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On the infinite: Spinoza's theory of the eternity of the mind and Cantor's set theory

REZUMATUL TEZEI DE DOCTORAT

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Because of its abstruse nature, the infinite escapes an unambiguous understanding, for which reason I believe an approach from different perspectives would offer interesting insights. Although the concept of infinity involves many possible approaches (see chapter 1), I have focused here on two theories which describe the nature of infinity as harmonious order of determinate elements, in actual and continuous manifestations, and denying the validity of a temporal and spatial perspective in any endeavor to understand its nature.

One is a theory focusing on the ethical existence of man – Spinoza's theory of the eternity of the mind – (chapter 2), considered in terms of a-temporal knowledge. The other is a mathematical theory – Cantor's set theory (chapter 3), interested in extending the number into the infinite, establishing the nature of the continuum, or offering a new foundation for mathematics. The first is an expression of a specific theory of the mind, based on metaphysics and physics, the other became the foundations of mathematics and involves (explicitly or implicitly) elements of the unique form of Spinoza's philosophy.

The purpose of this paper is to emphasize the constitutive function of the idea of order and the fact that both Spinoza and Cantor maintain the possibility of comprehending the nature of the (actual) infinite by a process of ordering or harmonizing the parts into a consistent whole. Besides its role in the constitution of a philosophical and mathematical theory, the infinite proves itself fundamental in the configuration of method.

I examine what I consider to be the building blocks of their theories, trying to emphasize that the common thread among these different developments is the idea of order, understood in terms of ethical transformation (as intellectual order) in the case of Spinoza, and the constitution of a consistent theory of sets using some unifying concepts such as well-ordering, set, power and ordinal, in the case of Cantor.

With the exception of an article, I did not find an extended account of these two thinkers taken together, foe which reason I also discuss (in the fourth chapter) fundamental consequences of their thinking and under common frameworks. I start by analyzing the possible connection between Spinoza's concept of mode and Cantor's idea of set, and then the possibility of, and the kind of relation existing between the finite and the infinite. I suggest that they meet in the way they are dealing with specific concepts and problems: the one and the many, the idea of unity, coherence, limit, determination, causality, or the nature of reality and the best method to understand it. Both theories accept that there are ontological, epistemological, ethical and mathematical differences involved by the idea of the (actual) infinite and to try to understand what exactly one could understand by these differences.

Besides being cause for all determinations and limits, the concept of infinity can be conceived of as a *method* for a radical change of perspective, among different determinations, or levels of determination, implying conceptual jumps (the diagonal argument, the power set), a 'liminal' perspective and the constitution of certain orders in relation to specific *causalities*. The last aspect also involves a criterion which can be God, Substance, Nature, the Absolute infinite.

By offering a new reading of Spinoza's theory of the eternity of the mind and locating Cantor's work in a more general, philosophical framework, especially related to Spinoza's philosophy, I hope that new perspectives on infinity in general, and of knowledge in particular, would become possible and acceptable.

Selective bibliography:

- CANTOR, Georg (1932): *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*. (ed. E. Zermelo) Springer, Berlin.
- SPINOZA, Benedictus de (2002): *Spinoza. Complete Works*, trans. Samuel Shirley, Hackett Publishing Company, Inc., Indianapolis/Cambridge
- BELL John L. (2008): *The Continuous and the Infinitesimals in Mathematics and Philosophy*. Polimetrica, International Scientific Publisher.
- CAVAILLÈS, Jean (1962): *Philosophie mathematique*. Paris, Hermann.

COHEN, Paul J. (1966): *Set Theory and the Continuum Hypothesis*. W. A. Benjamin, Inc., New York.

- CURLEY, Edwin (1988). *Behind the Geometrical Method. A Reading of Spinoza's Ethics*. Princeton University Press, Princeton
- CURLEY, Edwin & MOREAU, Pierre-François Moreau (eds.) (1990): *Spinoza. Issues and Directions. The Proceedings of the Chicago Spinoza Conference*. E.J. Brill, Leiden.
- DAUBEN, Joseph Warren ([1979]1990): *Georg Cantor. His Mathematics and Philosophy of the Infinite.* Princeton University Press
- DELLA ROCCA, Michael (2008): *Spinoza*. Routledge, New York.

- DEDEKIND, Richard ([1901]1963): *Essays on the Theory of Numbers.* Dover Publications, New York.
- DESCARTES, René ([1641]1979): *Méditations métaphysiques*. *Objections et réponses, suivies de quatre lettres* Flammarion, Paris.
- EHRLICH, Philip (ed.) (1994): *Real Numbers, Generalizations of the Reals and Theories of Continua.* Kluwer Academic Publishers, Dordrecht.
- EWALD, William ([1996] 2005): *From Kant to Hilbert: A Source Book in the Foundations of Mathematics.* I-II. Oxford University Press
- FERREIRÓS, José & GRAY, Jeremy J. (eds.) (2006): *The Architecture of Modern Mathematics. Essays in History and Philosophy*. Oxford University Press, New York.
- FERREIRÓS, José ([1999] 2007): *Labyrinth of Thought. A History of Set Theory and Its Role in Modern Mathematics*. Birkhäuser Verlag, Berlin.
- FRAENKEL, Abraham A. et al. ([1958]2001): *Foundations of Set Theory*. Elsevier, Amsterdam.
- FRANZEN, Torkel (2004): *Inexhaustibility. A Non-Exhaustive Treatment*. A K Peters Ltd., Wellesley, Massachusetts.
- FRANZEN, Torkel (2005): *Gödel's Theorem. An Incomplete Guide to its Use and Abuse*. A K Peters, Wellesley, Massachusetts.
- FREGE, Gottlob ([1884]1960): *The Foundations of Arithmetic. A logico-mathematical enquiry into the concept of number*. Trans. J.L. Austin. Harper Brothers, New York.
- FREGE, Gottlob ([1893]1964): *The Basic Laws of Arithmetic. Exposition of the System*. Trans. Montgomery Furth. University of California Press, Berkeley.
- GÖDEL, Kurt (1990): *Collected Works* II. Oxford University Press, New York.
GRATTAN-GUINNESS, Ivor (ed.) (2000): *From the Calculus to Set Theory, 1630-1910. An Introductory History*. Princeton University Press, Princeton.
- GRATTAN-GUINNESS, Ivor (2000): *The Search for Mathematical Roots. 1870 –1940. Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel.* Princeton University Press, Princeton.
- GRENE, Marjorie & NAILS, Debra (eds.) (1986): *Spinoza and the Sciences*. D. Reidel Publishing Company, Dordrecht.
- HALLETT, Michael (1984): *Cantorian set theory and limitation of size*. Clarendon Press, Oxford.
- HELLER, Michael & WOODIN, Hugh, W. (eds.) (2011): *Infinity. New Research Frontiers.* Cambridge University Press, New York.

- HUNTINGTON, Edward V. ([1917] 2003): *The Continuum and Other Types of Serial Order*. Dover Publications, Mineola, New York.
- IRVINE, Andrew (ed.) (2009): *Philosophy of Mathematics*. North Holland, Amsterdam.
- JECH, Thomas J. ([1978]2006): *Set Theory*. Springer
- KANAMORI, Akihiro (2009): *The Higher Infinite. Large Cardinals in Set Theory from Their Beginnings*. Springer.
- KANAMORI, Akihiro & FOREMAN, Matthew (2010): *Handbook of Set Theory*.I-II-III. Springer.
- KENNEDY, Juliette & KOSSAK, Roman (eds.) (2011): *Set Theory, Arithmetic, and Foundations of Mathematics: Theorems, Philosophies*. ASL & Cambridge University Press.
- KOISTINEN, Olli & BIRO, John (eds.) (2002): *Spinoza. Metaphysical Themes.* Oxford University Press, New York.
- LEIBNIZ, Gottfried Wilhelm, Freiherr von (1989): *Philosophical Essays.* Trans. by Roger Ariew & Daniel Garber. Hackett Publishing Company, Indianapolis
- MANCOSU, Paolo (1996): *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*. Oxford University Press, New York.
- MESCHKOWSKI, Herbert & NILSON, Winfried (eds.) (1991): *Georg Cantor. Briefe.* Springer-Verlag, Berlin Heidelberg.
- MOORE, Gregory H. (1982): *Zermelo's Axiom of Choice. Its Origins, Development, and Influence*. Springer-Verlag, New-York.
- RUSS, Steve ([2004]2006): *The Mathematical Works of Bernard Bolzano*. Oxford University Press, New York.
- RUSSELL, Bertrand ([1903]1996): *The Principles of Mathematics*. W. W. Norton & Comp., New York.
- SANDYWELL, Barry (1996): *Reflexivity and the Crisis of Western Reason. Logological Investigations.*I. Routledge, London.
- STILLWELL, John (2010): *Roads to Infinity. The Mathematics of Truth and Proof.* A K Peters, Ltd. Natick, Massachusetts.
WERSINGER, Anne Gabrièle (2008): *La Sphère et l'intervalle. Le scheme de l'Harmonie dans la pensée des anciens Grecs d'Homère à Platon*. Éditions Jérôme Millon, Grenoble.
- WHITE, Michael J. ([1992]2002): *The Continuous and the Discrete. Ancient Physical Theories from a Contemporary Perspective*. Clarendon Press, Oxford.
- ZERMELO, Ernst (2010): *Collected Works. Gesammelte Werke*. Springer-Verlag, Berlin Heidelberg.