BABEŞ-BOLYAI UNIVERSITY OF CLUJ-NAPOCA

FACULTY OF EDUCATION THE DOCTORAL SCHOOL OF

EDUCATION

The Contribution of a Learning Strategies Program to Students' Achievement and Problem – Solving Skills in Geometry

Long Abstract

CONDUCĂTOR DE DOCTORAT
DOCTORAL COORDINATOR
Prof. Vasile Chiş

Student-doctorand
Doctoral student
Kivkovich Nava

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Kivkovich, N., Chis, V. (submitted). The Contribution of Using Geometry Learning Strategies Program for All Participants in the Learning-Teaching Process. Fifth Edition of the Education, Reflection, Development 2017, Cluj- Napoca, Romania.

Kivkovich, N., Chis, V. (2016). Learning Abilities and Geometry Achievements. The European Proceedings of social & Behavioral Sciences, eISSN:2357-1330.

Kivkovich, N. (2015). A Tool for Solving Geometric Problems Using Mediated Mathematical Discourse (for Teachers and Pupils). *Procedia-Social and Behavioral Sciences*, *209*, 519-525.

ABSTRACT

The research engaged in the effect of use of a learning strategies program (LSP) to student's achievement and problem-solving skills in Geometry, their attitudes to mathematics in general and Geometry in particular and perceptions of self-efficacy. The researcher is a math teacher in high school for 21 years.

Research Goals: To investigate the contribution of a Learning Strategies Program (LSP) to students' achievements in Geometry; To examine the effects of using LSP in Geometry on students' problem-solving skills; To evaluate the contribution of LSP to students' beliefs about mathematics in general and attitudes towards geometry in particular and to their perceptions of self-efficacy.

Methodology: Mixed-methods research. Research Tools: Testing previous knowledge and understanding in geometry, Raven Test, Van Hiele Questionnaire, Achievement tests in geometry, Questionnaire with regard to perceptions about mathematic and geometry, self-efficacy of learning questionnaire and Depth interviews with students. The research included 77 participants, 15-16 years old students in three 10 grade classes, in a high school in central Israel.

Quantitative Findings: Data analysis found that the experimental Group received the highest average grade of three groups in the final test. Moreover, the average grade of experimental group increased the most between three groups in the intermediate test and final test. Improvements also found in the mean scores for the experimental group at the seconed van Hiele test in all levels. In conclusion, The experimental group improved its achievements the most.

Qualititative Findings: Analyzing the questionnaires regarding attitudes towards geometry and mathematics, found that experimental group, holds positive attitudes towards geometry. The interviews' analysis, found the theme employing a strategy characterized in part or whole by the 'Thinking Person' strategy was most widely used by the experimental group. Reinforcement found in the visual protocols.

Conclusions: Students intensive thinking processes can be improved by teaching thinking processes that include the 'Thinking Person' strategy. The effect of mastering this strategy can influence students in other areas, such as improved emotions, self-image and motivation. Teaching the strategy also contributes to raising levels of achievement.

INTRODUCTION

The significance of geometry is expressed in its being one of the central subjects in the curriculum for elementary and junior high schools. It is a subject with a deductive structure that is made up of investigation and thinking strategies.

Geometric shapes in general and polygons in particular comprise one of the main topics that accompany the student from elementary school to junior high school. Difficulties arise from the need to understand mathematical language in the field of geometry, while integrating it with prior knowledge, in parallel to the students' level of mental development. These difficulties require teachers with high discourse capacity, who are qualified and capable so that they can fulfill the task of mediating the material to the learner in a heterogenous group of learners, while using varied tools and skills and develping solution strategies that will constitute a significant and meaningful mediation tool. Geometry teachers need to develop expertise in the formal part of the subject and its combination with verbal explanations, which have special wording geometry(Kivkovich, 2015).

The current research is an action research in the field of education. For the purpose of this research, an intervention program - **Learning Strategies Program** (LSP) was constructed as a mediation tool for teachers to teach strategies of solving geometric problems, which includes the strategic mediation tool 'thinking person'.

Gap in Knowledge

Various studies (Hershkovitz 1992; Kramarsky 1996, Pelach-Borowitz 2004; Shalev, 2002) have dealt with the relationship between teaching strategies and student achievements. Strategies for solving geometric problems include: computer based practice strategies, using drawings in geometry and Van Hiele's levels of thinking model (Idris, 2009; Kutluca, 2013; Patkin, 1994a), and different strategies used by mathematics teachers in teaching computational geometry and trigonometry (Aydoğdu, 2014).

Some studies engaged in various factors and their connection with geometric problem-solving ability, such as motivation, emotion, drawing skills (Bailey, Taasoobshirazi & Carr, 2014), while others investigated the relationship between children's working memory capacity and mathematics achievements (Holmes & Adams, 2006). Additional studies were reviewed for the purpose of writing this

research. However, no studies were found on developing models of strategies teaching and geometry in high school and their use in high school. From this it is possible to see the contribution of this study in the area of teaching geometry in combination with a special teaching strategy. The program uses teaching strategies that have been found to encourage learning such as teaching based on visual demonstration; teaching that encourages problem based learning; teaching that encourages inquiry based learning and teaching that encourages reflective learning.

These strategies are expressed in the correct use of mathematical and geometric language in particular, while using visual grading, finding memory supports for prior knowledge, assessment and drawing conclusions and using them to reach a solution.

The goal of the research was to investigate the contribution of Learning Strategies Program (LSP) to student's achievement in Geometry; To examine the effects of using a LSP in Geometry on the student's problem solving skills; To evaluate the contribution of LSP to students' beliefs about mathematics in general and attitudes towards geometry in particular and to their perceptions of self-efficacy.

The Research Questions were:

1. In what way has the Learning Strategies Program (LSP) influence the students' achievements in geometry?
2. What is the contribution of the Learning Strategies Program (LSP) to the students' ability to solve problems in geometry?
3. What is the contribution of the LSP to students' beliefs about mathematics in general and attitudes towards geometry in particular and to self-efficacy perceptions?

The research hypotheses were: The experimental group will have higher results in a geometry test at the end of the program and will demonstrate greater improvement between the first and second test in geometry. Analysis of students' results will show a difference between the groups with regard to problem solving skills. The LSP will improve and demonstrate more positive beliefs about mathematics and attitudes towards geometry, and more positive perceptions of self-efficacy.

Research Limitations: This research was carried out with a small sample of students as enforced by the choice of school in which the researcher is an integral part to be part of the research. Accessibility to a larger school in which there are more classes and a greater number of students was not possible.

This research is a first groundbreaking research that integrates an intervention program that includes a school teaching program. As such this research was limited in time because it had to meet the needs of the planned school timetable for a teaching program. To overcome the limitation, the intervention program was built on the basis on the needs of research in coordination with the mathematics team to integrate LSP in the curriculum in an optimal manner. As such, the time limit for teaching the strategy, students internalizing and implementing it was a decisive factor in getting insignificant quantitative research results. It is possible that if the process of teaching the intervention program was expanded for a longer period of time, it would be possible to get more significant results.

Key words: Geometry, Student's attitudes, Learning Strategies Program (LSP), Strategies, Learning through Mediation, Scaffolding.

LITERATURE REVIEW

The literature review **confirms** that the program that integrates teaching strategies in geometry is based on Van Hiele's Theory (1959), which emphasizes the acquisition of skills as fundamental to the learning of geometry, whilst understanding that it is necessary for students to reach a level of mental development required for understanding geometry, as noted by Sarfaty and Patkin (2011), based on Van Hiele, advancing from one level to another is more dependent on teaching than on age or biological maturity. Van Hiele's Theory (1959) is the main theory engaging in the development of geometrical thinking. The theory discusses the various stages of mental development of geometry learners. Geometry learners' mental development can be hierarchically arranged on five levels. Sarfaty and Patkin (2011) listed the levels of geometric thinking: **Recognition**, **Analysis**, **Ordering**, **Deduction, Rigor**.

The important skills for studying geometry according to Hoffer (1981) are: **visual skills, verbal skills, sketching skills, scoring, logic skills and practical skills**.

According to Hoffer a combination of all the aforementioned skills in teaching geometry, will contribute to increasing students' interest in the subject and their understanding of learned material

Piaget (1960) and other researchers have emphasized the covert cognitive processes occurring within the learner, meaning the learner's individual development.

In contrast, Vygotsky (1978) emphasized social-cultural processes as the source of intrinsic cognitive change. Vygotsky argued that developmental processes and learning processes are not the same. The developmental process follows learning. The latter takes place in the zone of proximal development (ZPD) and the learning process becomes a developmental process (Miller, 2011). According to Vygotsky, what exists within the child as ZPD will appear in the future as actual development.

For a teacher to create quality mediated learning, use is made of dialogic discourse tools. Vygotsky (1978) shifted the focus of interest to the discourse developing between teachers and learners. Learning is a process that occurs in the interpersonal space between learners and significant others in the field where the learners wish to develop skills and knowledge. As mentioned, a teacher's control and strategic behavior in planning a lesson and in its course and assessing the situation at each stage is of the utmost significance.

Mathematical discourse requires a process of mediating the causality of mathematical phenomena between teacher and student (Regev & Shimoni, 2000). Therefore, the discourse is guided by presenting a problem and finding possible ways of solution.

The program uses discourse based on the **Commognitive** approach, which focuses on identifying changes in mathematical communication of learners and is suitable for present day changes in learning that advocates learning through participation. The communicative-socio-cultural approach, Vygotsky (1978) in particular, claimed that the process of development begins with child-adult interaction. In time this interaction is assimilated and becomes an internal cognitive tool. A learner's participation in dialogue with adults creates high level learning. The program relies on the notion that a student can acquire not only knowledge or skills, but also new cognitive structures. According to Feuerstein (1998), this is achieved through mediated learning, where a learner interacts directly with the environment. This direct interaction and exposure to stimuli provides a learner not only with knowledge or skill, but also ways of observation, approaches and ways of finding the link between them. Moreover, **the Commognitive** Approach (Sfard, 2007) relies on Vygotsky's theories. The approach refers to thinking and to language as communication based on similar patterns, where thinking is the individualization of interpersonal communication. This notion is based on Vygotsky's theory according to which high mental functions such as problem solving capability, drawing conclusions,

overcoming obstacles and others first develop in the social realm through language, which serves as a mediation tool that becomes an instrument that shapes the user's consciousness. Dialogic discourse in the program is of great significance, expressed in asking questions and active listening, which open the door to the possibility of negotiating developing knowledge. A teacher has to use responsible judgment regarding management of the process and use this judgment in questions, doubts and examination until a solution is reached. A student will then learn from this experience and expand it as a tool for coping with questions and other problems in the field of geometry in particular, and mathematics in general.

A strategy of teaching in stages, according to Galperin (1992a 1992b), serves as a guideline for teachers and not as a collection in a teaching guide that must accompany every individual. He particularly emphasized four stages (steps) of the teaching-learning process: (1) orientation, (2) communicated thinking, (3) dialogical thinking and (4) acting mentally. The nucleus of teaching and learning is found in conceptualizing mental process and abilities originating in significant shared activities, presenting children with such activities, giving them cognitive tools and directions to a process that leads to their development. These ideas can help develop a framework in which there is a more profound reference to mutual influences between teaching, learning and development.

The process of finding a solution by teacher and student together, with correct use of geometric language while using visual grading, finding memory supports for prior knowledge and its use, including asking questions, will advance students in authentic mathematical activity, bring them into contact with problems, methods and many solutions, advance their creativity and perhaps even serve as a bridge for students' thinking from concrete to abstract both in geometry and coping with abstract situations in life. Teachers in the approach are required to be intellectual, broadly educated who operate as legitimate and autonomous agents of culture and knowledge (Beaty, 2001). They have to be concerned with students' identifying with them and cultural values.

Teaching strategies are defined as ways that the teacher takes to accomplish the objectives of the lesson (Schroeder, Scott, Tolson, Huang & Lee, 2007). Learning strategies are a way to improve teaching and learning. In addition, they are a series of cognitive processes that influence information processing in order to provide students with tools that will help them learn, solve problems, and complete tasks

independently. Studies of learning strategies show a general agreement among researchers regarding the contribution of learning strategies to the improvement of the quality of learning. The more supportive and transferable a strategy is, the more it will benefit teaching and learning (Nisim, Barak and Ben-Tzvi, 2012).

Feuerstein (1998) believed the student is able to acquire not only knowledge and skills but also new cognitive structures the realizarion of which requires an investment of effort and resources. This ability is imparted to students through mediated learning in which he or she has direct interaction with the environment through direct exposure to stimulion. This direct interaction and exposure to stimulion provides the learner not only with knowledge or skill, but also ways of observation, approaches and ways of finding the link between them.

For the teacher to produce whole quality mediated learning, the program uses the tools of dialogic discourse. The teacher's mastery and strategy in preparing the lesson and during the lesson while performing assessments at each stage is of the utmost significnace. Mathematical discourse requires a process of mediating the causality of mathematical phenomena between teacher and student (Regev & Shimoni, 2000). In this program, the teacher has to choose problems that are suitable as being the focus of the geometric discourse. The teacher ought to be able to listen, know how to identify how the participants think, their level and the difficulties they encounter, and be able to respond to each student. Furthermore, the teacher has to pay attention to the different levels of participants and sense whether all participants have reached a state where they have understood the required minimum at the end of each cycle of presenting a problem, suggesting solutions, identifying and applying solutions.

Moreover, there are students' variables that can also influence the learning process and the teacher need to be aware to them in order to produce whole quality mediated learning, such as motivation. Skager (2014) defined intrinsic motivation as the ability to persevere with learning with neither reward nor external sanction, even without formal frameworks and despite other sorts of temptation. Intrinsic motivation is characterized by activities for which people receive no external remuneration, and whose performance is for the sake of such activities themselves.

Ames (1990), research showed that intrinsically motivated students tended more to carry out challenging tasks whereas extrinsically motivated students sufficed with less complicated tasks. Motivation and its effect can be controlled by self-regulation

using various strategies that everyone can employ. Students can overcome their extrinsic motivation for a task by promising themselves external prizes or positive activities such as: watching television or conversation with friends or to improve their intrinsic motivation to fulfill a task by making it more interesting or employing control orientation focused on learning (Pintrich, 2000).

An additional variable is meta-cognition, the realization of cognition, characterized as a high-level thought process, including active control and orientation of cognitive processes when resolving a task, whilst understanding how the task is performed (Garner, 1987). Meta-cognition is high level thinking responsible for active control of cognitive processes. We deal with meta-cognitive activities every day and at various opportunities in order to solve problems, overcome obstacles, achieve aims and succeed in studies. The simplest definition of meta-cognition is Flavell's (1979), knowledge and cognition of cognitive phenomena.

The process of combining meta-cognition is essential in supporting learning generally and solving problems in particular (Bransford, Brown & Cocking, 2000). Students have to understand the 'what' of a cognitive theory, but in addition, the 'how', 'why' and 'when' to use the appropriate strategy (Kramarsky & Gutman, 2006). Nonetheless, cognition and meta-cognition mostly do not suffice in order to advance students' achievements. As been reviewed, learners also need intrinsic motivation in order to want to use various strategies and direct their efforts correctly (Pintrich & De Groot, 1990).

Another important variable in learning process is Self-efficacy, a concept proposed by the American psychologist, Albert Bandura in 1977. This is a central concept in Bandura's overall theory regarding socio-cognitive learning. Socio-cognitive learning theory maintains that people act in society and react to what takes place around them using thought and evaluation processes. People do not just react to environmental stimuli and reinforcements, but also employ thought regarding these same stimuli, interpret them and according to their own interpretations, react. Self-efficacy is people's belief that they can successfully carry out certain required behaviors in order to promote certain desired outcomes (Bandura, 1977). Self-efficacy is based on self-perceptions of knowledge, personal ability, to perform and control, and is linked to specific future activities. To distinguish from personal qualities, self-efficacy depends on specific areas and aims as well as the complexity of a given task. self-efficacy beliefs influence how people think, feel, motivate themselves and act

(Zulkosky, 2009). Self-efficacy was found to be connected to a wide variety of adaptive learning results such as high levels of effort and perseverance in difficult tasks both in experimental and correlation studies, among students from different age groups and genders (Bandura, 1997; Pintrich & Schunk, 2002). Nonetheless it is not sufficient to recognize these factors in order to predict successful performance. This refers to people's beliefs in their ability to adjust their skills, cognitive resources and required ways of operating in order to advance their aims. This means that the greater one's perception of one's self-efficacy so is one's willingness to realize one's aims.

In order to improve students' abilities to solve geometric problems, external support or essential scaffolding is needed to make cognitive and metacognitive processes required to solve them easier. According to Greenfield (1999), scaffolding helps structured learning. Providing scaffolding is a process in teaching, when teachers provide students with cognitive, motivational and emotional support during learning in order to help students develop independence in learning. From a practical point of view, it appears almost impossible for any human to provide appropriate support individually to each and every student, therefore it is important to teach them to relay too on scaffolding and develop independent learning abilities. Scaffolding was first defined by Wood, Bruner & Ross (1976) as adults who control elements of a task beyond the abilities of learners, which allow learners to concentrate on those parts of the task that are within their ability range. Based on Vygotsky (1978) who argued that students learn with adults or colleagues with higher abilities than theirs and learning is done in the developmental area in which learners are found (ZPD). This area is defined as the distance between learners' true developmental level when solving a task, and their potential developmental level when they are helped by guidance from adults or colleagues with higher levels of ability. The aim of scaffolding is to bring about higher levels of ability in learners that they can achieve with the help of this scaffolding. In order to help learners be active, self-regulate their learning, much has been said in recent years about the need for various sorts of scaffolding. Since self-regulated learning is an acquired ability, a way must be found to train this ability in learners. By providing scaffolding, that is to say help or direction (crutches) at various stages as students require help, we can show them how to overcome difficulties. At later stages, students will no longer need crutches and will be able to self-regulate without external assistance.

Self-regulated learning, that has become one of growing interest following research findings indicating that self-regulated learners are more successful in their academic studies that those who do not (Zimmerman & Bandura, 1994). different definitions exist, but one can say that the common aspects of definitions of self-regulated learning are conscious and intentional use of specific processes; students' strategies or reactions; circular processes of self-feedback during learning, in which students monitor the effectiveness of their learning strategies and react to this feedback in diverse ways, starting with certain changes in their self-perceptions and ending with open changes in their behavior, such as changing a learning strategy (Birnbaum, 2000). Self-regulated learning refers to the ability of learners to consciously monitor their thoughts, emotions and behaviors during learning. The accepted definition of the concept includes meta-cognitive, motivational and behavioral characteristics that testify to the active participation of students in the learning process.

The program also included meta-cognitive learning that encourages awareness of thinking. One of the ways of implementing this is through **cooperative learning**, which is the opposite of frontal learning. It is democratic and meaningful learning, that takes place in small groups and is based on reciprocal relations and discussions between learners. Cooperative learning relies on two basic assumptions: meaningful learning is independent learning, and people learn how to learn when they are full partners in the learning process (Avinun, 2013). These learning colleagues encourage one another to abandon mistaken perceptions and search for better solutions, help pupils develop social processes alongside cognitive processes such as control and evaluation, and has a positive influence on pupils' academic achievements, collegial relationships, learners' self-confidence, approach to learned subject and reducing fear of the learned subject (Slavin, 2008; Tarim & Artut, 2004). Moreover, cooperative learning is a more effective learning environment that encourages mutual enrichment and thinking, and mathematical communication providing explanations and reasoning (Mevarech & Kramarsky, 1997). Additionally, it enables passing on information in writing and orally and as such creates an environment that emphasizes **mathematical discourse** between students. The purpose of mathematical discourse is to lead learners to organizing and reinforcing mathematical thinking and transferring it fluently and clearly to colleagues. Mathematical discourse enables analysis and evaluation of colleagues' mathematical thinking and strategies (NCMT, 2000).

Cesar (1999) found that discussions between pupils positively affect their cognitive development and create a positive relationship with mathematics. Studies indicate that meta-cognitive mathematical discourse between pupils in small groups with the help of meta-cognitive guidance improved their achievements in solving mathematical problems, searching for links between the mathematical discourse and providing substantiation (Kramarsky & Mevarech, 2003).

Therefore, the dialogic discourse is of great significance in a class where the teacher's control focuses on discourse and knowledge. All interaction between adult and child carries the nature of mediated learning experience. The ability to mediate depends neither on education nor on the technological level of culture. Different studies (Hershkovitz, 1992; Kramersky, 1996; Pelach-Borowitz, 2004; Shalev, 2002) have dealt with the link between using teaching strategies and pupils' achievements. Practice strategies based on computers, using drawings in geometry and Van Hiele's levels of thinking model (Idris, 2009; Kutluca, 2013; Patkin, 1994a) to solve problems in geometry as well as different strategies used by mathematics teachers in teaching computational geometry, trigonometry (Aydoğdu, 2014). There are those that dealt with different factors and thinks between them and the ability to solve problems in geometry, such as: motivation, feelings, drawing skills (Bailey, Taasoobshirazi & Carr, 2014). No studies were found that dealt with the construction of models of strategies in teaching geometry which constitutes a mediation discourse between teacher and pupil and their use in high school. As such, the researcher chose to conduct research whose aim is to examine the influence of a mediated strategic tool in geometry and its use on pupils' achievements in solving problems in geometry and their views with regard to mathematics and geometry in particular.

METHODOLOGY

This chapter describes the mixed methods research strategies, development of research tools, data analysis and ethical considerations. The research sought to investigate the contribution of a Learning Strategies Program (LSP) on students' achievements in Geometry, examine the effects of using a Learning Strategies Program (LSP) in Geometry on students' problem-solving skills, and evaluate the contribution of LSP to students' beliefs about mathematics in general and attitudes towards geometry in particular and to their perceptions of self-efficacy.

The research questions were (1) In what way has the LSP influenced the students' achievements in geometry? (2) What is the contribution of the LSP to the students' ability to solve problems in geometry? (3) What is the contribution of the LSP to students' beliefs about mathematics in general and attitudes towards geometry in particular and to self-efficacy perceptions?

It was hypothesized that the experimental group will have higher results in a geometry test at the end of the program, demonstrate greater improvement between the first and second test, that there will be differences between groups with regard to problem solving skills, and that the LSP will improve and demonstrate more positive beliefs about mathematics and attitudes towards geometry, and more positive perceptions of self-efficacy

Research Paradigm

Today two paradigms are popular among social scientists: positivism and post-positivism. A comparison of the two approaches reveals that the positivist approach is designed to test theories, according to the deductive approach, which starts with testing the premises of a theory by using empirical data. The constructivist approach, however, intended to construct theories and relies on the inductive approach that starts with data and attempts to construct a theory about a phenomenon from the observed data. It is important to note that these methods are not synonymous with qualitative and quantitative research concepts. However, the mixed-methods research that combines qualitative and quantitative data is often rather desirable (Bhattacherjee, 2012).

Research Approach

A mixed methods approach was used to examine the research hypotheses. The approach combines qualitative and quantitative research methods. Such a combination allows for effective research that is highly reliable, and combines a human aspect and a scientific one, by using various research methods in the different research stages (Tashakkori & Teddie, 2003). All this in order to help students overcome the difficulties in learning geometry in a program which combines the use of teaching strategies, such as proper use of mathematic and geometric language in particular, including visual grading and identifying memory supports to prior knowledge, using it in the process of finding a solution in solving geometry problem (Kivkovich, 2015).

Research Design

At the beginning of the research, a process of diagnosing and evaluating the subject with all its diverse aspects was carried out. After this, there was a need for a process by which to identify the diverse aspects of the subject, students and teachers. Only thereafter, could the second part of the research commence, which was building an intervention program - **Learning Strategies Program** (LSP). The LSP consists of six sessions of approximately four hours per session. Each session including teaching the geometry of polygons with an emphasis on a geometry learning strategy for solving problems in geometry, including: deductive reasoning methods, informed reading, scoring data, deducing data from existing data while using memory supports, drawing data, data in questions, concepts mentioned in mathematical language, to elicit prior knowledge. The lessons include a pedagogical sequence according to the learning program combined with a mediating strategy - 'Thinking Person' that was implemented in the experimental group.

The LSP intervention has two aspects: **Theoretical pedagogical** and **Mediating strategic**. The LSP includes: An outline of the **'Thinking Person'** tool and employing it while teaching the deductive language of geometry and teaching the tool and instructions of how to use it, **cooperative learning**, **mathematical discourse** and **meta-cognitive thinking.**

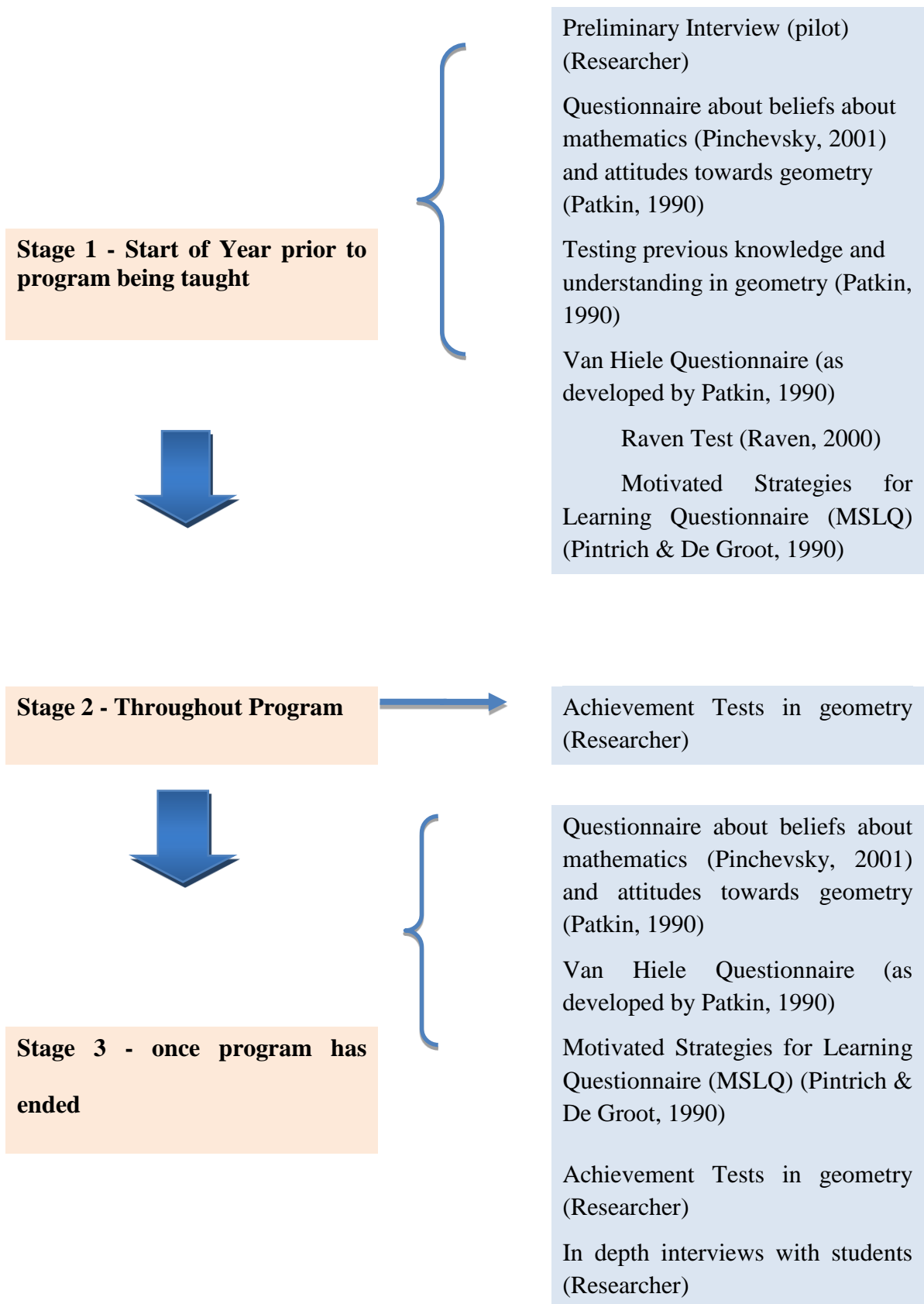The Strategic Tool - 'Thinking Person':

Visualization - The "Thinking Person" is a permanent symbol that constitutes a mediating tool between pupil and teacher during geometry lessons. The tool includes strategies for solving geometric problems and thinking. The tool is a mediating discourse tool between teacher and pupil and represents for them both a lots of information. During the process, the tool becomes an integral part of cognitive processes acquired by pupils as pointed out by Feuerstein (1998).

The symbol is a smiling face with a pair of horns, each of which has a meaning as a mediating dialogue tool - as thinking channels. One contains arches that demonstrate continuous thinking. The second characterizes the connection between conclusions derived from the arches in the first channel that represent data. This channel represents the stage of reaching widening conclusions - what is possible to be done with problem givens and the conclusions derived from these givens.

Linguistic meaning - in the first channel each arch is characterized by a geometric language regarding the logic of 'if' 'then'. This channel serves as a memory support to the continuing thinking process that pupils have to go through when in the thinking process while solving a problem (Hoffer, 1981) and according to progress in required levels of thinking through exercises presented to pupils (Van Hiele, 1959). When pupils build an arch in the first channel, they brainstorm in order to understand given meanings that appear in a problem. When they know that they have to create thinking arches, they internalize that they must carryout of process of thinking about prior knowledge. The givens allow them to focus on specific prior knowledge and they brainstorm about knowledge linked to this given, and bring them forward into their immediate memory, which allows them to draw from memory on prior knowledge in a way that will not require repetition or remembering sentence by heart. Pupils use the second channel of thinking people when they have to gather all the givens in a problem. After they have gone through all stage of building all possible arches using the first channel. In the second channel, pupils mix the information in order to draw conclusions that they are permanent processes: overlapping triangles (all sentences), and then it is possible to add various advance processes using the curriculum such as: triangles (all sentences), trigonometry - sines and cosines and more, according to the curriculum.

After this initial planning, it was possible to decide on the development or use of quantitative research tool for relevant measuring and evaluation as well as combining qualitative research tools to provide a more profound perspective of the research. Therefore, various quantitative tools were used - questionnaires were distributed as well as level and knowledge tests, and from the qualitative approach in-depth interviews as well as analysis of geometry tests. The following section will present these two research methods, as they were implemented in this research. After conducting the previous stages of constructing the intervention LSP and integrating the 'Thinking Person' intervention, and choosing suitable quantitative research tools, the next research stage was panned as presented in the following figure:

Stage 1 - Start of Year prior to program being taught	Preliminary Interview (pilot) (Researcher)
	Questionnaire about beliefs about mathematics (Pinchevsky, 2001) and attitudes towards geometry (Patkin, 1990)
	Testing previous knowledge and understanding in geometry (Patkin, 1990)
	Van Hiele Questionnaire (as developed by Patkin, 1990)
	Raven Test (Raven, 2000)
	Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich & De Groot, 1990)

| Stage 2 - Throughout Program | Achievement Tests in geometry (Researcher) |

Stage 3 - once program has ended	Questionnaire about beliefs about mathematics (Pinchevsky, 2001) and attitudes towards geometry (Patkin, 1990)
	Van Hiele Questionnaire (as developed by Patkin, 1990)
	Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich & De Groot, 1990)
	Achievement Tests in geometry (Researcher)
	In depth interviews with students (Researcher)

Figure 1: Research design

Research Population

The research participants were 15-16 years old pupils in three 10 grade classes, in a high school in central Israel. Pupils learn mathematics according to the curriculum determined by the Ministry of Education at a four point level[1]. Thus, the experimental design will include two groups:

Experimental group − one class that studied the subject according to the original program integrating strategies for solving problems in geometry- Group 3 (N=24)

Control group - two classes, the same age group that studied the issue in accordance with the regular curriculum without the integration of special teaching strategies - Group 1 (N=26) and Group 2 (N=27).

Research Tools

Quantitative Tools

A. Research Tools for Examining the Initial Level in the Three Groups

1. Testing previous knowledge and understanding in geometry, polygons, triangles, parallelisms (Patkin, 1990) - To test pupils; knowledge and understanding.

2. Raven test (Raven, 2000) - To examine general non-verbal cognitive levels, and evaluate development of thinking processes from the simplest level to analogous thinking.

3. Van Hiele Questionnaire (as developed by Patkin, 1990) - To examine different levels of thinking in geometry in accordance with Van Hiele's theory. This test was administered in the beginning to assess initial level, but also at the end of the program.

B. Research Tools for Examining Achievements in Geometry

Geometry achievements tests (Test 1, Test 2) - To examine pupils' achievements during the interaction and at its end with emphasis on level of achievement and use of mediating tool.

C. Research tools for Examining Attitudes towards Mathematics

We employed Pinchevsky's (2001) tool, the purpose of the questionnaire is to examine beliefs regarding mathematics according to three measurements: belief

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[1] Israeli high school subjects are taught on the basis of points, or credits − the highest level is 5 points; 4 points is the second highest level etc.

regarding the nature of mathematics, beliefs regarding means of teaching and learning resolving mathematical problems and beliefs regarding self-efficacy in mathematics learning. The questionnaire will be distributed to participants at the beginning and end of the intervention program.

D. Research Tool for Examining Attitudes towards Geometry

The questionnaire was developed by Patkin (1990), its items refer to a number of issues in the cognitive and effective areas, where pupils are asked to express their opinion on the subject of geometry and its importance, on teaching methods, on achievements and the relationship of pupils in general and each individually to the subject. The questionnaire will be distributed to participants at the beginning and end of the intervention program. The questionnaire's reliability is Cronbach's alpha = 0.84.

E. Research Tool for Examining Students' Perceptions of their Self-Efficacy of learning

Motivated Strategies for Learning Questionnaire (MSLQ) was put together by a group of researchers under Professor Pintrich & De Groot (1990). It examines orientations, motivations and use of different learning strategies among junior high and high school students. The questionnaire is structured from two key sections. The one relates to motivation and includes three scales meant to evaluate students; aims, beliefs and fears of tests in relation to their success in a particular subject (geometry in this case). The second part refers to learning strategies and includes two scales, relating to the use of cognitive and meta-cognitive strategies to motivate themselves to learn. The questionnaire consists of 44 items measured on a seven step Likert scale, where the highest score (7) represents full agreement with a statement and the lowest (1) representing complete disagreement.

Qualitative Tools

- **Semi-structured interviews** with visual examination and verbal protocols including solving a geometric problem, with a sample of students from each of the groups were used. The interview included various areas chosen and processed within the interview. It included a geometric problem that students had to solve while emphasizing the thinking processes they went through. The visual protocol, cognitive thought, thinking processes appearing in the strategy as well as additional elements that appeared in various questionnaires derived from this too. The aim was to reach an understanding of the gaps between

different groups according to various criteria examined in the questionnaires as well. Interviews held with a sample of 8 students from each research group that chosen randomly from each group, totaling 24 students.

- Geometry tests carried out during and after the intervention program in all three groups, the answers were analyzed according to geometric thought as well as 'Thinking Person' strategy criteria.

Validity, Reliability and Generalizability of Research

Classical criteria for examining the authenticity of scientific research findings are validity, reliability and objectivity (Sever, 2005). Validity is defined as the correspondence between the scientific explanations and description of a social situation as a phenomenon. Validity refers to the way a research is conducted, if the truth is depicted, it will apply to similar phenomena and in this case, the research is valid (Shlasky & Alpert, 2007; Shkedy, 2003). In this research, in order to verify the research and to prevent doubts regarding the validity of explanations of the research results, support is presented explicitly, and an unequivocal result of the research, through the use of accurate and reliable methods - in order to meet the expectations of the research.

Reliability refers to the reliability of the research tools chosen for research: the extent to which the result will recur after repeated tests, and whether the tools are not misleading or bias the research (Shkedy, 2003). There are two mechanisms that will ensure the reliability of qualitative research: triangulation and reflection. The research filed interfaces with all the domains of her life as a researcher. She is a mathematics teacher, and she teaches students by using varied teaching strategies, in particular in the field of geometry. All these help her involvement in the research itself, to reflection ability and to acquaintance with the students' needs.

Triangulation is the use of multiple sources of information for establishing a multi-dimensional and holistic description of the research environment and its different components. Triangulation allows for validating the data, and means the use of a variety of sources of information to increase the validity of findings. The purpose of triangulation is to strengthen the overall research findings and provide a better understanding of the researched phenomenon (Shimoni & Levine, 2002; Shkedy, 2003). In addition, reliability is expressed in the researcher's personal responsibility which is based on the findings, hence its being ethical-moral. The reliability of

findings is established by their ability to make us act upon them. Triangulation was achieved on a number of levels: using various research methods – case study, action research and statistical analysis of data in addition to interviews with students and teachers. This allow for cross referencing data and drawing conclusions in a more accurate manner. The fact that this research is based on a number of research methods, it will be possible to generalize its findings and apply the conclusions to geometry students. This research reflects the voices of participants without bias.

A qualitative researcher cannot expect other researchers reconstruct the course of the research and its findings. Therefore, the researcher has to reveal to colleagues and readers the details of research and decisions made so that they could judge the quality of the research and its logic. This presentation allows researcher and readers to examine the reliability of research (Merric, 1999; Arksey & Knight, 1999). In this research, the reliability was implemented both within and outside of academic research.

Generalization is the ability to copy the results onto different contexts, different environments and other people (Shkedy, 2003). In this research, Generalization was implemented through various research methods, and also trough theories compatibility with data.

The Researcher's Position

The research is action research in which the researcher studies his or her own actions to bring about professional improvement (Carr & Kemmis, 1983 in Kinney, 2006) in three aspects: the researched context, the participants' professional development and systemic change. Action research is a cyclic process that includes self-exploration, and combines action and reflective thinking about doing, and making improved assumptions for further action (Globman & Kula, 2005; Shlasky & Alpert, 2007).

The current research is an action research in the field of education. For the purpose of this research, an intervention program - **Learning Strategies Program** (LSP) was constructed as a mediation tool for teachers to teach strategies of solving geometric problems, which includes the strategic mediation tool 'Thinking Person'.

The research questions in the current research examined a teaching program integrating problem solving strategies in the field of geometry and the degree of its influence on students' achievements. Some achievements are measurable in the

qualitative approach and some in the quantitative approach. Therefore, the research combined both methods. The qualitative approach was used to collect information on the behavior of students as reflected in interviews. In addition, quantitative data was collected to describe the teachers' use of teaching strategies, students' level in geometry, achievements before during and after the program, examine general non-verbal cognitive levels, beliefs about mathematics and attitudes regarding geometry.

Since the research questions referred to a process that occurs over a long period of time, the researcher's continuous involvement in all stages of research was required. The researcher is a teacher at the school where the research was conducted, and thus had high access to the field. This allowed for monitoring the whole system of participants' behavior which developed naturally over the research period rather than set the data in defined time points, as is reflected in other research methods.

Ethical Aspects

Since this research deals with high school students, the researcher obtained the parents' consent in writing, and thus makes sure the parents agree for their children to participate and be interviewed. Additionally, all details remain confidential. Participants were identified by false names and the data will be presented as is, with no bias. Moreover, the researcher asked for the parents' permission for any type of publication, and did not enclose photographs or any detail that might expose the students' identity.

FINDINGS

The findings will be presented and analyzed according to research questions and hypothesis. But before, due to the heterogenic of the groups it will be presented analyzed data that examined the students' starting level.

Knowledge and Understanding of Geometry before the LPS intervention

Table 1: Variance results of comparison of groups in Geometry level test

|	Group 1 N=26		Group 2 N=27		Group 3 N=24		total		group	
	Mean	Std. deviation	Mean	Std. deviation	Mean	Std. deviation	Mean	Std. deviation	F	Sig.
Knowledge	10.077	3.7728	11.444	2.2927	9.917	3.4125	10.506	3.2388	1.871	0.161
Comprehension	15.038	2.6904	16.370	2.1865	15.458	4.0323	15.636	3.0432	1.358	0.264
Application	10.077	1.9985	9.185	1.8405	8.375	3.4365	9.234	2.5541	2.900	0.061
Analysis & synthesis	13.000	6.1838	17.667	4.7150	16.250	4.9454	15.649	5.6166	5.191	**0.008**
Total	48.192	10.5528	54.667	6.1080	50.000	13.6286	51.026	10.6487	2.821	0.066
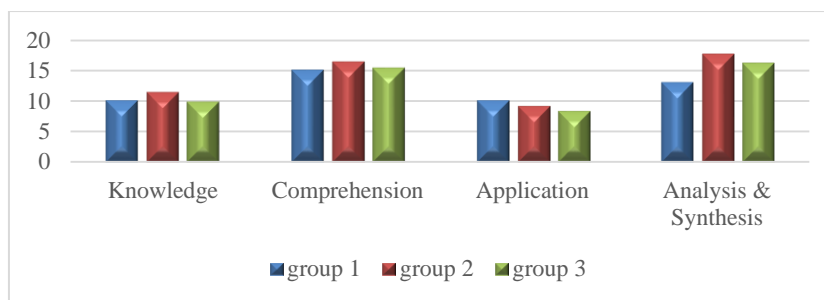
Figure 2: Geometry level test - comparison

Data analysis was carried out according to van Hiele's levels of thinking. It was found that in three of the four criteria, control group 2 received the highest scores in relation to the other two groups. Differential analysis was carried out on each of the variables, the results showed that there is no significant difference in the achievements at each level (Knowledge, Comprehension, Application), with the exception of Analysis & Synthesis, where a significant difference was found in the level of thinking $[F(2.73)=5.191; p=0.0080]$ between the groups. Bonferroni post hoc test fund that group 1 differed from group 2 [M=17.667 (Sd=4.7150), M= 13.000 (Sd= 6.1838) respectively].

Table 2: Results of Raven test for comparison between groups

Group 1 N=26		Group 2 N=27		Group 3 N=24		total			
mean	Std. deviation	Mean	Std. deviation	Mean	Std. deviation	Mean	Std. deviation	F	Sig.
71.000	20.88444	76.3704	20.04063	65.8333	23.46258	71.2727	21.58864	1.689	0.192
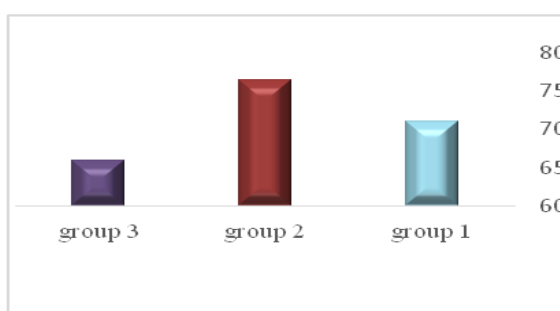
Figure 3: Raven test – comparison of groups

In order to examine differences between the three groups, a one-way ANOVA analysis was carried out between the groups. It was found that there were no significant differences between the groups, however it is possible to say that control group 2 got the highest score (M=76.370 (Sd.=20.040)) in the test and experimental group 3 got the lowest score (M=65.833 (Sd.=23.462).

Findings Related to the First Research Question: in what way has the Learning Strategies Program (LSP) influenced students' achievements in geometry?

Table 3: Geometry Tests During and After the LSP Intervention

	Group 1 N=26		Group 2 N=27		Group 3 N=24		total			
	Mean	S.D	Mean	S.D	Mean	S.D	Mean	S.D	F	Sig.
Test 1	56.923	37.923	75.556	35.445	72.083	30.924	68.182	35.493	2.033	0.138
Test 2	66.346	34.163	77.778	34.566	84.167	23.759	75.909	31.899	1.829	0.168
time									2.118	0.150
Time*group									0.322	0.726
group									3.129	**0.050**

It was found that experimental group 3 received the **highest** score on average in the test after the program (84.167). Moreover, the average score of experimental group 3 increased the most (12.084) of all groups between the intermediate test and test on completion of the program. It can be concluded that Hypothesis 1a was confirmed: the experimental group got higher results in a geometry test at the end of the program.

Table 4: Results of examination of differences between groups before and after their interaction – Van Hiele test

	Group 1 N=26		Group 2 N=27		Group 3 N=24		total			
	Mean	S.D	Mean	S.D	Mean	S.D	Mean	S.D	F	Sig.
Pre-van	3.538	2.611	5.629	4.271	5.416	3.832	4.857	3.719		
Post-van	5.5769	3.951	6.333	2.801	6.875	5.058	6.246	3.980		
time									3.308	**0.073**
group									2.033	0.138
Time*group									0.548	0.580
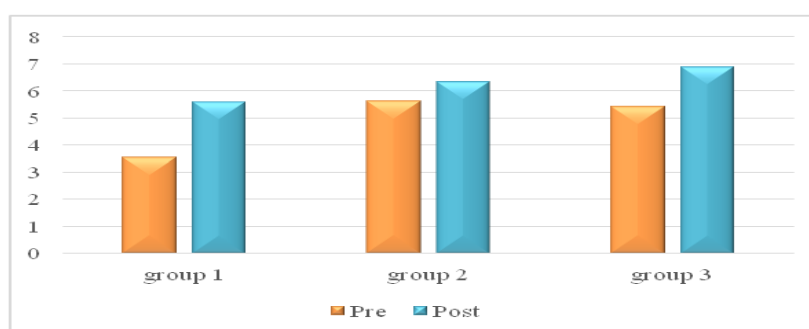
Figure 4: Comparison of groups Van Heile test before and after the intervention

In order to examine differences between the three groups before and after and the interaction between time and group, a two-way ANOVA analysis was carried out between the groups. It was found that there were no significant differences between

the groups [F(2.73=2.033; p=0.138]. The time-group interaction did not yield significance either [F(2.73-0.548; p=0.580]. However, it was found after the intervention, the experimental group received the highest score relative to the two control groups.

Thinking Levels of Van Hiele Test Before and After the Intervention:

Data analysis was carried out according to Van Hiele's levels: ***Recognition, Analysis, Ordering and Deduction***. At each level, there was improvement in the experimental group. Furthermore, in the level ***Deduction -*** experimental group **improved** its achievements significantly both before and after the intervention program in comparison to control group 2, who at this level achieved ***lower grades*** after the intervention program

Quantitative Analysis of Geometry Test 1 & 2:

The quantitative data was based on quantitative analysis of the two tests according to **criteria of the 'Thinking Person' Strategy**: An identical level was maintained in ***Comprehension***. Additionally, Improvement was found in the following categories: ***Reading, Application, Grade, Analysis and Synthesis.***

It can be concluded that hypothesis 1b: The experimental group demonstrate greater improvement between the first and second test in geometry was confirmed.

Findings Related to the Second Research Question: What is the contribution of the Learning Strategies Program (LSP) to the students 'ability to solve problems in geometry?

Analysis was based on four visual protocols identified and characterized in the interviews. Visual protocols were constructed from the interviews reflecting interviewees' verbal thoughts while solving a geometric problem: At the end of the interview students were asked to describe the process they had undergone in solving geometry problems. This description was used to construct a verbal protocol for each student. A visual protocol was constructed to demonstrate the verbal protocol.
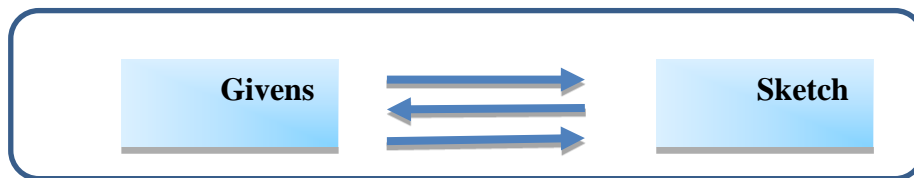
Protocol 1:
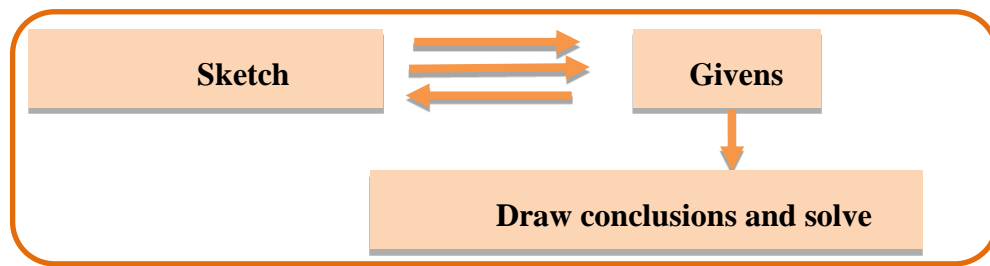
Figure 5: Visual description of Protocol 1

Visual Protocol 1 was used similarly by group 2 and experimental group but the most by group 1. In this protocol the student observing the givens parallel to observing the sketch. At protocol 1 conclusion derives from the givens only, and the problem is solved without observing the sketch again.
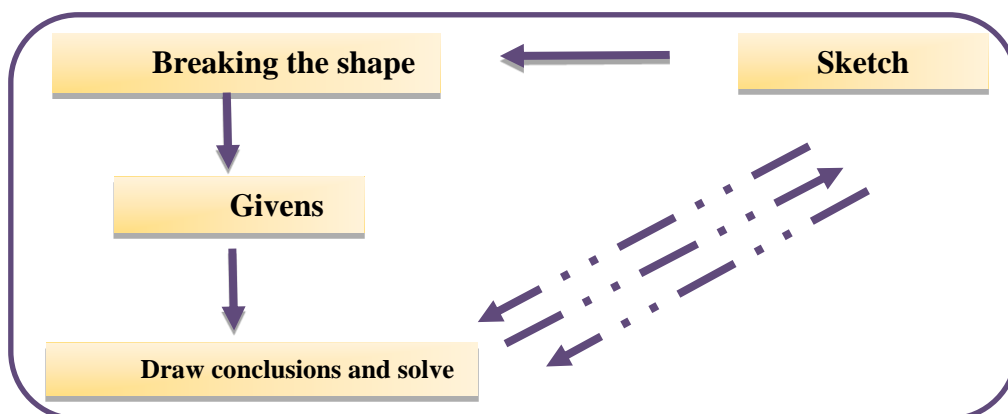
Protocol 2:

Figure 6: visual description of Protocol 2

Visual Protocol 2 used only by **one** interviewee from control Group 2. The student observing the givens parallel to observing the sketch. The use of this thinking protocol **does not** lead students to drawing conclusion and solving the problem. It can be stated that this solution process characterizes a student who does not master the thinking process required for solving problems in geometry.
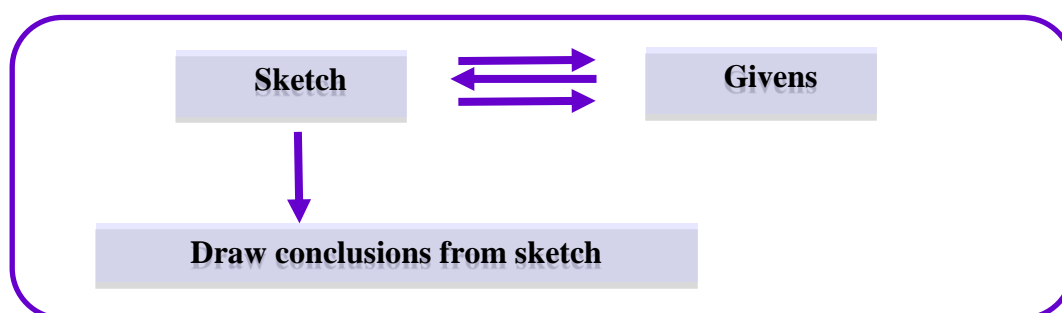
Protocol 3:

Figure 7: Visual description of Protocol 3

Visual Protocol 3 was used similarly by group 1 and group 2 but **the most by experimental group**. This Visual protocol describes interviewees' *complex thinking*

during the process of solving a geometric problem. Observed the **sketch** only and **break shape** into various components observed the problem and **drew conclusion** from the **givens**, while integrating them with givens in the sketch parallel to observing the figure and reach the solution. **This visual protocol is the *most similar* to the thinking processes in 'Thinking Person' strategy**, as a result it can be said that the research group used this strategy tool learned in the LSP, when the need arose to solve a problem in geometry.

Protocol 4:

Figure 8: Visual description of Protocol 4

Visual Protocol 4 was used similarly by all three groups. The student observing the givens parallel to observing the sketch. At protocol 3 conclusion derives from the sketch only without re-observing the givens and without drawing conclusions from the givens in the process. As such it is possible to say that hypothesis 2: There are difference between the groups in problem solving skills was confirmed.

<u>Findings Related to the Third Research Question: Are there differences between the experimental group and the control groups regarding beliefs about mathematics in general, attitudes towards geometry in particular and self-efficacy perceptions?</u>

With regard to the third research hypothesis, A two-way ANOVA analysis was carried out for Motivated Strategies for Learning Questionnaire (MSLQ), Attitudes Towards Geometry Questionnaire & Beliefs about Mathematics Questionnaire showed there was *no significant between the groups* with regard to capability, intrinsic motivation or self-direction. In addition, there was no difference between the groups in the tests prior to and after conducting the program with regard to beliefs about mathematics, learning mathematics and capability. It can be said that Hypothesis 3 The experimental group will demonstrate more positive beliefs about

mathematics and attitudes towards geometry, and will have more positive sense of self-efficacy was not confirmed.

Qualitative findings based on the interviews, were collected and categories chosen. It was found, in the category of learning methods, the theme of independent learning was employed most by experimental group, in the category of employing a strategy to solve geometric problems, the theme employing a strategy characterized in part or whole by the 'thinking person' strategy was found to be employed most in experimental group and there were no reports of not employing a strategy among interviewees from experimental group. The experimental group students emphasized the following points in the interviews: (1) **Motivation to Learn:** *"I compared myself to how I used to solve geometry problems and how I solve them now. Now it is more fun and easier. Now I am really enjoying geometry and want to succeed….I use what we learnt in class"* (2) **Employing the 'Thinking Person' Strategy**: *"Once, too I kind of did not like geometry, and then, when I sort of got used to it really …and this 'Thinking Person' helped gradually…now I am really love geometry"* (3) **Positive feeling towards mathematics**: *"You have made it (mathematics) more fun, sort of, more experiential, and not only numbers or grades"*

In new analysis of the qualitative findings from the attitude towards geometry and beliefs about math questionnaires, Statements were chosen from the two questionnaires and placed in categories chosen from the qualitative research, the Statements were grouped by criteria and placed in tables. From the tables it was possible to compare the three research groups and identify significant statement that scored the highest or the lowest in the experimental group. The aim was to reach and understanding the gaps between different groups according to various criteria examined in the questionnaires.

Table 5: Findings from "Attitudes toward Geometry" and "Beliefs about Mathematics" Questionnaires

Category	Statement	Before	After
Ways of learning and solving exercises	"I find it *difficult* to concentrate in geometry class lessons"	↑	↓
Independent learning ability	"I believe that independent learning contributes as much to the ability to solve math problems as teachers' instruction"	↓	↑
Positive approach to geometry	"I do **not** like geometry as a subject "	↑	↓

The change emphasizes the contribution of the LSP to the link between a sense of self-efficacy and students' level of involvement. This sentence emphasizes the teacher's important role as a mediator, who makes skills and thinking processes accessible to learners, thus turning them into independent learners. It indicates a direct influence of the intervention program on positive attitudes to geometry. As such it can be said that a qualitative analysis of the questionnaires found a difference between the groups, and particularly Group 3, which was found to hold positive attitudes towards geometry. This finding supports the third research hypothesis.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions will be described as factual, as answers to the research questions, and conceptual, as the researcher refers to the research questions.

First research question "In what way has the Learning Strategies Program influenced students' achievements in geometry?"

Factual Conclusion: The research group that underwent the intervention program got better grades in the geometry test at the end of the program as well as showing improvement in their Van Hiele thought levels. From this, it can be concluded that using the 'Thinking Person' strategy integrated into a teaching program significantly improves students' achievements in geometry as well as bringing them to higher thought levels.

Conceptual Conclusion: One can conclude and point to the fact that mediation characteristics included in the intervention program, are those that led to higher achievements. In addition, it can be concluded that the LSP intervention program includes means of developing students' thinking processes regarding the correct manner of solving problems in geometry. Moreover, it can be concluded that the environmental support from appropriate ways of teaching learning, while ensuring strict correct use of the strategy and its mediation by teachers, help bring students to higher thought levels.

In addition, this mediated discourse tool constituted a progress tool in teaching geometry and scaffolding in teachers' hands to advance students. It can be concluded that there is also importance in integrating this tool in teaching arrangements in curricula at the first stages of teaching deductive geometry because it is a basic, practical strategic tool that includes employing the language of geometry. Therefore,

it is important to use this tool in the initial stages of teaching arrangements for teachers throughout all stages of teaching geometry, so that it will be assimilated and internalized by students and become an intuitive structured tool in their thought process.

Second research question "What is the contribution of the Learning Strategies Program to students' ability to solve problems in geometry?"

Factual Conclusion: One can conclude that the intervention program and mediation of the 'Thinking Person' strategy affected thought criteria in solving geometric problems. These thought criteria are integrated into the intervention program and constitute a basis for the 'Thinking Person' strategy. It can also be concluded that the LSP intervention program improved the skills demanded for solving geometric problems integrated in the 'Thinking Person' strategy, such as visual, verbal skills, drawing skills and logic. Therefore, it can be concluded that the 'Thinking Person' strategy is a tool for significant collaborative activities that provide cognitive and directional tools for learners' development.

Conceptual Conclusion: Mastering the 'Thinking Person' strategy constitutes a positive influencing factor on the sense of concentration in geometry lessons, changing perceptions of self-efficacy, and significant improvement in attitude toward ability to learn independently. It can be concluded that mastering this strategy is what produced the change and brought students to a higher level of self-regulated learning, including meta-cognitive characteristics, motivational and behavioral elements, which constitute a basis for students' active participation in the learning process. Improving self-regulated learning abilities, the ability of learners to control their thoughts, emotions and behaviors during learning.

It can also be concluded that mastering the strategy significantly improves students' positive attitudes toward teachers' functioning as mediators and raised the great importance there is in linking between the role of teachers as mediators for students and success in mathematics in general and solving geometric problems in particular.

In addition, it can be concluded that one of the contributions of the LSP intervention program is that it provides tools to cope with geometric problems and raised implementation and meta-cognitive abilities, and this can be seen in the implementation of thought protocols parallel to the 'Thinking Person' strategy widely employed by the research group.

Additionally, one can conclude that using the strategy changed students' perceptions of the learning process; learning became more meaningful, through the use of memory supports, a greater ability to access immediate memory of studied subjects, use of ongoing thinking, significantly improved ability to break forms, process information and ability to understand forms, which are characteristics of the strategy.

Third research question "Are there differences between the experimental group and the control groups regard to attitudes to mathematics in general and geometry in particular and in self-efficacy perceptions?"

Factual conclusion: It can be concluded that although findings were not significant, support of the intervention program and teaching the mediated tool was still identified as influencing positive attitudes towards Geometry, a sense of satisfaction and success regarding achievements in Geometry, led the students to a high level of belief in their conceptual knowledge and personal capabilities, belief in professional knowledge that influences success in geometry problem solving and a high level of motivation.

Conceptual Conclusion: It can be concluded that there is a need to examine the issue of students' attitudes toward mathematics and geometry as well as their sense of self-efficacy. This because the research question was only partially upheld, therefore it is possible that there are additional factors that influenced the absence of significant differences between the research groups, for example the relatively short time range of the research. Therefore, it can be concluded that this issue should be examined over a longer term so as to see whether significance will be found. It should be noted that interviewees' reports helped greatly to conclude that despite the absence of significance, a difference was found in the research group's approach, in contrast to the other groups, to mathematics, geometry and self-efficacy. It can be concluded that indeed the intervention program does influence the more positive students approach in the context of positive feeling toward geometry, with reference to belief in subject knowledge as affecting success in solving geometric problems as well as high motivation to do so.

Recommendations for Implementation: It is possible to develop training arrangements for teachers that include developing a gradual teaching kit based on the geometry curriculum. Evaluation will be adjusted to age and content and integrates the 'Thinking Person' strategic tool in lesson arrangements. Evaluation will include

transition to content from the curriculum, detailed explanations of ways of teaching the strategy, in stages while teaching learned subjects.

In addition, it is recommended to expand the usages of teaching methods used in the intervention program in this research, among math teachers and geometry in particular, by raising teachers' awareness of their teaching methods, instill knowledge and information on the topic, as well as getting support from the Ministry of Education and assimilated the program's contents according to adjust age and content characteristics. Moreover, awareness must be raised about the importance of the connection between teachers and students and its influence of students' achievements. Therefore, this should be given a place in the curriculum both instrumentally and emotionally to fully realize students' potential.

Research Limitations: This research was limited in *time* because it has to meet the needs of the planned school timetable for a teaching program To overcome the limitation, the intervention program was built on the basis on the needs of research in coordination with the mathematics team to integrate LSP in the curriculum in an optimal manner.

Recommendation for Further Research: It is recommended to carry out broader and longer term research about this study on the use of an intervention program that includes the 'Thinking Person' strategy, after appropriate teacher training. To measure and evaluate the effects of the program over the years, students study geometry from the beginning. In this framework, it will be possible to use the research tool, the 'Thinking Person' strategy and integrate it into intervention program arrangements as well as using semi-structured interviews, throughout the research process.

Additionally, it would be worthwhile to carry out comparative research in various countries, adding elements such as gender, developed versus developing countries, adapting the strategy to mold independent learners in special education, by developing a teaching program that integrates the 'Thinking Person' strategy.

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