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Emergence of Structure in Complex Systems

Summary of the PhD Thesis

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Contents

Publications Related to Thesis

The following papers related to this Thesis have been published:

- Bartha, A. and Dumitrescu, D. (2011) Perturbation in Population of Pulse-Coupled Oscillators Leads to Emergence of Structure, *International Journal of Computers Communications & Control* 6(2):222–226. [Bartha and Dumitrescu, 2011]
- Dumitrescu, D. and Lung, Rodica Ioana and Nagy, Réka and Zaharie, Daniela and Bartha, Attila and Logofătu, Doina. (2010) Evolutionary Detection of New Classes of Equilibria: Application in Behavioral Games In *Proceedings of the 11th international conference on Parallel problem solving from nature: Part II* Lecture Notes in Computer Science, pages 432–441 Berlin, Heidelberg, Springer-Verlag. [Dumitrescu et al., 2010]

A number of patent applications related to the topics studied have been filed to OSIM:

- D. Dumitrescu, A. Bartha, M. Cremene. (2011) *Procedeu pentru rezolvarea problemei NP complete subset-sum.* Cerere brevet OSIM nr. A 00828/23.08.2011. [D. Dumitrescu, 2011]
- M. Cremene, A. Bartha, D. Dumitrescu. (2011) *Dispozitiv electronic pentru rezolvarea problemei NP complete subset-sum.* Cerere brevet OSIM nr. A 00884/12.09.2011. [M. Cremene, 2011]
- A. Bartha, D. Dumitrescu, M. Cremene. (2011). *Identificarea partilor sumei pentru probleme NP complete de tip subset-sum folosind un dispozitiv electronic digital.* Cerere brevet OSIM nr. A 00987/30.09.2011. [A. Bartha, 2011]

Chapter 1

Introduction

In this Thesis a number of topics related to complex systems and emergent behavior are studied. The notion of *emergence* is broad one with more or less different interpretations in Natural and Social Sciences and Philosophy. The common denominator is perhaps expressed as 'the whole is greater than the sum of the parts'.

From the point of view of this Thesis *emergence* is used in the context of complex systems where a collection of similar agents interact and as a consequence a new behavior, not found in individual agents, can be observed.

Chapter 2 of this Thesis is related to efficiently finding game equilibria using evolutionary computation methods. The concept of Generative Relations is used to transform the problem of finding game equilibria in an optimization problem where evolutionary computation methods can show their strength. Mathematical properties of different Generative Relations are studied and the importance of transitivity is put in evidence.

The Prisoner's Dilemma Game is often used to study social interaction because it captures the essence of a situation where agents have to choose between cooperation and betrayal. A multi-stage (extensive form) version of the Prisoner's Dilemma, called The Centipede Game is also well studied in literature. The Centipede game can be played both in extensive form and in normal form. In Chapter 3 an experiment is reported in which the effect of presenting the Centipede Game in normal form versus extensive form is studied. Results indicate that players are more likely to cooperate if the game is presented in extensive form.

In Chapter 4 a new model of spatial pulse-coupled oscillators is proposed. A particular aspect of this model is the clear separation of the mechanism responsible for the intrinsic (isolated) frequency of an oscillator and the mechanism responsible for oscillator interaction. Numerical experiments indicate that in certain conditions synchronization waves emerge in the oscillator population. The most interesting result of this study is that a required condition for the emergence of synchronization waves is the presence of noise in the intrinsic frequencies of the oscillators.

Chapter 5 proposes the concept of Cascade Systems as a generalized model of computation. It is shown how this model can be particularized into the familiar models of logic circuits, neural networks and cellular automata. A Cascade System is constructed to model the NP-complete Subset-Sum decision problem.

1.1 Thesis Structure

The topics studied in this Thesis are organized in separate chapters from Chapter 2 to Chapter 5. In Chapter 6 an overview of the principal results is presented and directions for further research is indicated.

Chapter 2

The properties of Generative Relations are studied, in particular the transitivity, which is important for developing efficient evolutionary methods for equilibrium search. A Generative Relation is in general not transitive. However, as demonstrated in this Thesis, the transitive closure of a Generative Relation induces the same equilibrium as the original relation. This can be used in practice to evolutionary methods for game equilibrium search.

Chapter 3 In practice people do not play games as the mathematical properties would suggest. The Centipede game introduced by Rosenthal is a well-studied game in the context of behavioral games. Experiments show that in general people do not play Nash equilibrium.

An experiment where students played the centipede game in two different setups is presented. The subject played the centipede game in two different setups: as an extensive form game and as a normal form game.

Experimental results indicate that people who play the centipede game in the normal form are using strategies closer to Nash equilibrium than people who play the same game in extensive form.

Chapter 4 Many physical and biological systems can be described by mathematical models of coupled oscillators [Mirollo and Strogatz, 1990]. Pulse-coupled oscillators are a special case when the interaction between oscillators is pulsatile. Each oscillator can enter a firing state when emits a pulse of physical signal. This signal is received by other oscillators in the population and as a result they may change their own oscillation frequency.

A population of pulse-coupled oscillators on a 2D grid is considered. Oscillators (grid cells) interact only if the distance between them is less than a given radius. The interaction strength decreases with the squared distance between the interacting cells.

Each oscillator is composed by two parts: one which is a classical integrate-and-fire oscillator and another one, which is sensitive to the signal emitted by other oscillators and integrates the received signal.

In the proposed model, extensive simulations indicate that a total synchronization of the population does not occur. Instead, waves of synchronously firing oscillators emerge.

Chapter 5 A concept called Cascade System is proposed intended to model complex systems. The principal goal of Cascade Systems is the conceptual separation of computational aspects from the interactions between the system components. This is accomplished by placing computational power in *nodes*, while the interactions are modeled as *arcs* connecting the nodes.

The Cascade System concept is used to build a system consisting of simple, similar modules capable of solving the NP-complete Subset-Sum problem.

1.2 Contributions

The main contributions of this Thesis are the following:

- A new Generative Relation inducing the Nash equilibrium.

  The proposed generative relation is based on the concept of an *ascendant path*, defined as a sequence of substitutions in the strategy profiles of the game, such that at each step only one player changes her strategy. A sequence of such substitutions is called ascendant if the sequence of payoffs associated with the player changing her strategy is not decreasing, and at least for the first step the inequality is strict. A dominance relation between strategy profiles is defined where a strategy dominates another if there is an ascending path from the latter to the former. It is demonstrated that this dominance relation induces the Nash equilibrium of the game in question.

- The transitive closure of a Generative Relation is shown to induce the same equilibrium as the original Generative Relation.

  A Generative Relation is not transitive in general. It is possible to consider the transitive closure of a Generative Relation as another relation which induces a game equilibrium. It is demonstrated that the game equilibrium induced by the transitive closure is the same as the equilibrium induced by the original relation.

- It is shown empirically that presenting the Centipede game in extensive form as opposed to normal form increases the probability of cooperation among players.

  The Centipede game is traditionally played in extensive form, where players take turns. The game can also be played in normal form where the players choose their actions once, independently of the opponent. The effect of different modes of play have been investigated in a series of experiments. The experimental results indicate a greater level of cooperation when the game is played in extensive form.

- A spatial model of pulse-coupled oscillators where emergence of synchronization waves is observed.

  A model of pulse-coupled oscillators on a regular two-dimensional grid is proposed. The oscillator model incorporates properties found in natural systems:

  - The interaction strength between oscillators decreases with euclidean distance, modeling the propagation of physical signals such as sound or light waves.
  - The interaction radius is limited to a maximum distance. Taken together with interaction strength decay, this is equivalent to a threshold of sensitivity under which the oscillators do not interact. This sensitivity threshold is often found in biological systems.

  - The mechanism responsible for the intrinsic oscillator frequency is separated from the mechanism responsible for oscillator interaction. This parallels the usually different mechanisms found in biological systems for sensory input and information processing.

  The results of numerical simulations of the proposed model indicate that synchronization waves appear in the oscillator population in the presence of noise in the intrinsic frequencies of the individual oscillators.

- A computational model called Cascade System with application to Subset-Sum problem.

  A computational model called Cascade System is proposed. As an application, a Cascade System for the NP-complete Subset-Sum problem is constructed. The model consists of a collection of simple, similar modules which work in parallel in order to provide a solution to the Subset-Sum problem. The simplicity of the modules suggest the possibility of a simple hardware implementation using commonly available circuits.

Chapter 2

Generative Relations

2.1 Introduction

Non-cooperative game theory provides models for strategic interactions. Normal form games describe situations where players' utilities depend on the actions of all other players.

The players interact simultaneously and the available actions and utilities of all players are common knowledge. The players are rational (utility maximizers) and each player is certain about the possible actions of the other players.

The starting point of standard game theory is a solution concept describing a stable situation of the game. Nash equilibrium ([Nash, 1951]) is the most known solution concept describing a steady-state situation in the game.

Several refinements of Nash equilibrium aiming to supply a complete characterization of (strategic) rationality have been proposed [Nagy et al., 2011], [Nagy et al., 2012]. Detecting Nash and its relative equilibria is a challenging computational task.

An approach where a given equilibrium concept may be characterized by a binary relation (Generative Relation) on the set of game strategies was considered by Lung and Dumitrescu [Lung and Dumitrescu, 2008], [Dumitrescu et al., 2009]. The equilibrium states of a game are the set of non-dominated strategies with respect to a Generative Relation. The Generative Relation inducing a particular type of game equilibrium is not unique.

In a population of strategies each strategy may be characterized by the number of strategies it dominates. Non-dominance associates a quality measure to each strategy and thus an evolutionary optimization technique may be used for equilibrium detection. A population of strategies is evolved using dominance-based fitness assignment or other appropriate quality concept related to the Generative Relation. The population is expected to converge towards an acceptable approximation of the game equilibrium.

In this section of the Thesis several aspects of using Generative Relations for game equlibria detection are studied. In particular, methods of defining Generative Relations based on unary or binary quality measures are considered. Also, it is demonstrated that the transitive closure of a Generative Relation induces the same equilibrium as the original relation.

2.2 Basic Notions

Definition 1 *A finite non-cooperative strategic game is defined as a system*

$$G = (N, S, u_i), i = 1, ..., n$$

where

- *N is a set of n players, $N = \{1,2,\dots,n\}$;*
- *for each player $i \in N$, S_i represents the set of available actions (or pure strategies);*
- *$S = S_1 \times S_2 \times ... \times S_n$ is the set of all possible situations of the game;*
- *an element of S is a strategy profile (or pure strategy) of the game;*
- *for each player $i \in N$, $u_i : S_i \to \mathbb{R}$ represents the payoff function.*

Let s^* be a strategy profile. Denote by (s_{i_j}, s^*_{-i}) the strategy profile obtained from s^* by replacing the strategy of player i by s_{i_j}:

$$(s_{i_j}, s^*_{-i}) = (s^*_1, s^*_2, ..., s^*_{i-1}, s_{i_j}, s^*_{i+1}, ..., s^*_n).$$

2.3 Nash Equilibrium

The concept of Nash equilibrium is based on the idea of stability against unilateral deviation. Each player has no incentive to deviate unilaterally as she cannot improve her payoff by modifying her pure strategy (action) while the others do not modify theirs. The strategy profile s^* is a Nash-equilibrium if the inequality

$$u_i(s^*) \geq u_i(s_i, s^*_{-i})$$

holds for every action s_i of each player i, $s_i \in S_i$.

2.4 Dominant strategies equilibrium

The standard domination concept of Game Theory between individual strategies is considered [Gintis, 2009].

Definition 2 *Let s'_i and s''_i be two pure individual strategies (actions) of the player i. The individual strategy s'_i weakly dominates the individual strategy s''_i if and only if the following conditions hold:*

$$u_i(s'_i, s_{-i}) \geq u_i(s''_i, s_{-i}), \forall s_{-i} \in S_{-i}$$

$$\exists s_{-i} \in S_{-i} : u_i(s'_i, s_{-i}) > u_i(s''_i, s_{-i})$$

Definition 3 *The individual strategy s_i' strictly dominates the individual strategy s_i'' if and only if the condition*

$$u_i(s_i', s_{-i}) > u_i(s_i'', s_{-i})$$

holds for each $s_{-i} \in S_{-i}$.

Definition 4 *The strategy $s_i \in S_i$ is said to be strictly dominant if s_i strictly dominates every other possible strategy of the player i.*

Definition 5 *Strategy $s_i \in S_i$ is weakly dominant if s_i dominates all other strategies of the player i, but some are only weakly dominated.*

Definition 6 *A dominant strategy (DS) equilibrium is a strategy profile $s \in S$ such that each player is playing a dominant individual strategy in s.*

In a DS equilibrium each player i plays a dominant strategy $s_i \in S_i$ regardless of the strategies of other players. Therefore in a DS equilibrium each player plays a best response that does not depend on the strategies of other players.

The iterated elimination of strictly dominated strategies results in a unique strategy for each player. The corresponding strategy profile represents the dominant strategy (DS) equilibrium.

2.5 Generative relations induced by quality measures

Many equilibrium concepts for pure strategies have been introduced A challenging problem is how to characterize equilibrium by a relation on the strategy set.

Such a relation may be called the generative relation of the equilibrium. Generative relations may offer algebraic and computational tools for describing and detecting equilibria. A refinement of a generative relation may be conceived for selecting an equilibrium for games having several equilibria.

2.5.1 Binary quality measures

Let S be the set of strategies. Assume there is a mapping $Q : S \times S \to \mathbb{R}$ that associates a relative quality to each pair of strategies.

Let x and y be two strategies (strategy profiles), $x, y \in S$. Then $Q(x, y)$ measures the relative quality of strategy x with respect to the strategy y.

In what follows we assume that a smaller value stands for higher quality.

The relative quality indicator Q can be used to define a relation $\leq_Q$ (or R_Q) on the set of strategies. The relation $\leq_Q$ is defined as

$$x \leq_Q y$$

if and only if

$$Q(x, y) \leq Q(y, x)$$

A strong version of the relation $<_Q$ can also be defined as:

$$x <_Q y$$

id and only if

$$Q(x, y) < Q(y, x)$$

Binary quality measures can be defined in several ways.

2.5.2 Unary quality measures

It is possible to consider unary quality measures as inducing generative relations.

Let us assume a mapping

$$A : S \to \mathbb{R}$$

associates a quality measure to each strategy profile.

We may interpret $A(x)$ as measuring the degree x supplies an approximation of a certain equilibrium E.

Let us define the relation $\leq_A$ as

$$x \leq_A y$$

if and only if

$$A(x) \leq A(y)$$

We may consider $\leq_A$ as a candidate for the generative relation of the equilibrium E.

2.5.3 Characterizing equilibria by generative relations

Let R be a binary relation on the strategy set S, i. e. $R \in S \times S$. If $(x, y) \in R$, we may say that the strategy x dominates y with respect to relation R.

Definition 7 *A strategy x is said to be non-dominated with respect to relation R if and only if the following condition holds:*

$$\nexists y \in S, y \neq x, (y, x) \in R$$

Let us denote by NDR the set of non-dominated solutions with respect to the relation R. Hence we have

$$NDR = \{s \in S, \nexists x \in S : (x, s) \in R\}$$

A subset $S' \subset S$ is said non-dominated with respect to the relation R if and only if

$$S' \subset NDR.$$

Consider a finite non-cooperative game G. Let R be a binary relation on the set S of game strategies. Denote by $E^* \subset S$ the equilibrium set of the game G.

Definition 8 *The binary relation R is said to be a generative relation for the equilibrium E^* if and only if the set of non-dominated strategies with respect to R equals the set E^* of strategies. Otherwise stated, R is a generative relation for E^* if and only if the equality*

$$NDR = E^*$$

holds.

2.5.4 Evolutionary equilibria detection

Assume it is possible to construct a generative relation R for the equilibrium E. Equilibrium detection is usually a difficult computational task.

Generative relations allow an evolutionary optimization/search technique to be applied for equilibrium detection. A sequence of approximations of the equilibrium set may be constructed by using selection based on generative relation and variation (search) operators.

The proposed approach can be essentially represented as a scheme:

- Find a relation R on the strategy set S such that the set of non-dominated elements with respect to R equals (or approximates) the equilibrium E. (Preferably R is a transitive relation); If a quality measure Q with respect to E can be associated to each strategy then a fitness assignment based on Q can be considered: $eval(x) = Q(x)$;
- Use an evolutionary optimization procedure to evolve successive approximations of the equilibrium set E;
- A selection procedure relying on the relation R is used. Usually this is a non-dominance based ranking mechanism. A direct fitness assignment process can also be considered in some cases.

2.5.5 Defining new equilibrium concepts by generative relations

We have indicated how a generative relation can be fruitful in detecting the equilibria it describes (or generates). The proposed approach can be captured by the scheme:

- Establish a generative relation of the equilibrium E (a formal algebraic characterization of the equilibrium E).
- Assign a fitness to each strategy based on the generative relation.

- Rank-based fitness assignment (inducing an appropriate selection operator) or other appropriate selection mechanism.
- Evolutionary detection of the equilibrium E.

2.5.6 A structural generative relation of Nash equilibrium

Let us consider a game G and two strategy profiles $a, b \in S$. Suppose there is a way to transform the strategy profile b into a by replacing (stepwise) in b the corresponding positions from a.

Let us denote by $r_1, r_2, ..r_m$ the positions in b replaced by the corresponding positions in a. Positions from a replaced in b are thus $a_r 1, a_r 2, ..a_r m$. We call $p = (r1, r2, ..rm)$ a *transformation path* or simply a *path*. Consider a process so that at the i-th step an elementary transformation involving the position r_i is applied. The transformation path is said to be *ascendant* if for each player r_i the payoff of that player increases by applying the elementary transformation $r_i, i = 1, ..., m$.

Hence for the player r_i the payoff associated to the strategy at step i is less than the payoff of the mutated strategy. For simplicity let us denote

$$(x_i, y_-) = (x_i, y_{-i})$$

The transformation process may be viewed as a procedure for constructing a sequence

$$c^0, c^1, ..., c^m$$

where the first element is the strategy profile b and the last equals the strategy a. This process is described as the iterative procedure:

$c^0 = b$

$c^1 = (a_{r_1}, b_-)$

$c^2 = (a_{r_2}, c^1_-)$

...

$c^k = (a_{r_k}, c^{k-1}_-)$

...

$c^m = (a_{r_m}, c^{m-1}_-)$

Definition 9 *We say that* $p = (r_1, r_2, ..., r_m)$ *is an* ascendant path *if and only if the payoffs corresponding to the modified positions are not decreasing, and at least for the first step the inequality is strict:*

$u_{r_1}(b) < u_{r_1}(c^1)$

$u_{r_2}(c^1) < u_{r_2}(c^2)$

...

$u_{r_m}(c^{m-1}) < u_{r_m}(a)$

The existence of an ascending path indicates an interesting relationship between strategies.

Definition 10 *Let $a, b \in S$. We say that strategy profile a dominates strategy profile b, and we write $a < b$, if and only if there is an ascending path from b to a. Let us denote by ND the set of strategy profiles that are nondominated with respect to the relation $<$, i.e.*

$$ND = \{x \in S | \nexists y \in S : y < x\}$$

Strategies that are non-dominated with respect to the relation $<$ may have some interesting properties. The main properties are established in the next Propositions, proved in the Thesis.

Proposition 1 *Each nondominated strategy profile (with respect to the relation $<$) is a Nash equilibrium, i.e.*

$$ND \subseteq N$$

where N denotes the Nash equilibrium set of the game G.

Proposition 2 *Each Nash equilibrium strategy is a non-dominated strategy (with respect to $<$), i.e. the inclusion*

$$N \subseteq ND$$

holds.

Proposition 3 *The Relation $<$ induced by the ascendant path is a generative relation of Nash equilibrium, i.e.*

$$N = ND$$

.

2.5.7 A unary quality measure inducing a transitive generative relation

Consider a strategy profile $x \in S$. Let us denote by $c(x)$ the number of positions the strategy profile x can be improved by unilateral deviations. We may express the quality measure $c(x)$ as

$$c(x) = m(\{i | \exists s_i \in S_i, u(s_i, x_{-i}) > u_i(x)\})$$

where m is a measure such as $m(A) > 0$ for each nonempty A. A simple and useful measure m is the cardinality, i.e.

$$m(A) = card(A)$$

The unary quality measure may define a binary relation

$$<_N \in S \times S$$

in a straightforward way:

$$x <_N y$$

if and only if the inequality $c(x) < c(y)$ holds.

The following propositions are proved in the Thesis:

Proposition 4 *A strategy profile x is a Nash equilibrium if and only if $c(x) = 0$.*

Proposition 5 *If x is a Nash equilibrium then x is non-dominated with respect to $<_N$.*

Proposition 6 *If there exists a Nash equilibrium then each strategy profile non-dominated with respect to the relation $<_N$ is a Nash equilibrium.*

Proposition 7 *If there exists a Nash equlibrium then the relation $<_N$ is a generative relation for the Nash equilibrium.*

Proposition 8 *The relation $<_N$ is transitive.*

2.6 Transitive closure of generative relations

Let R be a generative relation of a certain equilibrium E. The transitive closure R^+ of the relation R is defined as

$$R^+ = \cup_{n \geq 1} R^n$$

where

$$R^1 = R$$

and

$$R^k = \{(a, b) | \exists x : (a, x) \in R^{k-1} and\ (x, b) \in R^{k-1}\}$$

The following propositions are proved in the Thesis:

Proposition 9 *For any $n \geq 1$, if $(a, b) \in R^n$ then there exists x such that $(x, b) \in R$.*

Proposition 10 *If $(a, b) \in R^+$ then there exists x such that $(x, b) \in R$.*

2.6.1 Transitive closure as a generative relation

The following proposition is proved in the Thesis:

Proposition 11 *If R is an intransitive generative relation of an equilibrium E then the transitive closure R^+ is also a generative relation for E, i.e. the set NDR of non-dominated strategies with respect to R equals the set of non-dominated strategies with respect to R^+:*

$$NDR = NRD^+ = E$$

2.6.2 A measure based technique

Let R be an intransitive generative relation of the equilibrium E. If x and y are two strategy profiles such that $(x, y) \in R$ then x dominates y with respect to R. Let us denote by $DR(x)$ the set of strategy profiles dominating x (with respect to R).

$$DR(x) = \{y \in S|(y, x) \in R\}$$

The set $DR(x)$ may supply an indicator of how dominated strategy x is. If x is non-dominated then

$$DR(x) = \emptyset$$

The smaller the set $DR(x)$ is, the closer the strategy x is to the equilibrium. We may thus define a quality indicator $g(x)$ of a strategy x as

$$g(x) = m(DR(x))$$

where m is an appropriate measure concept.

It is easy to observe that $x \in NDR$ if and only if $g(x) = 0$.

We may now define a new binary relation Q on $S \times S$ as

$$(x, y) \in Q$$

if and only if

$$g(x) < g(y).$$

If $(x, y) \in Q$ we say that x dominates y w.r.t. Q.

The set of non-dominated strategies with respect to Q is denoted by NDQ.

Proposition 12 *If NDR is not empty, then the set of non-dominated strategies with respect to Q equals the set of non-dominated strategies with respect to R:*

$$if NDR \neq \emptyset \; then \; NDQ = NDR = E.$$

Therefore R and Q are generative relations of the equilibrium E.

Chapter 3

Experimental Games

The concept of Nash equilibrium plays an important role in Game Theory. Many situations can be modeled by the game-theoretical formalism where the Nash equilibrium offers a comprehensive explanation of experimental observations.

Studies have shown that people do not play the Nash equilibrium strategy in a wide variety of games. This 'irrational' behavior may have many reasons, such as psychological, social, educational or cultural.

3.1 Rosenthal's Centipede Game

The centipede game introduced by Rosenthal is one of the most studied games, which demonstrates that people don 't play always Nash, people don 't think in the real world in a rational manner. In the two-player version of the game players can choose between two possibilities: take the sum or pass. If the player chooses take, then the game ends. In each step the sum of the prize for the players increases but has a different distribution. A typical setup of the centipede game is presented in Figure 3.1.

Studies have shown that the players have a certain amount of trust and the game is played several steps before one player decides to end the game by taking the pool of money. Different models have been proposed for explaining how the players think, what strategy they follow while playing the Centipede game.

We have conducted an experiment where students played the centipede game in two different setups; in one version a classical extensive form was used, in the other version the

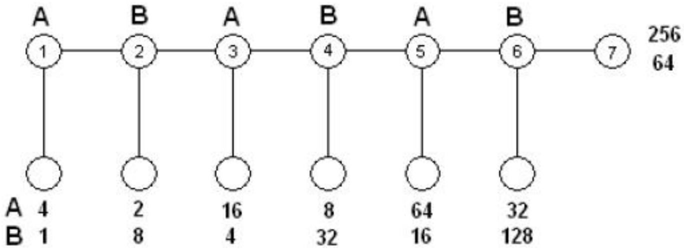

Figure 3.1: Two-player Centipede game.

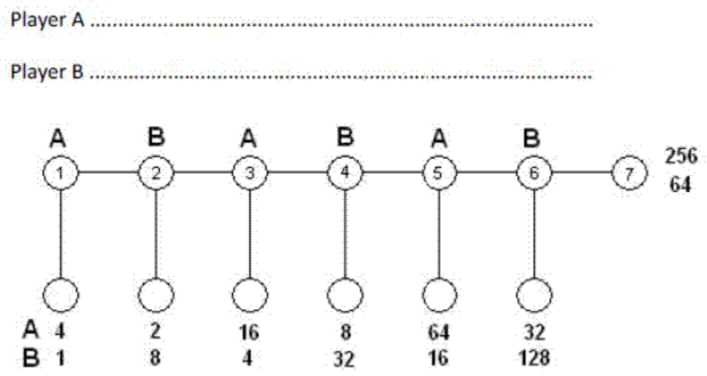

Figure 3.2: Paper-form handed to students who played the Extensive Form of the Centipede game.

game was played in the normal form. Our results show that the way the game is presented has measurable effect on how people play the centipede game.

3.2 Experimental Setup

The experiments were designed to be carried out in a classroom using very simple means.

The students are divided in two equal groups, named group A and group B. In case of odd number of students, one student is left out of the experiment, or, one of the experimenters can take the role of a student. Each student from group A plays one or more games against each student from group B.

Before each game the students are paired so that one player is from group A and the other from group B. This matching is changed after each game, so that at the end of the experiment each student had played against every student from the opposing group. The games are played in 'rounds'.

Before a round starts, the students are matched by changing their places at their desks so that the current opponents sit at the same desk. Each student is handed out a form printed on paper on which they will record their moves.

There are two different printed forms, one for the extensive form and one for the normal form of the game. These forms are presented in Figures 3.2 and **??**.

The students are asked to write their names on the headers of the forms. When all students are ready to start to play the current game, the start of the round is announced. The students start to record their choices on the provided forms.

There is no time limit for a round; everyone can take his/her time to make decisions. After each pair of students finished playing the game, the forms are collected. The matching is changed, so that each student is paired with the next opponent, and another round is started.

The experiment terminates after all rounds are played.

Player A .... Player A ....

Player B .... Player B ....

A B

Figure 3.3: Paper-form handed to students who played the Normal Form of the Centipede game.

class	nr. of experiments
X.A	100
X.B	90
XI.A	81
XI.B	64
XI.C	56
XII.A	144
XII.B	132
XII.C	56
IX.A	182
IX.C	132

Figure 3.4: Experiments played in each class.

3.3 Methods of Play

The first method of play is playing the game in extensive form. This implies the players choosing their move in turn. The second method of play is playing the game in normal form. In this mode each player has to choose independently of the other player the number of step at which he wishes to make a 'take' move.

3.4 Experimental Results

Totally 1037 games were played, from which 474 iterative (extensive form) and 563 non-iterative (normal form) games. Table 6 presents the number of the experiments in each class.

The data referring to the step at which the students ended the game is summarized in Table 7. The step number which occurred most frequently is indicated in bold.

The average of take steps is calculated for both types of game playing mode. The graph of this average relative frequency is presented in Figure 4. From the graph is apparent that the step number at which the 'take' move was chosen by the students depends on the mode of the game was played; extensive vs. normal form. This result shows that people, though they play a mathematically identical game might respond differently based on how the game is presented.

extensive	1	2	3	4	5	6	7
X.B	0.00	0.00	0.01	0.29	**0.43**	0.20	0.07
XI.B	0.06	0.14	**0.30**	0.27	0.19	0.03	0.02
XI.C	0.02	0.11	0.27	**0.29**	0.23	0.05	0.04
XII.B	0.09	0.06	0.19	**0.25**	0.24	0.11	0.06
IX.C	0.03	0.08	0.21	**0.33**	0.11	0.14	0.11
normal	1	2	3	4	5	6	7
X.A	0.15	0.15	0.19	**0.26**	0.17	0.08	0.00
XI.A	**0.25**	0.20	0.17	0.16	0.12	0.05	0.05
XII.A	0.29	**0.31**	0.26	0.10	0.03	0.01	0.01
XII.C	0.04	0.09	0.27	**0.34**	0.21	0.05	0.00
IX.A	0.24	**0.34**	0.20	0.09	0.09	0.02	0.02

Figure 3.5: The relative frequencies of the ‘take’ steps as played by the students.

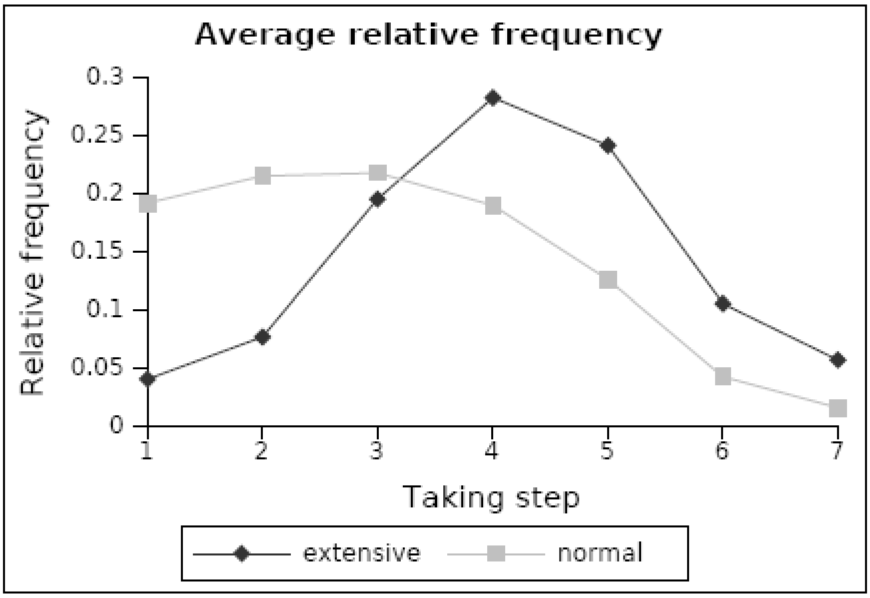

Figure 3.6: The average relative frequencies at which the students chose the ‘take’ step. The graph indicates that the most frequently chosen ‘take’ step is different for the extensive form and for the normal form.

Chapter 4

Emergence of Structure in Oscillator Populations

4.1 Introduction

A population of pulse-coupled oscillators on a $2D$ grid is considered. Changes in each oscillator occur at discrete time moments. Oscillators (grid cells) interact only if the distance between them is less than a given radius. The interaction strength decreases with the squared distance between the interacting cells.

Each oscillator is composed by two parts: one which is a classical integrate-and-fire oscillator and another one, which is sensitive to the signal emitted by other oscillators and integrates the received signal. The integration is "leaky": the accumulated energy decays according to an inverse exponential law.

In many models of pulse-coupled oscillators a total synchronization of the population is eventually observed. However, for the proposed model, extensive simulations indicate that a total synchronization of the population does not occur. Instead, waves of synchronously firing oscillators emerge.

4.2 Proposed Oscillator Population Model

4.2.1 Grid model

A population of pulse-coupled oscillators arranged on a 2-dimensional grid is considered. The oscillators states are synchronously updated at regular intervals, called generations. The structure of each oscillator consists of a classical integrate-and-fire mechanism combined with a separate mechanism for sensing and integrating the signal emitted by neighboring oscillators. The integrator has a constant slope (which is a parameter of the oscillator). It generates a signal which increases proportionally with time.

A separate mechanism in each oscillator is responsible for sensing the signals emitted by other oscillators. The energy of the detected signals is integrated by a leaky integrator, i.e. the accumulated energy of the integrator decays exponentially in time.

An oscillator meats the firing condition if the output value of the integrate-and-fire mechanism summed with the weighted value of the sensing integrator is greater than one.

The proposed model limits the interaction distance between two cells to a certain value called the interaction radius. This corresponds to a sensitivity threshold of a cell.

4.2.2 Oscillator state and update rules

The internal state of an oscillator is represented by a set of three variables (P, S, F) with the following meaning :

P, called potential, represents the current value of the constant-slope integrator;

S, called signal, represents the current value of the sensing integrator;

F, called firing state, is a boolean value which is ?true? when the oscillator is in firing state in the given generation.

The model uses the following parameters :

- At each generation the state variables of each oscillator are updated according to the following rules:
  - The potential P is incremented with a constant value E. The value of E is an oscillator-specific parameter and determines the slope of the integrator.

    $$P := P + E$$

  - The signal S is calculated by summing the intensities of the signals emitted by all oscillators in the interaction radius which are in firing state, then decrementing S by an amount proportional to the value of S in the previous generation.

    $$S := \sum_k (1/d_k^2) - DS$$

    where d_k is the distance between the oscillator and a neighboring oscillator k in the interaction domain $d_k \leq R$ which is in firing state; D is an oscillator-specific decay parameter.
  - The firing state F of an oscillator is determined by comparing the weighted sum of the potential P and signal S with a constant threshold:

    $$F := true\, if\ P + KS \geq 1$$

    $$F := \ otherwise$$

    where K is an oscillator-specific *coupling* parameter.

A cell is in firing state for only one generation. In the next generation returns to normal state and the potential P is reset to zero:

$$P := 0$$

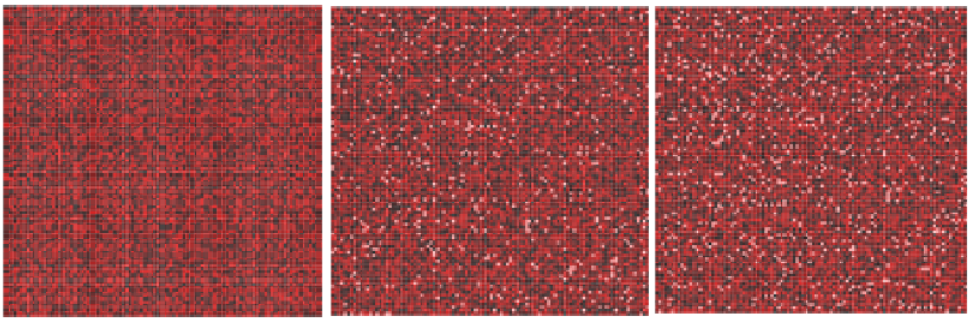

Figure 4.1: Population of 100 x 100 oscillators simulated with $E = 0.05$, $D = 0.7$, $K = 0.1$ and $Radius = 30$.

if $F = true$ in the previous generation.

4.3 Experimental Results

4.3.1 Steady-state behavior

The proposed model is simulated for different values of the parameters E, D and K. Different interaction radii R are also considered.

The potential P of each oscillator is initialized to a random value uniformly distributed in the interval [0, 1). The starting value of the signal S is set to zero for each oscillator.

In each experiment the decay and the coupling parameters D and K are set to the same value for each cell.

In the following diagrams the potential values of the cells are displayed in red color with intensity proportional to the potential. Cells in the firing state are indicated by a bright color.

Running the simulation with different parameter values shows random firing of oscillators. The population of oscillators maintains a dynamic equilibrium state where the initial random phases of cells are preserved in time, in the sense that no correlation of phases – and thus synchronous firing – is observed for many thousands of generations.

A situation of this kind is presented in Figure 5. The simulation used the following parameter values: E = 0.05, D = 0.7 and K = 0.1 and R = 30 for a grid of 100 x100 oscillators.

4.3.2 Emergence of synchronization waves

In the experiments described in the preceding section the parameter E which determines the intrinsic firing frequency of the cell was chosen to have the same value for each cell. In other experiments the E value is perturbed by a small random noise, so that each cell is initialized with different values of E.

In the experiments where the parameter E is perturbed in a few hundred generations clusters of firing cells emerge. Waves of synchronously firing cells emerge typically after

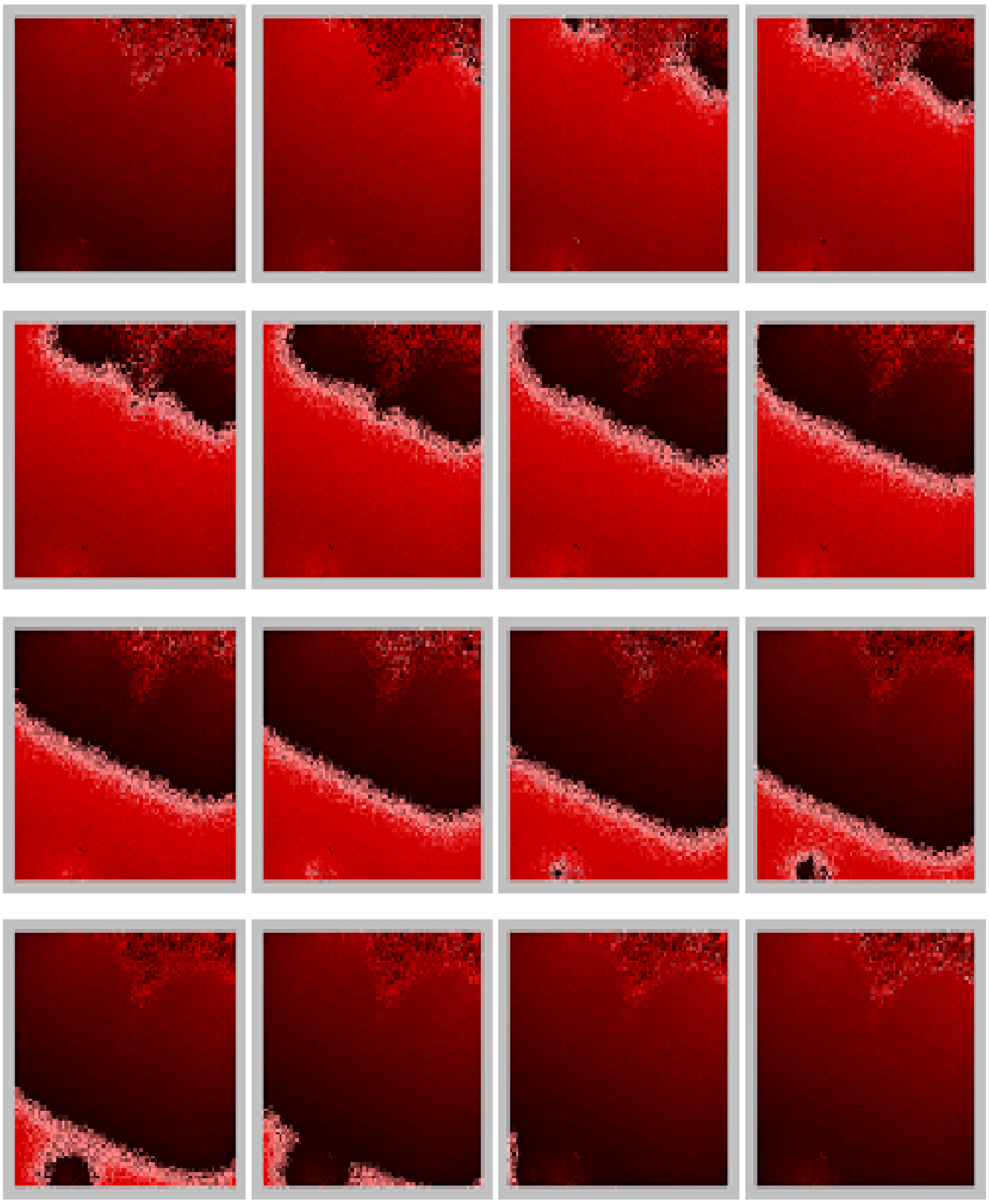

Figure 4.2: Sequence of a typical evolution of the oscillator population with perturbed E parameter. Setting: $E = 0.05$, $D = 0.7$, $K = 0.2$, $R = 30$. The variance of the E parameter was set to 0.005.

600-1000 generations.

Figure 6 depicts a typical evolution of the automaton on a grid of 150 x 150 cells, when the intrinsic firing frequencies of cells differ. Parameter values are E = 0.05, D = 0.7, K = 0.1 and R = 30. The initial value of parameter E for each cell is perturbed by a random value in the range [0, 0.005] around the central value of 0.05.

The size of a local group is of the same magnitude as the value of the interaction radius R. This is to be expected, since the influence of an oscillator cannot extend further. Even in a group, the action range of the cell is limited.

In the case where the population array is small enough, only one local group can form. In these cases after a thousand generations the cycle of the local group stabilizes.

If the dimension of the cell grid is much larger than the interaction radius, then many

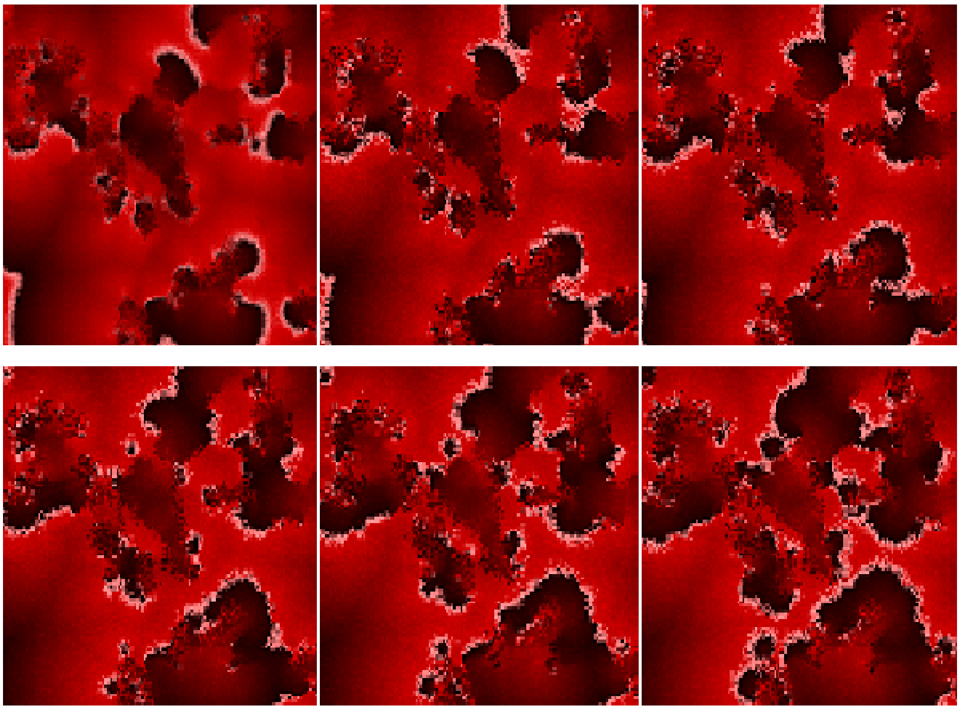

Figure 4.3: Consecutive states of an oscillator population of 800 x 800 cells. Setting: E = 0.05, D = 0.7, K = 0.2, R = 30. The variance of the E parameter was set to 0.005. The formation of many local groups is visible.

local groups can form. Waves emitted by local groups clash with each other at the boundaries of the groups.

A series of images demonstrating the evolution of an oscillator population of 800 x 800 cells is presented in Figure 3. The parameters used are the same as for the preceding experiment : E = 0.05, D = 0.7, K = 0.1 and R = 30 and E is perturbed by [0, 0.005] around the central value.

4.3.3 Mechanism of wave emergence

The phase of an oscillator can be considered to be the composite value of the potential and the signal. When this value reaches 1 it determines the firing of the cell :

$$Phase = P + K * S.$$

The phase value is in the interval [0..1). In some papers the phase of an oscillator is considered to be in the range $[0..2 * pi)$, but other sources use the [0..1) interval, which will also be used here.

At the start of the simulation the phases are randomly distributed in the [0..1) interval since the value of signal is zero. When an oscillator accumulates energy by sensing the effect of nearby firing cells, the phase increases by the phase shift H

$$H = K * S.$$

This phase shift makes the oscillator fire earlier.

The intrinsic firing period T of an oscillator is determined by the value of E (the slope of the potential increase). This is given by

$$T = round(1/E)$$

where the function "round" gives the nearest integer greater than 1/E. The intrinsic firing period is expressed in number of generation.

In other words, if an oscillator is not influenced by any other oscillator, then it will periodically enter firing state at every T generations.

If, however, the oscillator is accumulating energy, the current phase shift H can make the oscillator fire by one generation earlier, in the case that H is greater or equal with E.

Thus, the phase shift necessary for an oscillator to reduce its firing period by one generation is

$$E = H = K * S$$

Since the signal is decaying at each generation with D * S, in order to this period change to become permanent, the oscillator needs to accumulate an amount of energy to cover this loss in the signal value.

The necessary accumulated energy in one generation for the firing period to be reduced by one generation is

$$U = K * S * D$$

Chapter 5

Cascade Systems

5.1 Introduction

A problem is said to be NP (nondeterministic polynomial time) if any given solution of the problem can be verified in polynomial time. NP-complete is a subset of NP, namely the set of such problems to which any other NP problem can be reduced by a polynomial-time transformation.

The question whether NP-complete problems are solvable in a polynomial time by a deterministic Turing machine is a key open problem, known as the P versus NP question.

Godel claimed that a linear- or quadratic-time procedure for solving NP-complete problems would have consequences of the greatest magnitude, as it would clearly indicate that [] the mental effort of the mathematician in the case of yes-or-no questions could be completely replaced by machines [Sipser, 1992].

The connection between NP-complete problems and the physical reality has also been investigated. The question whether NP-complete problems can be solved in polynomial time using the resources of the physical universe can yield new insights about nature [Aaronson, 2005].

A new computational concept called Cascade Computing (CC) aiming to cope with complex problems is proposed.

5.2 Cascade System

A Cascade System is defined as the tuple $C = (N, A, S, F)$, where

- N is a set of nodes;
- $A \subseteq NN$ is an ordered set of arcs, connecting nodes;
- S is a nonempty set of node states. Each node is in a given state $s \in S$ in any moment;
- F is a set of transition functions. Each node n has an associated transition function $f_n : S^{m+1} \to S$, where m is the indegree of node n.

The transition function $f_n \in F$ is a function of arity $m+1$, where m is the indegree of node n (the number of arcs ending in n). The arguments for f_n are the state values of the predecessors of node n and the state of the node itself. If $m > 0$ then the order of the arguments in the transition function f_n is implicitly given by the ordering of the inbound arcs of the node n defined by the order relation defined on set A.

According to the problem or method involved, the state set S may be continuous or discrete. Time may also be considered to be continuous or discrete.

5.2.1 Transition functions

The transition function associated to a node can be any function which maps Sm+1 into S. The interesting cases are those where the set of transition functions is restricted to a particular class of "simple" or computationally cheap functions.

5.2.2 Particularizations and Turing Completeness

The general model of Cascade System can be particularized to some well-known computing models, such as cellular automata, neural networks and logic circuits.

Each of the above computing models is known to be able to simulate Turing machines. It is thus clear that Cascade Systems can also simulate any Turing machine, thus being Turing-complete.

5.2.3 Discrete Time Cascade System

A Discrete Time Cascade System (DTCS) is a particular Cascade System where the time is considered to be discrete. The node states are synchronously updated according to the transition functions.

5.3 Solving Subset Sum with DTCS

The subset-sum is an important problem in the class of NP-complete problems. A method for solving subset-sum using a DTCS is proposed in the following sections.

5.3.1 Solutions of Subset Sum

The subset-sum problem $P = (A, Sum)$ can be formulated as follows:

- Let $A = \{a_1, a_2, .., a_N\}$ be a set of N positive integers.
  For a given Sum, is there a subset $B \subseteq A$ such that the sum of all elements of B equals Sum?

A classical approach for solving the subset-sum decision problem is an algorithm using dynamic programming [Cormen et al., 2001].

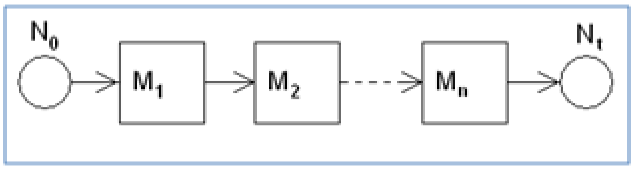

Figure 5.1: Modular structure of the subset-sum cascade system.

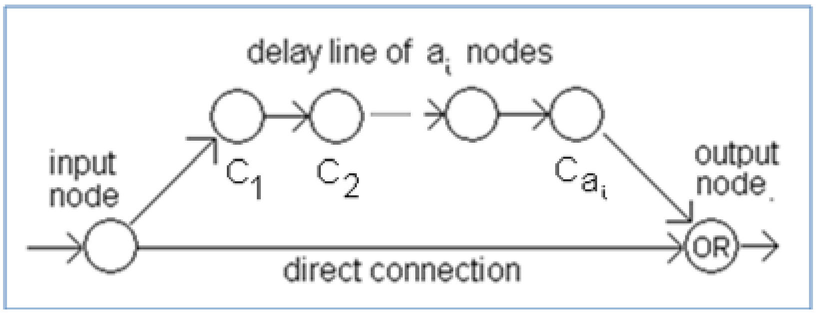

Figure 5.2: The module M_i of the subset-sum cascade system

Approaches for solving the subset-sum problem based on "natural computing" are known ([Oltean and Muntean, 2009], [Leporati and Gutiérrez-Naranjo, 2008]).

5.3.2 Decision Problem. System Architecture

A Discrete Time Cascade System with Boolean transition functions for addressing the NP-complete subset-sum problem is introduced.

The proposed system is able to decide in polynomial time (with respect to Sum) whether a solution exists (decision problem) and to detect an effective solution.

The proposed cascade system is a concatenation of similar modules preceded by an initialization node N_0. The output of the last module is connected to a reading (or test) node, N_St as shown in Figure 5.1.

The proposed cascade system has a corresponding module M_i for each positive integer $a_i \in A$. The number of nodes in the delay line of the module M_i is exactly a_i.

The structure of a DTCS module is depicted in Figure 5.2.

5.3.3 Detecting a Solution

In the original formulation, the subset-sum problem is a decision problem, asking the question whether the required sum can be obtained by summing a subset of the given numbers. In case of affirmative answer, we may ask how to detect the exact subset(s) which provide the solution.

The proposed DTCS model can be extended for effectively detecting a solution subset.

Let X denote the set of solutions for a subset-sum problem $P = (A, Sum)$, and $x1$ be the maximum of the least elements of any solution. Let Y be the subset of X where the least element is $x1$. No number less than $x1$ belongs to any set from Y. Thus Y is the set of solution-sets for the subset-sum problem $(A\ \{x1\}, Sum)$. If $x1$ is removed from A, then the subset-sum problem P has no solution.

Thus $x1$ can be determined by successively removing the least elements from the set A, until the subset-sum problem P has no solution; $x1$ is the last number removed.

Removing a number from the set A is equivalent to removing from DTCS the module associated with the given number. Thus $x1$ can be determined by successively removing the modules corresponding to the least elements of A. For each module removed, the procedure for determining the solution for the subset-sum decision problem must be applied.

If $x1$ has been determined then finding the rest of a solution to the original subset-sum problem can be reduced to another problem $Q = \{A\ \{a|a <= x1\}, Sumx1\}$, which can be solved in a similar manner. If Z is a solution for Q, than x1 U Z is a solution for the original problem P.

The following algorithm can be used to determine a solution set for the subset-sum problem P.

Algorithm 1 Find a solution set of the subset-sum problem P(A, Sum)

```
if there is no solution for the subset-sum decision problem P(A, Sum) then
  return Empty Set
end if
Solution = Empty Set
Sort A in ascending order
while A is not empty AND Sum is not zero do
  X = first element of A
  remove X from A
  if there is a solution for subset-sum decision problem P(A, Sum) then
    add X to Solution
    Sum = Sum - X
  end if
end while
return Solution
```

5.4 Model Complexity

5.4.1 Time complexity

It is shown in the Thesis that the time complexity of solving the subset-sum decision problem (A, Sum) using the proposed DTCS is O(Sum).

5.4.2 Space complexity

The space requirement of the DTCS model is proportional to the total number of nodes (the sum S_A of all elements of the set $A + 2N + 2$).

Since SA is typically much larger than N, the space complexity of the model can be considered as being $O(S_A)$.

Chapter 6

Conclusions

The study of evolutionary techniques have opened a way to a more general approach of defining game equilibria, by defining dominance relations on pairs of strategies. Properties of generative relations are studied. In particular the transitivity of the generative relation is important in order to develop an efficient evolutionary method for equilibria search. Defining game equilibria by generative relations makes possible to mix different types of equilibrium concepts in a new one. A composition of Nash and Pareto equilibria it is studied and applied to behavioral games.

The theoretical conclusions of game theory do not always apply in practice when people play games in real life. An experiment using the centipede game was conducted in a school which has shown that the method of presenting the same game in different ways can result in significant differences in how people play. The centipede game is played closer to Nash equilibrium if played in normal form than if it is played in extensive form.

A new model of pulse-coupled oscillators is proposed and investigated. A population of pulse-coupled oscillators on a 2D grid is considered. A total synchronization of the population is eventually observed in many models of pulse-coupled oscillators. Simulations indicate that in the proposed model total synchronization of the population does not occur. For certain parameters emergence of synchronization patterns can observed. These emergent patterns are observed as waves of synchronously firing oscillators.

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