to be estimated;
- ε_{t-1} represents the error in the mean equation at time t-1;

This model has the advantage of incorporating other classes of models for specific values of the parameters δ and γ_1. Thus, this model can be transformed into a simple GARCH if $\delta = 2$ and $\gamma_1 = 0$, into TS-GARCH for $\delta = 1$ and $\gamma_1 = 0$, into GJR-GARCH if $\delta = 2$ and $0 \leq \gamma_1 \leq 1$, into NGARCH for $\gamma_1 = 0$ and in TGARCH when $\delta = 1$ and $0 \leq \gamma_1 \leq 1$ (Bollerslev, 2010). Due to its usefulness, this model has been used repeatedly and in various combinations in this paper.

The semi-parametric methods include volatility-weighted historical simulation, filtered historical simulation, autoregressive conditional VaR models (CAViaR) proposed by Engle and Manganelli (2004), methods from extreme value theory, and methods based on Monte Carlo simulation method.

One of the semi-parametric methods used in this paper is the filtered historical simulation proposed by Barone-Adesi et al. (1999) and it consists in combining historical simulation with volatility modeling. This method is applied according to the following steps:

1) Let there be a series of financial returns r_t for which conditional volatility is estimated. The authors exemplify the method using a GARCH(1,1) model.
2) After determining the volatility series, the series of normalized residuals is calculated by dividing them with the volatility for the coresponding period:

$$z_t = \frac{\varepsilon_t}{\sqrt{\sigma_t^2}}$$

where:

- ε_t represents the estimation error in an AR type model;;
- z_t are standardized residual values from an AR type model;;
- σ_t^2 is the volatility obtained in step 1.

3) Bootstrap simulations are performed on the series constructed in the second step, thus obtaining a sample for the distribution of z_t;
4) The empirical distribution of the simulated returns is determined. Their shape is given by:

$$r_t = \mu_t + z^* \sigma_{t+1} \tag{1.1}$$

where:

- r_t represents the returns;
- σ_{t+1} is the forecasted variation for the next period;
- z is the bootstrap distribution obtained in the previous step.

Value at risk is obtained by extracting the quantile at the chosen confidence threshold α, from the simulated distribution of r_t. With the help of filtered historical simulation, results of increased accuracy were obtained at both univariate and multivariate level.

In order to test the accuracy of VaR, methods such as failure rate, the quadratic loss function introduced by Lopez (1999), Kupiec's unconditional coverage test (1995), the conditional coverage test proposed by Christoffersen (1998), the dynamic quantum test of Engle and Manganelli (2004), the loss function introduced by Gonzalez-Rivera et al. (2004) were used in this paper.

Chapter 2 – Resume

In this chapter, the research objectives are to analyze the accuracy of several risk quantification methods using the VaR measures, from the parametric and semi-parametric family of methods, for financial time series that show the daily evolution of a set of stock market indices. The study highlights the following aspects:

- Proposes the use of a combination of filtered historical simulation (FHS) and the APARCH model introduced by Ding et al. (1993) for modeling the dynamics of volatility, an approach less used in literature, but which, leads to high accuracy results, as it will be shown in this chapter;
- Covers several geographical areas and various types of financial markets (developed, emerging and border);
- It captures both, periods of financial stability and periods of instability such as the financial crisis that began in 2007-2008.
- Evaluates the accuracy of the selected methods for a set of stock indices that contain the most representative assets for the capital market of the selected countries.

A set of 14 international stock market indices from 14 financial markets with different degrees of development were selected. For every index, the data were collected from January 1, 2006 to July 31, 2016, obtaining an average number of 2643 days per index.

The value at risk was estimated using 6 methods: the first two use the filtered historical simulation in combination with an APARCH (1,1) and GARCH (1,1) models. The following 2 models use the extreme value theory for estimating VaR. One approach uses the generalized Pareto distribution and VaR is determined from the following equation (Marimoutou et al., 2009):

$$VaR_{\alpha} = u + \frac{\sigma}{\xi}\left[\left(\frac{n}{N_u}(1-\alpha)\right)^{-\xi} - 1\right]$$

where :

- u represents the selection threshold of financial returns;;
- σ is the scale parameter of the generalized Pareto distribution;
- ξ is the shape parameter of the generalized Pareto distribution;
- n represents the number of observations;
- N_u denotes the number of observations above the threshold u;
- α represents the confidence level at which VaR is calculated.

The following method of estimating VaR is a Monte Carlo version of the previous one. The last 2 methods come from the parametric sphere and consist in the use of an APARCH model (1,1) and a GARCH (1,1) model respectively, for estimating volatility and its integration in the VaR calculation:

$$VaR_{\alpha} = \mu + z_{\alpha}{}^{*}\sigma$$

Three tests were used in order to evaluate the accuracy of the methods: failure rate, quadratic loss function (Lopez, 1999) and Kupiec's unconditional coverage test (1995). A VaR model is considered adequate from the perspective of the failure rate, if it records a value as close as possible to the threshold α; according to the quadratic loss function, its value must be as small as possible and the p-value related to the unconditional coverage test must be as large as possible.

From the used methods, the combination of filtered historical simulation and APARCH model (FHS-APARCH) was the most accurate, the results being presented in the table below.

Table 2.4 –FHS-APARCH accuracy

Index	**FR**	**QLF**	**LR Stat**	**P-val**
BET	1.1480%	0.0114827	0.350	0.554
SPX	0.6623%	0.0066229	2.173	0.140
CAC	0.8792%	0.0087945	0.262	0.609
ATX	1.0487%	0.0104887	0.038	0.845

BUX	1.0989%	0.0109914	0.157	0.692
FTSEMIB	1.0695%	0.0107006	0.080	0.777
IBEX	1.0018%	0.0100237	0.000	0.994
RTSI	0.8637%	0.0086434	0.319	0.572
WIG	1.0322%	0.0103239	0.017	0.896
DSM	1.2099%	0.0121006	0.690	0.406
IBX	0.9299%	0.0093003	0.082	0.775
NKY	1.3810%	0.0138153	2.088	0.148
KOSPI	0.5549%	0.0055490	3.870	0.049
SET100	0.8233%	0.0082338	0.530	0.467

Source: own computations in R environment

Note:

FR = Failure rate;
QLF = average value of the quadratic loss function;
LR state = unconditional coverage test statistics;
P- val = probability associated with the unconditional coverage test.

The used method takes into account the information asymmetry found in the financial returns series and the results indicate a sufficiently high accuracy.

The purpose of the study presented in this chapter was to compare several methods for estimating VaR measure, taking into account a set of stock indices from a variety of capital markets, during 2006-2016. As the results indicate, by filtered historical simulation high-accuracy VaR measures were obtained in all markets regardless of their type (developed, emerging or frontier). Another important aspect to note is that taking information asymmetry into account (by using an asymmetric model for volatility - APARCH (1,1)) leads to an increase in forecasts' accuracy.

Chapter 3 – Resume

This chapter is dedicated to estimating VaR for an asset portfolio, in a multivariate approach. Its objectives are:

a) comparing the accuracy of several VaR estimation methods for a portfolio of financial assets. The variance-covariance matrix of the individual asset returns will be considered both, constant and time-varying;

b) decomposition of portfolio risk into components, in order to obtain the contribution of every financial asset to the total portfolio risk.

In the study developed in this chapter, it is proposed an approach that makes use of dynamic conditional correlation in asymmetric form and combines this method with filtered historical simulation, a combination applied with the rolling window method. This approach was implemented for a portfolio stocks on the Romanian market, which according to the FTSE Russell classification (September 2019), is considered a frontier market. The dataset contains information on 11 stocks listed on the Bucharest Stock Exchange (BVB) between 2014-07-08 and 2019-10-04, thus resulting in a series of 1319 daily returns for every financial asset.

In order to estimate the value at risk, ten methods were selected: the first six are parametric methods, and the last four come from the semi-parametric sphere. The first method is based on the portfolio theory or Riskmetrics methodology (Morgan, 1996), which is based on the assumption of normality of financial returns' distribution. The second method is based on the approach proposed by Favre and Galeano (2002), which determines the value at risk using a Cornish-Fisher type expansion. The authors called it "Modified value at risk ".

The following four methods were selected from the parametric sphere and they are built on the idea of letting the variance-covariance matrix vary over time, instead of assuming that it is constant. They are based on families of multivariate GARCH models, using the dynamic conditional correlation (DCC) introduced by Engle and Sheppard (2001) and the asymmetric dynamic conditional correlation (aDCC), as found in Capiello et al. (2006). The GARCH - Bollerslev (1986) - and APARCH -

Ding et al. (1993) specifications in combination with the DCC and aDCC methods were used.

The last four models are based on a semi-parametric technique. They use a combination of the aforementioned multivariate methods, DCC and aDCC, respectively and filtered historical simulation (FHS) introduced by Barone-Adesi et al. (1999) which was described in the previous sections.

To make back testing possible, all the above mentioned methods were applied using the rolling window method. Given that the sample had a length of 1319 trading days, the length of the window was selected at 500 days, thus remaining with a test sample of 819 days. In the approaches that use FHS, the bootstrap window length was set at 300 observations.

In order to assess the accuracy of VaR models, the following five methods were used: failure rate, Kupiec's unconditional coverage test (1995), Christoffersen's (1998) conditional coverage test, dynamic quantile test introduced by Engle and Manganelli (2004) and the average value of the loss function as found in Gonzalez-Rivera et. al (2004). A model is considered appropriate if the failure rate is as close as possible to the significance threshold at which the VaR was estimated (in this case 1%). According to the 3 tests, the model is adequate if the p-value is above the null hypothesis rejection limit (H0: the model predicts VaR correctly), and according to the loss function, the best model is the one with the lowest loss value. Their results can be found in the table below.

Table 3.4 – Back testing results

Model	Fail_rate	UC_Pval	CC_Pval	DQ_Pval	Loss
Riskmetrics	1.829%	0.03245	0.07675	0.05993	0.00043494
Modified VaR	0.244%	0.00910	0.03316	0.29167	0.00054135
GARCH-DCC	1.220%	0.54137	0.73336	0.00270	0.00036479
GARCH-aDCC	1.098%	0.78221	0.87092	0.92204	0.00036143
APARCH-DCC	1.585%	0.12059	0.24305	0.00008	0.00035742
APARCH-aDCC	1.220%	0.54137	0.73336	0.78223	0.00034940
FHS-GARCH-DCC	0.854%	0.66573	0.85756	0.98442	0.00043263
FHS-GARCH-aDCC	0.854%	0.66573	0.85756	0.97139	0.00042622

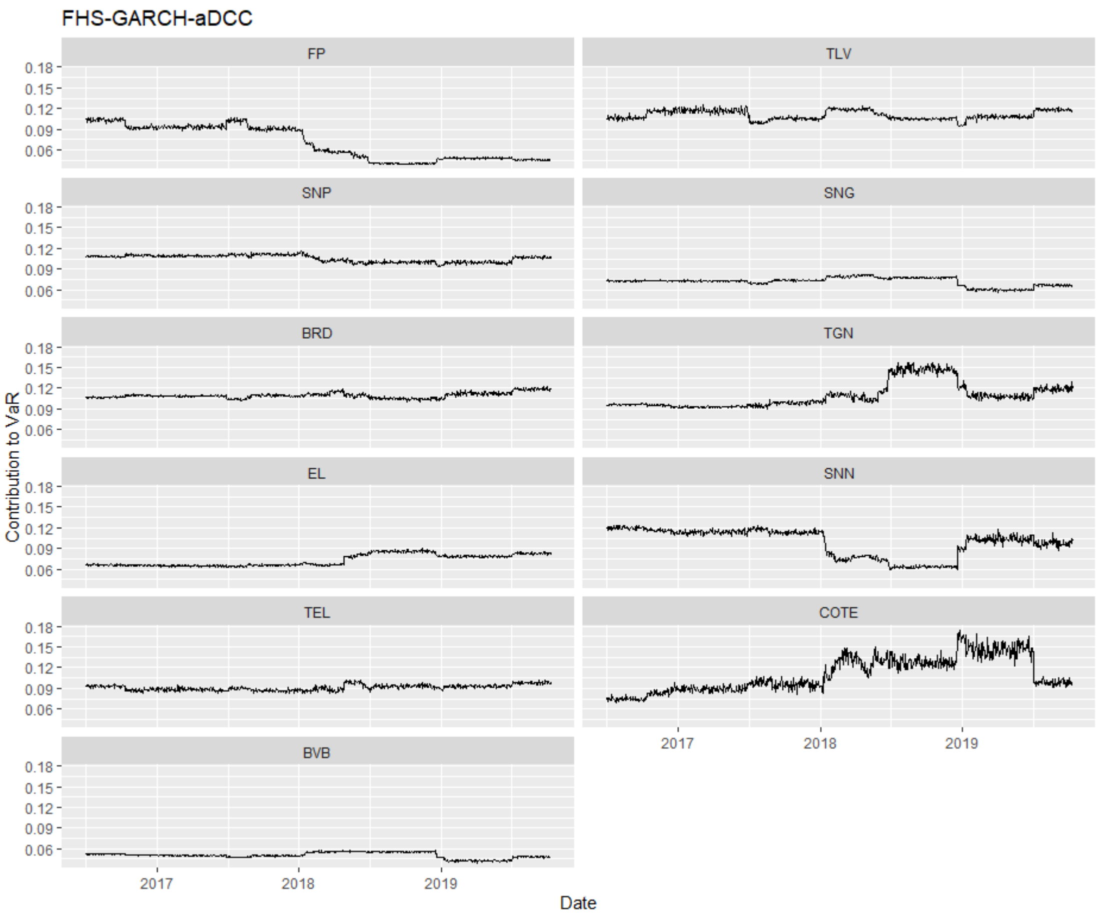
FHS-APARCH-DCC	0.732%	0.41746	0.68866	0.88348	0.00042632
FHS-APARCH-aDCC	0.854%	0.66573	0.85756	0.98356	0.00043407

Source: own computations in R environment

Note: Fail_rate = failure rate;
LRuc_Pval = P-value from the unconditional coverage test (Kupiec, 1995);
LRcc_Pval = P-value from the conditional coverage test (Christoffersen, 1998);
DQ_Pval = P-value from the dynamic quantum test (Engle and Manganelli, 2004);
Loss = value of the loss function (Gonzalez-Rivera et al., 2004);

The best models are those that use a GARCH model for univariate series and that take information asymmetry into account at a multivariate level (aDCC). From the FHS model category, the GARCH-aDCC specification records the lowest loss value.

Figue 3.2 – Risk contribution according to FHS-GARCH-aDCC



Source: own generation in R environment

The risk value of the portfolio was decomposed using the incremental VaR methodology found in Mina and Xiao (2001). Given the way in which the rolling window method was used, it was possible to obtain the contribution of every asset to the total portfolio risk, on a daily basis, starting with observation 501. The values of the individual contributions to portfolio risk for the FHS-GARCH-aDCC method are shown in Figure 3.2.

The techniques presented in this chapter may be useful to portfolio managers and financial institutions for risk management as well as to regulatory authorities for setting market risk policies and other regulations.

Chapter 4 – Resume

The VaR measure is a quantile value, therefore it is important to model the tail of the profitability distribution. Extreme value theory focuses precisely on the behavior of the tails of a distribution, therefore providing adequate statistical tools for risk estimation and prediction. Daníelsson et al. (2012) point out the fact that the value at risk tends to violate the property of sub-additivity in fewer cases if semi-parametric methods combined with elements of extreme value theory are used in the estimation process. This aspect is specific to the tail region of the distribution of financial returns. Several studies in the literature use methods from the extreme value theory (EVT) to estimate VaR.

Chen and Yu (2020) apply EVT techniques in combination with the APARCH model in order to estimate VaR. Compared to their study, in this chapter an improvement of the research methodology is proposed:

a) the rolling window method is used, which allows a more adequate assessment of the methods' accuracy;
b) the data set includes 20 stock market indices from developed, emerging and frontier markets;
c) in addition to the methods that combine the generalized Pareto distribution with GARCH / APARCH type models, five quantile regression models are included in the comparative analysis (four of them being the specifications of the CAViaR model introduced by Engle and Manganelli, 2004);
d) several test procedures were used.

The dataset contains daily financial returns for 20 international stock indices, collected from 01-01-2006 to 2019-09-30. The observation period is approximately 13 years and it includes both stable periods and also periods of economic instability.

In this chapter, the value at risk was estimated using 12 models. The first is the one proposed by Morgan (1996), also known as the "Riskmetrics Method". This was implemented using the rolling window method. Every window has a length of 1000 observations, the first one containing the observations from 1 to 1000. Value at risk is estimated using the information from the first 1000 days. Then the window is moved one step forward (including all records from 2 to 1001) and the calculation is redone. The process is repeated until the last financial return in the series is reached. In this way, a sample of over 1000 observations is available for back testing (this is different for every index, depending on the length of the individual series). Due to its popularity and simplicity, the VaR model estimated using the Riskmetrics approach was selected as a reference model.

The following three models are based on the method proposed by McNeil and Frey (2000). This approach combines elements of extreme value theory ('peaks over threshold') with conditional volatility models. First, a model from the GARCH family is applied on the of financial returns series, thus obtaining standardized residual values. At the next step, the parameters of a generalized Pareto distribution (GPD) are estimated using the data in the tail of the previously obtained residual distribution. Value at risk is Obtained from the following equation (Marimoutou et al., 2009):

$$VaR_{\alpha} = u + \frac{\sigma}{\xi}\left[\left(\frac{n}{N_u}(1-\alpha)\right)^{-\xi} - 1\right] \tag{4.2}$$

where:

- u represents the threshold that delimits the tail of the financial returns distribution;;
- σ is the scale parameter of the generalized Pareto distribution;
- ξ is the shape parameter of the generalized Pareto distribution;
- n represents the sample size;
- N_u is sample size above threshold u;
- α represents the significance level selected for VaR estimation.

The approach based on the generalized Pareto distribution (GPD) is combined with several models from the GARCH family, as follows: a simple GARCH (1,1), an APARCH model proposed by Ding et al. (1993), estimated starting from the normality assumption and an APARCH model based on the assumption of an asymmetric normal distribution, thus resulting in three VaR models.

Unlike the classic methods encountered in the literature, in this chapter, the three approaches mentioned (GPD + GARCH / APARCH / APARCH skewed) are implemented using the rolling window method (GPD + GARCH / APARCH / APARCH skewed + rolling window). For every model, value at risk is estimated according to equation (4.2). The length of the window was set at 1000 observations.

The last models are based on quantile regression approaches. The first model represents a simple quantile regression. The other models were estimated using the four specifications of the CAViaR model introduced by Engle and Manganelli (2004)..

In order to evaluate the performance of the above presented VaR models, a battery of five tests was used: the failure rate, the unconditional coverage test proposed by Kupiec (1995), the unconditional coverage test introduced by Christoffersen (1998), the dynamic quantile test proposed by Engle and Manganelli (2004) and the loss function of Gonzalez-Rivera et al. (2004).

The empirical results are included in the following table only for the GPD-APARCH model applied using the sliding windows method - the one based on asymmetric normality.

Table 4.4 – Back testing VaR accuracy (selection)

GPD-APARCH skew roll	**Fail_rate**	**LRuc_Pval**	**LRcc_Pval**	**DQ_Pval**	**Loss**
BET	1.02%	0.911	0.082	0.014	0.0004
SPX	1.10%	0.629	0.000	0.000	0.0003
SPTSX60	0.98%	0.919	0.784	0.217	0.0002
DAX	0.97%	0.860	0.779	0.087	0.0004
UKX	1.05%	0.799	0.008	0.000	0.0003
CAC	1.19%	0.347	0.431	0.002	0.0004
ATX	0.95%	0.820	0.004	0.000	0.0004
BUX	1.03%	0.882	0.523	0.472	0.0004
FTSEMIB	1.09%	0.672	0.680	0.072	0.0005
IBEX	0.76%	0.203	0.385	0.657	0.0004
RTSI	0.87%	0.498	0.317	0.082	0.0006

WIG	0.98%	0.938	0.006	0.000	0.0003
DSM	1.07%	0.744	0.091	0.003	0.0004
IBX	0.83%	0.403	0.261	0.141	0.0004
SHSZ300	0.98%	0.930	0.793	0.757	0.0005
NKY	0.97%	0.889	0.005	0.000	0.0005
KOSPI	0.88%	0.533	0.684	0.533	0.0003
SET100	0.72%	0.152	0.317	0.558	0.0003
SASEIDX	1.21%	0.361	0.007	0.000	0.0004
SMI	1.06%	0.761	0.008	0.000	0.0003

Source: own computations in R environment

Note: Fail_rate = failure rate;
LRuc_Pval = P-value from the unconditional coverage test (Kupiec, 1995);
LRcc_Pval = P-value from the conditional coverage test (Christoffersen, 1998);
DQ_Pval = P-value from the dynamic quantum test (Engle and Manganelli, 2004);
Loss = value of the loss function (Gonzalez-Rivera et al., 2004);
The extension "skew" next to the model name indicates the use of the asymmetric normal distribution in the estimation process;
The "roll" extension next to the model name indicates the use of the rolling window method.

Similar to the results obtained by Chen and Yu (2020), the GPD-APARCH model has a good performance when applied to the 20 stock indices from markets with various degrees of development. The threshold u, which delimits the tail of the distribution, was chosen using the empirical quantile value of 99% for all distributions, for all models.

In this chapter, the GPD-APARCH approach has been improved with the rolling window method and compared with its original form. Also, in the present study, the GPD-APARCH model was estimated using the assumption of a normal asymmetric distribution and it was compared with other types of models, such as quantile regression and CAViaR models, respectively.

Another major difference between the two studies is the data set used. Chen and Yu (2020) used the HIS futures index (Hong Kong), during 2006-2012. In the current study, a longer time period (2006-2019) was considered, and the sample contains 20 stock indices from several international markets. The study presented in this chapter backs up the results obtained by Chen and Yu (2020), when analyzing the accuracy of VaR estimation on the capital markets.

As showed by the obtained empirical results, the GPD-APARCH model, applied using the rolling window method and estimated under the assumption of asymmetric normality, is one of the most accurate, having a performance at least as good as the CAViaR-SAV approach, in forecasting VaR.

Chapter 5 – Resume

This chapter is dedicated to VaR estimation methods for intraday data. Some of these techniques are specific to high frequency data, and others require prior aggregation of data to daily frequency.

The current chapter contains three case studies conducted on 2 sets of high frequency data (intraday). The first set consists of 12 companies listed on the Bucharest Stock Exchange, whose quotations were collected during July 5, 2010 - June 28, 2013, between 10:00 and 17:00. A database containing 689,809 observations was constructed. The data was grouped in 2-week time intervals. The second data set is identical to the one used in the first case study, the grouping of data being done in a similar way. Periods of 2 weeks were considered, but in this case, the last period was used to assess the accuracy. The third dataset contained the intraday values of the BET index from 04.01.2010 to 31.12.2016, resulting in a series of 454640 observations.

In the first case study, six methods were used to determine VaR: a simple historical simulation, a simple parametric method (based on normal distribution), a parametric method based on realized volatility, a Monte Carlo method, the Monte Carlo method combined with realized volatility and a liquidity-adjusted VaR method. The accuracy of the methods was tested using the failure rate and the quadratic loss function (Lopez, 1999). Table 5.1 shows the failure rate for the 6 methods.

Table 5.1 – Failure rate; Liquidity adjusted VaR

Stock	Hist	Param	Param RV	MC	MC RV	Liq Ajd VaR
BIO	3.90%	5.19%	5.19%	3.90%	5.19%	2.60%
BRD	3.90%	5.19%	2.60%	2.60%	1.30%	0.00%
BRK	0.00%	0.00%	1.30%	0.00%	0.00%	5.19%
BVB	2.60%	3.90%	2.60%	1.30%	1.30%	0.00%
SIF1	3.90%	7.79%	1.30%	2.60%	0.00%	0.00%
SIF2	1.30%	3.90%	1.30%	1.30%	0.00%	0.00%
SIF3	5.19%	1.30%	0.00%	1.30%	0.00%	0.00%
SIF4	3.90%	6.49%	1.30%	1.30%	0.00%	0.00%

SIF5	11.69%	11.69%	3.90%	7.79%	1.30%	0.00%
SNP	3.90%	10.39%	1.30%	6.49%	1.30%	0.00%
TEL	2.60%	3.90%	3.90%	2.60%	2.60%	0.00%
TLV	9.09%	12.99%	11.69%	7.79%	9.09%	0.00%

Source: own computations in R and Excel environments

The second case study uses 2 methods applied to the same data set (as in the first case study). One of these uses an ARFIMA model specific to intraday data, and the other uses an APARCH model for the volatility series. Both are subsequently integrated into a filtered historical simulation procedure. The results of the failure rate for the FHS-APARCH method is shown in Table 5.4.

Table 5.4 – Failure rate; FHS-APARCH

Symbol/Day	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
BIO	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BRD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BRK	0.00	0.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BVB	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SIF1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SIF2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SIF3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SIF4	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00
SIF5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SNP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TEL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TLV	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	88.84

Source: own computations in R and Excel environments

Four models of VaR estimation are used in the third empirical study. The first is a deviation from the classical parametric model, which assumes that the distribution of financial assets returns is normal. The difference is that the volatility of returns was estimated using the realized volatility. The second is a Monte Carlo version of the first method. Methods 3 and 4 use filtered historical simulation combined with the ARFIMA and APARCH techniques in a manner similar to the one found in the second case study.

The accuracy of the models was tested using three techniques: the failure rate, the quadratic loss function (Lopez, 1999) and the unconditional coverage test (Kupiec, 1995). The results of these procedures are shown in Table 5.7.

Table 5.7 – VaR back testing results; BET index

Model	**Fail Rate**	**QLF**	**Unconditional coverage**	
			LR Stat	**P-val**
FHS-APARCH	1.2113%	0.012117	0.314	0.575
FHS-ARFIMA	1.8843%	0.018847	4.658	0.031
Param-RV-MC	1.4805%	0.014808	1.509	0.219
Param-RV	6.4603%	0.064613	100.245	0.000

Source: own computations in R environment

As the results show, taking the liquidity of financial assets into account leads to more accurate estimates of value at risk. The use of realized volatility also leads to good results, but not to those expected. In this case, the use of more advanced measures is recommended such as the ones found in Barndorff-Nielsen and Shephard (2004)..

Another method that stood out for its increased accuracy is filtered historical simulation. This method has proven to be accurate in both case studies in which it has been used.

The limits of the empirical studies in this chapter are mainly given by the insufficiency of the data and by the impossibility of updating the database. Another limitation of these applications is the insufficient number of back testing methods for the VaR measures. In order to increase the quality of assessments, with the exception of the third case study, it is recommended to add additional back testing procedures to measure the accuracy of VaR.

General conclusions and future research directions

The present paper was focused on the identification of useful approaches in estimating the VaR measure, approaches that lead to results with a high degree of accuracy, starting from specific characteristics of financial data. Empirical studies were conducted using several data sets, on both the international capital markets and on the Romanian market. Daily and intraday data types were considered. Given the characteristics of financial returns, in this paper, the proposed VaR measures take into account aspects such as: *heteroskedasticity, fat tails, asymmetric information, dynamic correlation between returns of an asset portfolio, serial correlation of quantile values and liquidity of financial assets*. At a general level, it has been observed that the measures which take into account the previously mentioned characteristics have a higher degree of accuracy.

In the univariate empirical analysis, six VaR estimation methodologies were applied, for a set of 14 stock indices, relevant for various international markets, over a period of approximately 10 years. Developed, emerging and frontier markets were included in the study. Two of the methods were purely parametric, using GARCH and APARCH volatility models, the following were based on the filtered historical simulation (FHS) in combination with the two volatility models mentioned above, and the last two used a generalized Pareto distribution - in a parametric version, respectively in a Monte Carlo simulation approach. The highest accuracy was obtained by the FHS-APARCH model, which was tested using the failure rate, Lopez's quadratic loss function and Kupiec's unconditional coverage test. It has been observed that taking into account the asymmetry from financial returns series leads to more accurate forecasts. The VaR predictions determined on the basis of the APARCH model were more accurate than those based on a GARCH type specification.

This paper also exemplifies estimation of value at risk methods for a portfolio of financial assets. 10 multivariate VaR models were selected and applied on a portfolio of 11 securities from the Bucharest Stock Exchange. Most models are based on multivariate GARCH approaches or combinations of these with filtered historical simulation (FHS). The best model was obtained by using a FHS-based method, which was combined with an asymmetric dynamic conditional correlation model (a multivariate GARCH which takes into account the asymmetry). Again, it is confirmed that the accuracy of risk forecasts is higher when the methods used are well adapted to the characteristics of financial returns. We propose the use of this approach for other financial assets portfolios in future researches. The accuracy of the models was tested using the following methods: the failure rate, Kupiec's unconditional coverage test, the conditional coverage test proposed by Christoffersen, the dynamic quantile test introduced by Engle and Manganelli, and the loss function proposed by Gonzalez-Rivera et al. Also, for value at risk estimated for a portfolio, the risk decomposition was possible. This was done with the incremental VaR method, therefore making it possible to observe the riskier assets in the portfolio and adjust every weight according to the needs and risk aversion of the investor.

The empirical results from the study developed in Chapter 4 are robust from the perspective of the data set, the analyzed estimation methods and the financial returns characteristics which were incorporated in the estimation methodology. The dataset contained 20 international stock market indices, and covers a period of over 12 years. 12 VaR estimation methods from the extreme value theory and from the quantile regression sphere were applied. The VaR estimation methodology that distinguished itself through superior accuracy consists of a combination of the generalized Pareto distribution (GPD), the APARCH model (estimated based on the asymmetric normal probability distribution) and their application in a rolling window approach. An advantage of this method is given by the fact that the tail of the financial returns distribution is modeled directly and its asymmetry is also taken into account. The APARCH model also incorporates the heteroskedasticity present in the series of financial returns. High accuracy forecasts were also obtained with some quantile regression models, and the model proposed in this paper, GPD-APARCH skew applied with the rolling window method obtains similar results in terms of accuracy as the CAViaR models (based on quantile regression). The data covers a variety of financial markets: developed, advanced emerging, secondary emerging, and frontier, according to the FTSE Russell classification.

This paper also includes applications for intraday data. One of the data sets considered in the study consisted of 12 assets from Bucharest Stock Exchange - intraday data. The VaR measure was estimated on this set, using both classical and specific methods for high frequency data. Higher accuracy was recorded for measures based on realized volatility and for a VaR measure that takes liquidity into account. The major limitation in this study was the small size of the data set, and the impossibility of updating it due to data unavailability. However, it has been observed that liquidity has a strong influence on the accuracy of VaR measures and it is recommended to test these measures on a larger sample of high frequency data.

When estimating VaR for the intraday return series of the BET index, among the analyzed estimation methods, a VaR method based on an ARFIMA model, applied directly to intraday data, and a combination of filtered historical simulation and an APARCH model (applied after data aggregation at a daily frequency) were noted. Similar to the previous case, the major limit is given by the data insufficiency. The time series covers a long period of time, but the analysis was conducted only on the BET index, due to the unavailability of other data series. The accuracy of the models was assessed by the failure rate, Lopez's quadratic loss function, and Kupiec's unconditional coverage test.

One limit of the empirical studies conducted in this paper is given by the limited availability of data for the Romanian capital market, especially intraday data. The portfolio of financial assets considered in Chapter 3 could also be expanded in a future research. The accuracy of VaR estimation methods was analyzed only for capital market data. Possible future research directions would be to apply the methods of estimating value at risk, identified in this paper as being the most appropriate, for data on oil markets, commodity markets, exchange rates and even on digital currency markets. Another direction for extending the research would be the application of VaR techniques on composite indices, as in this paper only stock indices containing the most liquid assestes were considered. At the same time, research can be directed to the study of other risk measures, such as Expected Shortfall (ES) or even to the use of both measures, VaR and ES, in order to have different perspectives in risk measurement.

One of the key aspects to be taken into account in statistical modeling is the correct highlighting of data characteristics. If this aspect is neglected, the results will be poor in terms of quality and, at the same time, inaccurate. In the case of financial returns, it is necessary to take into account heteroskedasticity - by using ARCH-GARCH family models, asymmetric information - the models proposed in this paper take into account this feature, as well as fat tails. This last aspect, if not treated properly, can lead to biased results and even to risk underestimation. Some methods used, such as those from extreme value theory, model the tail of the distribution directly. Given the above arguments, it can be noted that classical methods of risk quantification (such as the Riskmetrics approach) are not adequate for financial returns. Precisely for this reason, it is recommended to use methods that have a higher degree of complexity and take into account as many aspects of the financial returns series as possible.

In the current paper the methodology based on filtered historical simulation in combination with an asymmetric model used for volatility was repeatedly highlighted, at both the univariate (FHS-APARCH) and the multivariate levels (FHS-GARCH-aDCC). The combination of the generalized Pareto distribution, the APARCH model (estimated based on the asymmetric normal probability distribution) and the rolling windo method was also highlighted. All these techniques have a relatively high degree of accuracy in the process of forecasting value at risk.

An important aspect of this paper is the applicability of the presented methods. These methodologies can be useful for both individual investors and companies who want to manage their risk by anticipating their potential losses. Some methods are applicable to individual financial assets, while others can be used to assess and decompose the portfolio risk. VaR estimation techniques can also be used to adjust or establish the risk policies of entities operating in the field of financial markets, they can be used by banks in order (but not limited) to establish their risk appetite, but they are also useful, to regulators. The latter could use VaR estimation methods for the simulations performed to set and/or impose limits or restrictions for the financial market participants.

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