Babes-Bolyai University
Faculty of Mathematics and Computer Science
Department of Computer Science
Centre for the Study of Complexity

Mihai Suciu

Evolutionary Optimization and Strategic Interactions

Abstract

Supervisor: Prof. D. Dumitrescu

Thesis Committee:
Prof. Daniela Zaharie, West University, Timisoara
Conf. Rodica Ioana Lung, Babes Bolyai University, Cluj-Napoca
Conf. Marcel Cremene, Technical University, Cluj-Napoca

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Contents

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Publications Related to Thesis

- **Mihai Suciu**, Rodica I. Lung, Noémi Gaskó, and D. Dumitrescu. *Differential evolution for discrete-time large dynamic games.* In 2013 IEEE Conference on Evolutionary Computation, volume 1, 21082113, 2013. DOI=10.1109/CEC.2013.6557818. http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6557818
- **Mihai Suciu**, Denis Pallez, Marcel Cremene, and D. Dumitrescu. *Adaptive MOEA/D for QoS-based web service composition.* EvoCOP'13, Martin Middendorf and Christian Blum (Eds.). Springer-Verlag, Berlin, Heidelberg, 73-84, 2013. http://dx.doi.org/10.1007/978-3-642-37198-1_7
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- D. Dumitrescu and **Mihai Suciu**. *Resonance Search.* IEEE Transactions on Evolutionary Computation. (submitted)

Chapter 1

Introduction

Optimization is a major part of everyday life. One always tries to optimize each aspect of life as best as it can. In a computational context solving an optimization problem implies that the problem has an exact mathematical description. But there are problems where one cannot accurately define, in a mathematical sense, the optimization function or this problem is a dynamical one, i.e. problem characteristics change with time. In these cases exact algorithms are not able to give a solution or the time needed to compute a solution is not practical, however nature inspired optimization techniques give a good approximation of the optimal solution.

Almost all real life optimization problems have multiple conflicting objectives that need to be simultaneously optimized, e.g., minimize cost while maximizing efficiency. The solution of such problems is always a compromise between the conflicting objectives. Since 1970 [Rosenberg, 1970] Evolutionary Algorithms have been investigated for solving optimization problems with multiple conflicting objectives. *Evolutionary Multiobjective Optimization Algorithms (EMOAs)* are population based optimization techniques able to find a good approximation of the solutions.

When solving an optimization problem with multiple conflicting objectives using an evolutionary technique there are mainly three approaches that can be applied: Pareto dominance based algorithms, indicator based algorithms, and decomposition based algorithms. Pareto dominance based algorithms scale badly when applied to many-objective problems (problems with more than 4 objectives) [Farina and Amato, 2002; Hughes, 2005; Khare et al., 2003]. Indicator based approaches scale relatively good with the number of objectives albeit this happens only when the performance indicator can be computed [Wagner et al., 2007].

This Thesis addresses the optimization problem of many conflicting objectives through an evolutionary approach based on decomposition, and explores its applications in the field of Service Oriented Computing. We also address the problem that arises when Pareto dominance is used (exponential increase of the number of solutions found with increasing number of objectives) by using the concept of *Lorenz dominance* to concentrate the search in the equitable region of the Pareto front.

A competing player will always try to predict, in a trial and error fashion or through depth analysis, how opponents will react to his own actions. Game theory, as a mathematical field, tries to answer such a question - it studies strategic interactions and tries to predicts the outcome of interacting players. If these players are rational and play the best strategy, most rational one, then the outcome of this interaction is predictable.

Nash equilibrium [Nash, 1951] is the most common solution concept for a game. A player maximizes its payoff by choosing a strategy that is the best response to the one chosen by his opponent, so players have no reason to change their strategies. But this solution concept has some disadvantages: does not always assure the highest payoff and in some situations a game has multiple Nash equilibria.

Within a one-shot non-cooperative game [Osborne, 2004] players make their decisions simultaneously, the game consisting only from one decision-making process. But the characteristics of the game may change over time. Within Game Theory a discrete-time dynamic games capture changes within game parameters (payoffs).

Dynamic Games are mathematical models of the interaction between independent decision makers (players, agents) who are controlling a dynamical system [Haurie and Krawczyk, 2000]. Such a game represents a more realistic real-world decision model. A static non-cooperative game can be considered a special case of a dynamic game that represents a more complex and realistic notion [Basar and Olsder, 1982].

Thesis also investigates other equilibrium concepts, $\epsilon-$*Berge-Zhukovskii* and *Pareto-optimal Nash*, that overcome some of *Nash equilibrium* disadvantages. We concern ourselves with the detection and tracking of *Nash* and *Berge-Zhukovskii equilibrium* in dynamic games. Standard game models are overly simplified - they allow interactions between players with the same rationality. With the aid of multicriteria games we investigate a more realistic model that simulates players behavior in repeated interactions.

Thesis Structure

Chapter 2 gives an introduction into Evolutionary Optimization, MultiObjective Optimization, and performance indicators used to evaluate evolutionary multiobjective optimization algorithms. Some basic notions about Game Theory such as Nash and Berge-Zhukovskii equilibria, the Generalized Nash Equilibrium Problem, fitness assignment mechanism used to guide an evolutionary algorithm towards the Nash equilibrium, and some non-cooperative games are also presented.

Chapter 3 explores the problem of many-objective optimization. Some scalarization techniques are analyzed, comparative test with classic scalarization techniques on benchmark test problems are performed. Pareto dominance scales poorly with the increasing number of objectives, Lorenz dominance concept is investigated as an alternative. A new evolutionary multi-population model based on decomposition is presented for multiobjective and many-objective optimization problems.

In Chapter 4 the application of multiobjective optimization algorithms to the problem of web services composition based on *QoS* properties is analyzed. A hybridization between a decomposition based multiobjective optimization algorithm and an adaptive technique is proposed to cope with the dynamic nature of the problem. The concept of Lorenz dominance is applied to the web service composition problem for reducing the approximation set found by the optimization algorithm in order to aid the Decision Maker in choosing a final solution. The performance of some state of the art multiobjective optimization algorithms is analyzed.

Chapter 5 investigates detection of ϵ-Berge-Zhukovskii and Pareto-optimal Nash equilibria with the aid of an evolutionary heuristic. A computational method for detecting these equilibria is proposed. A game situation where some constraints are applied to players strategies is also explored.

Chapter 6 proposes an evolutionary technique for detecting and tracking the equilibrium of a dynamic game. Constrained/unconstrained, symmetric and asymmetric dynamic games are analyzed.

Chapter 7 deals with iterated spatial games. The dynamics of an economical model played in an iterated spatial framework is studied. An interaction model based on hyper-graphs for modeling inter-group cooperation is proposed. In decision making real life players rely on more than just payoff. Standard game theory allows interactions between players with the same rationality. We propose a more realistic model for players interactions. The concept of multidimensional games is investigated for promoting cooperation in the context of iterated games.

Chapter 8 concludes this thesis and gives some future research directions.

Figure 1.1 presents the general outline of Thesis.

Contributions

The main contributions reported in Thesis are:

- A new multi-population optimization algorithm based on decomposition for multiobjective and many-objective optimization problems (Section 3.3).

  Multiobjective problems with more than three objectives are usually referred as *many-objective* problems. The application of standard dominance-based EMOAs to many-objective

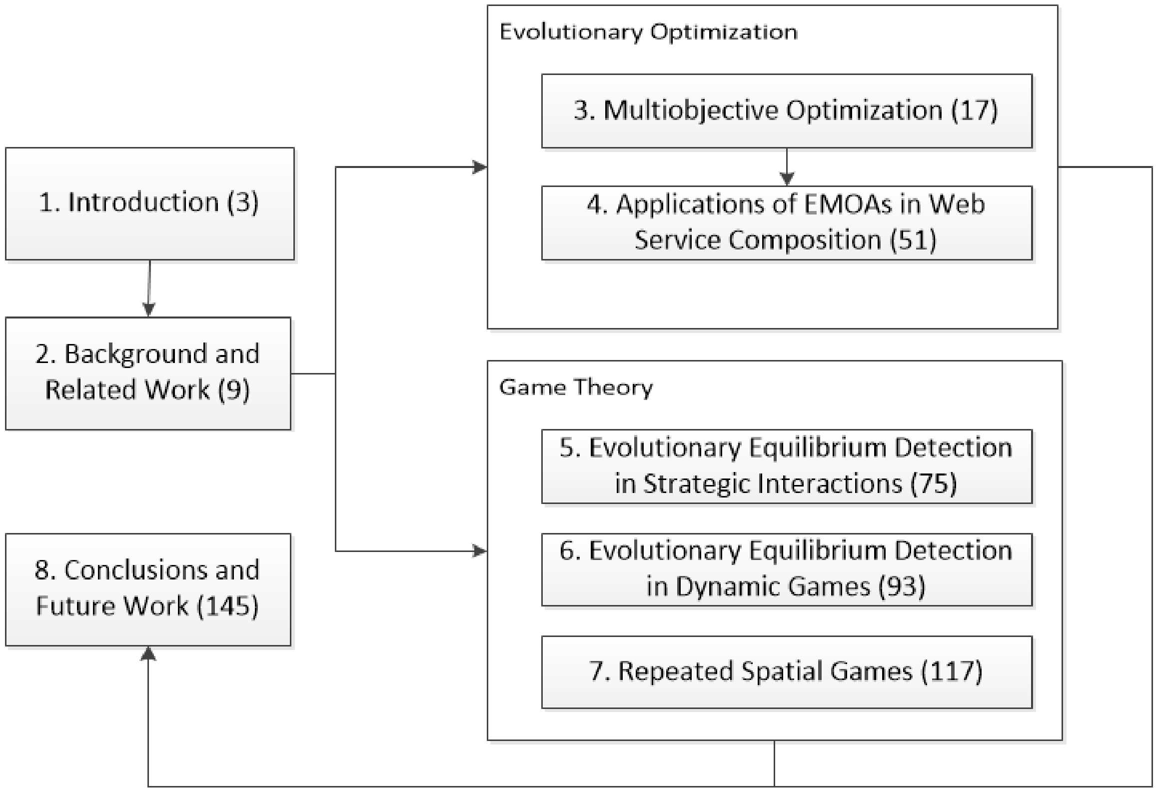

Figure 1.1: Thesis structure.

problems is drastically limited by the tendency of feasible solutions to become non-dominated when the number of objectives increases [Farina and Amato, 2002]. The selection pressure towards the Pareto set is thus severely deteriorated.

Several approaches have been proposed in order to cope with many-objective problems. A class of techniques focuses on replacing Pareto dominance with weaker dominance concepts [Sato et al., 2006; Farina and Amato, 2004]. Another class of approaches relies on decomposition techniques [Benders, 1962] - many-criteria problems are decomposed in a class of single-objective problems. A scalarizing function and a set of weight vectors decompose the optimization problem in many single-objective problems [Miettinen, 1999; Kathrin and Tind, 2007; Zhang and Li, 2007; Engau and Wiecek, 2008; Engau, 2009]. The main advantages of this approach are computational efficiency and good scalability with the number of objectives.

We consider a new model according to which the Pareto set is approximated by several instances of a *Dynamic Fitness Landscape* (DFL). Each DFL instance is described by an aggregated function. This aggregated function serves for fitness assignment and guides the exploration of DFL instance. Several implementations of this model are possible. We consider a very simple technique able to offer some indications concerning the potential of the DFL model. The proposed approach, called *Resonance Search* (RS) is a multi-population evolutionary technique. Each population uses one or several search techniques, mainly based on scalarization.

- A new model that includes other player preferences (players identity) in iterated spatial games (Section 7.3).

  Standard Game Theory makes some simplifying assumptions: players are rational, know that their opponents are rational, and have common knowledge about the game played [Osborne, 2004; Gintis, 2009]. So a players only goal is to maximize its own gain. We believe that this is only partially true. Real players, besides maximizing their payoff, take into account other criteria such as: reputation, morality, etc. These criteria capture the identity of a player and weigh heavily in their decision.

In most cases these criteria are not measured by the same unit, e.g. payoff and morality, so they can not be just aggregated into one single criterion. Real life agents do not make their decision based only on actual payoffs. Beyond the actual payoffs their identity determines their choices. For example an agent with a cooperator identity is less likely to defect even though rational thinking implies defection regardless of the strategy of the opponent.

Standard game theoretic models allow interactions between only players that have the same rationality type. This is an unrealistic restriction, in real life situations players within a game rarely think and act in the same manner. Thus these game models are overly simplified and do not model real life decision making adequately.

Our goal is to develop a model that overcomes these limitations. For these purpose we study the Prisoner's Dilemma game within the context of multidimensional/multicriteria games. The standard model is extended by adding a second criterion that models the identity of a player (this second criterion captures the fairness or morality of a player). In this manner a psychological factor can be considered in the decision making process, besides actual payoffs. Real life players are diverse, thus our goal is to allow interactions between players with different identities. By introducing the second criterion in the Prisoner's Dilemma game we want to better model the irrational nature of real life players.

- Model for intergroup cooperation (Section 7.2) [Suciu et al. *Evolutionary dynamic for intergroup cooperation*. ROMJIST - submitted].

  An important question in Evolutionary Game Theory [Fisher, 1930] is the emergence of cooperation. Darwinian evolutionary theory favor defectors, selfish individuals. Despite this, cooperative individuals can benefit from cooperation. High levels of cooperation can be achieved in well integrated environments even in well mixed groups. Whereas in a segregated society cooperation does not appear [Goette et al., 2006; Goette and Meier, 2011]. Increasing group diversity can induce cooperation [Goette and Meier, 2011].

  Emergence of cooperation has been studied only for interactions in a group [Szabó and Fáth, 2007]. Our aim is to study the interactions between groups. In order to analyze the emergence of cooperation between heterogeneous groups we concentrate on interactions between groups, ignoring intra-group interactions. A hypergraph interaction model is proposed to describe interactions between players and groups of players. When playing a n-person Prisoner's Dilemma game an interaction model based on a hypergraph seems more natural. The interaction between different groups can be described in this way. The proposed hypergraph model ensures a higher flexibility in the sense of the interactions between players (players can interact in each round with other random opponents).

- A computational method for tracking equilibrium in dynamic games (Chapter 6) [Suciu et al., 2013b, 2014; Gaskó et al., 2013].

  If an optimization problem changes its characteristics over time then it is considered a *dynamic problem*. Dynamic nature of the problem may be reflected in the change of the fitness function, problem constraints, or problem instance [Yang et al., 2007].

  Within Game Theory a discrete-time dynamic games capture changes within game parameters (payoffs). Such a game represents a more realistic real-world decision model. Within a one-shot non-cooperative game [Osborne, 2004] players make their decisions simultaneously, the game consisting only from one decision-making process. In some cases a static non-cooperative game is considered just a special case of a dynamic game that represents a more complex and realistic notion [Basar and Olsder, 1982] with multiple applications in economics [Long, 2010], engineering [Lasaulce and Tembine, 2011] and others.

  Dynamic Games are mathematical models of the interaction between independent decision makers (players, agents) who are controlling a dynamical system [Haurie and Krawczyk, 2000]. Although there is an extensive amount of research on evolutionary optimization techniques for dynamic environments [Jin and Branke, 2005; Plessis and Engelbrecht, 2013; Cheng et al., 2013; Yang et al., 2012; Nguyen et al., 2012; Cruz et al., 2011] to the best of our knowledge there are no previous attempts to adapt the search of an evolutionary algorithm to dynamic games.

We propose a simple method, called *Dynamic Equilibrium Tracking (DET-DE)*, able to track the Nash equilibrium and adapt to changes in the environment.

- An adaptive evolutionary optimization technique based on decomposition applied to web service composition (Section 4.2) [Suciu et al., 2012, 2013a,c].

  Despite the fact that the *web services composition based on Quality of Service parameters* optimization problem is multiobjective by nature few approaches based on multiobjective algorithms can be found in the literature [Li et al., 2010; Ross, 2006; Taboada et al., 2008; Wada et al., 2008; Yao and Chen, 2009]. In most cases single-objective algorithms are used to solve this problem.

  There are many variants of Evolutionary Algorithms with different control parameters: population size, operators used, crossover and mutation probabilities, etc. Selecting appropriate values is mainly done based on empirical studies, often a "trial and error" fashion is used for adjusting these values. Typically one parameter is adjusted at a time, which may lead to sub-optimal choices, since often it is not known how parameters interact. Such an approach is time consuming. In the last couple of years there has been an increasing interest in designing methods that self-adapt these parameters [Chakhlevitch and Cowling, 2008; Eiben et al., 1999; Neri and Tirronen, 2010].

  The *QoS*-based service optimization is a NP hard combinatorial multiobjective optimization problem. Decomposition techniques have good performance for combinatorial problems, another advantage is the small computational load. One drawback to decomposition evolutionary approach is the dependency between problem type and algorithm parameters. The same algorithm must solve different instances of this problem. Different web services are described by different workflows, the search space changes for each workflow making it a very dynamical problem. A set of parameters that work for one particular problem instance may not yield good results for another workflow.

  A self-adaptive technique seems the obvious choice for this kind of problem. The hybridization between a decomposition based multiobjective optimization algorithm and an adaptive technique is proposed. An algorithm based on a decomposition technique [Benders, 1962; Kathrin and Tind, 2007] seems appropriate for solving this problem.

- Evolutionary detection of Pareto-optimal Nash equilibrium (Section 5.2) [Gaskó et al., 2012].

  Equilibrium detection in non-cooperative games is an essential task. Decision making processes can be analyzed and predicted using equilibrium detection. The most known equilibrium concept, *Nash equilibrium* (NE) [Nash, 1951], has some limitations: if a game has multiple Nash equilibria a selection problem can appear, it does not always give the highest payoffs. Refinements of *Nash equilibrium* such as: Aumann equilibrium [Aumann, 1959], coalition proof Nash equilibrium [Bernheim et al., 1987], modified strong Nash equilibrium [Ray, 1989; Gaskó et al., 2011], etc., address this limitations.

  Pareto-optimal Nash equilibrium is one refinement of the Nash equilibrium, it selects the NE that is Pareto non-dominated with respect to the other NEs of the game. A computational method for detecting the Pareto-optimal Nash equilibrium is proposed.

Citations

Florin-Claudiu Pop, Denis Pallez, Marcel Cremene, Andrea Tettamanzi, Mihai Suciu, and Mircea Vaida. *QoS-based service optimization using differential evolution.* In Proceedings of the 13th annual conference on Genetic and Evolutionary Computation (GECCO '11), 1891-1898, 2011.

Cited in:

- Cavalca, U.; Mesquista, C.; Pereira, A.C.M.; Carrano, E.G.; *A methodology for traffic shaping multiobjective optimization for next generation network.* Computational Aspects of Social Networks (CASoN), 2012 Fourth International Conference on , 345-350.

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Chapter 2

Background and Related Work

Thesis tackles the multiobjective optimization problem with some applications in the field of Service Oriented Computing and the problem of game equilibrium detection. As these problems are computational hard ones we try to solve them with the aid of *Evolutionary Algorithms* (EA). EAs are population based metaheuristics able to find a good approximation of the solution to an optimization problem in a single run, coping well with various problem features.

2.1 Evolutionary Algorithms

Evolutionary Algorithms represent a simple and efficient nature inspired optimization technique [Holland, 1992; De Jong, 2006]. In essence EAs are stochastic techniques that use an element of randomness to find the solution for the optimization problem. A set of random solutions is evolved with the help of some variation operators, only good solutions will be kept in the population with the help of selection for survivor operators. The basic procedure of an EA is presented by Algorithm 1.

Algorithm 1: Evolutionary Algorithm

```
Initialize population P;
Evaluate P;
repeat
    Generate new individual using selection and variation operators;
    Evaluate new individuals;
    Select survivors;
until termination condition is met ;
```

2.2 Multiobjective Optimization Problem

Many real-world problems are multi-objective by nature. Their solution represents a compromise between the conflicting objectives, for instance minimize the cost of a service while maximizing utility and efficiency. When optimizing all objectives simultaneously we have a *Multiobjective Optimization Problem (MOP).*

For a MOP the m objectives are represented by a set $\{f_i\}_{i\in\{1,...,m\}}$ of functions where $f_i : S \rightarrow \mathbb{R}$ maps a solution s from the decision space $S \subseteq \mathbb{R}^n$ to the objective space $\mathbb{R}$. $F : S \rightarrow \mathbb{R}^m$ represents the objective vector that needs to be minimized $F(x) = (f_1(s), f_2(s), ..., f_m(s))$.

Without loss of generality a minimization multiobjective optimization problem can be stated as follows:

$$\begin{cases} min & F(x) = (f_1(x), f_2(x), ..., f_m(x)), \\ s.t.: & g_i(x) \geq 0, \ i = 1, 2, ...k, \\ & h_j(x) = 0, j = k+1, ..., q, \\ & x \in \Phi, \end{cases} \tag{2.1}$$

where we have $m \geq 2$ objective functions, g_i and h_j represent problem constraints. $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$. $\Phi \subseteq \mathbb{R}^n$ represents the decision space or the search space, $x = (x_1, ..., x_n)$ is the decision vector.

Usually two solutions are compared using the *Pareto dominance* relation. For any two decision vectors $x, y \in \Phi$ we say that x is better than y or x *dominates* y, if:

$$f_i(x) \leq f_i(y) \forall i \in \{1, ..., m\} \; and \; \exists j \in \{1, ..., m\} \; such \; that \; f_j(x) < f_j(y).$$

Usually the ideal solution vector x that optimizes simultaneously all objective functions does not exist. When solving a multiobjective problem usually one does not find a single solution that best approximate F but a set of solutions that approximate the *Pareto optimal set*. The Pareto optimal set is formed by the *Pareto non-dominated* solutions which represent the best trade-offs among the m objectives.

A solution $x \in \Phi$ is *Pareto-optimal* if and only if $\nexists y \in \Phi$ such that $y \prec x$.

The *Pareto-Optimal Set* (POS) is defined as the set of all Pareto-optimal solutions

$$POS = \{x \in \Phi | \nexists y \in X, y \prec x\}.$$

The *Pareto-Optimal Front* (POF) is defined as the set of all objective functions values corresponding to the solution in POS.

$$POF = \{F(x) | x \; is \; non-dominated\}.$$

Evolutionary Multiobjective Optimization Algorithms have been successfully used for solving such problems [Kukkonen and Lampinen, 2005; Coello Coello et al., 2007; Zhou et al., 2011]. As they are population based metaheuristics they represent a good choice because by evolving a set of possible solutions (the population) a good approximation of the Pareto front can be found in a single run while the shape, continuity or other mathematical properties of the true front do not hinder the search.

2.3 Game Theory prerequisites

Game Theory (GT) tries to capture an agents behavior in strategic situations. Individuals make choices depending on the choices of others. In order to analyze a strategic interaction some assumptions about the agents are made:

- All players are rational - they try to maximize their gain;
- Players know their opponents are rational and know the rules of the game played.

Games can be viewed as cooperative and non-cooperative. In this thesis we will concern ourselves only with non-cooperative games. A set of players, a set of possible actions associated with each player and their corresponding gain are the elements of a strategic game. Mathematically, a game is a system $G = ((N, S_i, u_i), i = 1, ..., n)$, where:

- N represents a set of players, and n is the number of players;
- for each player $i \in N$, S_i is the set of actions available to him and

  $$S = S_1 \times S_2 \times ... \times S_n$$

  is the set of all possible situations of the game. An element $s \in S$ is called a situation (or strategy profile) of the game;
- for each player $i \in N$, $u_i : S \rightarrow R$ represents the payoff function of i.

2.3.1 Nash equilibrium

One of the most popular and used equilibrium concept in standard non-cooperative Game Theory is the Nash equilibrium [Nash, 1951]. It represents a steady state in a strategic game.

Playing in Nash sense means that no player can unilaterally change his strategy in order to increase his payoff. In other words a player maximizes its payoff by choosing a strategy that is best response to the one chosen by his opponent, so players have no reason to change the chosen strategies.

Formally, a strategy profile $s^* \in S$ is a Nash equilibrium if the inequality holds:

$$u_i(s_i, s^*_{-i}) \leq u_i(s^*), \forall i = 1, .., n, \forall s_i \in S_i.$$

Here (s_i, s^*_{-i}) denotes the strategy profile obtained from s^* by replacing the strategy of player i with s_i.

2.3.2 Berge-Zhukovskii equilibrium

Considering a strategic game G, strategy s^* is a *Berge-Zhukovskii (BZ)* equilibrium, if the payoff of player i does not decrease considering any deviation of the other $N - i$ players. Playing in *BZ* sense means that each player wants to maximize the payoff of the other players. This equilibrium concept can be interpreted as capturing cooperation in a non-cooperative game.

More formally:

Definition 1 *A strategy profile $s^* \in S$ is Berge-Zhukovskii equilibrium if the inequality*

$$u_i(s^*) \geq u_i(s^*_i, s_{N-i})$$

holds for each player $i = 1, ..., n$, and $s_{N-i} \in S_{N-i}$.

2.3.3 Generalized Nash Equilibrium Problem

Game theory models strategic interactions among players with conflicting interests in which the payoff of each player depends on the choices of all other partners. A constrained, generalized game, is a general model of a decision situation where the strategies of the players are not independent, i.e. they not only affect each others payoffs but also their choices can be restricted by each other.

A formalization of this idea is called Generalized Nash Equilibrium Problem (GNEP) introduced in the 50'ies in [Debreu, 1952; Arrow and Debreu, 1954]. A GNEP is a generalization of the classical equilibrium problem, in which players' strategies depend on the other players' strategies. From a computational point of view, one of the main challenges in solving a GNEP is to detect the entire set of generalized equilibria in a single run.

Thus a GNEP is a generalization of the classical Nash equilibrium problem [Nash, 1951], in which players strategies depend on the other players strategies.

Many real-world situations can be formulated as GNEP: oligopoly models with shared resources, energy markets [Cardell et al., 1997], spatial oligopoly electricity model [Jing-Yuan and Smeers, 1999], a game theoretic interpretation of joint implementation of environmental projects [Breton et al., 2006], electrical market games [Contreras et al., 2004]. The commonly accepted solution of a GNEP is the generalized Nash equilibrium [Han et al., 2012].

A GNEP can be described as a system $G_{GNEP} = ((N, K, u_i), i = 1, ..., n)$, where N represents a set of players, and n is the number of players. For all players $i \in N$ the **common strategy set** is formalized as follows:

> let $s = (s_1, s_2, ..., s_n)$ be the vector formed by all decision variables of the game (strategy profile); let us denote by s_{-i} the vector formed by each players strategy except of the ith player.

Let $S_i \in \mathbb{R}^n$ be the strategy set of player i, $S = \prod_{j \in N} S_j$, $S_{-i} = \prod_{j \in N, j \neq i} S_j$, which means S_{-i} represents the full S set, except the ith player's set.

Let $K_i : S_{-i} \rightarrow S_i$ be a point-to-set mapping which means, that all players j can affect the feasible strategy of player i. Then $K_i(s^*) \subseteq S_i, \forall s^* \in S_{-i}$ Let $K = \prod_{i \in N} K_i(s_i)$ the mapping formed from the K_i.

For each player $i \in N$, $u_i : grK_i \rightarrow R$ represents the payoff function of i, where grK_i represents the graph of the mapping K_i.

Formally, a generalized Nash equilibrium (GNE) is a strategy profile $s^* \in S$ such that the inequality holds:

$$u_i(s_i, s^*_{-i}) \leq u_i(s^*), \forall i = 1, .., n, \forall s_i \in K_i(s_{-i}).$$

The definition of the GNE differs from the normal Nash equilibrium only in the feasible strategy of each player.

Chapter 3

Multiobjective Optimization

Many real-world problems are multiobjective by nature. Their solution represents a compromise between the conflicting objectives e.g. minimize the cost of a service while maximizing utility and efficiency.

Evolutionary *Multiobjective Optimization Algorithms (EMOAs)* are population based optimization techniques able to find an approximation of the true Pareto front for the *Multiobjective Optimization Problem* (MOP) in a single run.

When solving an optimization problem using an evolutionary technique there are mainly three approaches that can be applied: Pareto dominance based algorithms, indicator based algorithms, and decomposition based algorithms. While Pareto dominance based algorithms [Deb et al., 2002; Kukkonen and Lampinen, 2005] perform very well for solving MOPs (2-4 objectives) they scale badly when applied to many-objective problems (more than 4 objectives) [Farina and Amato, 2002; Hughes, 2005; Khare et al., 2003]. This drop in performance is due to exponential growth of non-dominated solutions, all individuals in the population become indifferent to each other (they do not Pareto dominate each other), even in early generations.

Decomposition approaches [Ishibuchi and Murata, 1998; Zhang and Li, 2007; Engau and Wiecek, 2008] rely on a decomposition technique [Benders, 1962]: a scalarizing function and a set of weight vectors decompose the optimization problem in many single-objective problems. The main advantages of this approach are computational efficiency and good scalability with the number of objectives. By decomposing the problem the selection pressure is lover than within Pareto based algorithms. The selection pressure is transferred to weight vector diversity - one needs to generate sufficient weight vectors and assure their. But the number of sub-problems required to approximate the Pareto front also grows exponentially with the number of objectives.

3.1 Scalarization Techniques for EMOAs

When faced with a many-objectives optimization problem *(MOP)* one of the challenges that arises is the exponential growth in the number of solutions required to properly approximate the Pareto front [Farina and Amato, 2002]. Another challenge is the rapid growth of the number of non-dominated solutions in the current population. The effectiveness of the selection for survivor operators is thus severely reduced. Moreover the large solution diversity decreases the performance of recombination operators [Ishibuchi et al., 2012].

Decomposition approaches seem to be a good choice when solving many-objective optimization problems. It is easier to solve multiple single-objective problems. The main scalarization function used as decomposition technique are: *Weighted Sum* and *Weighted Tchebycheff* [Miettinen, 1999]. But there are are other scalarization techniques such as Weighted L_p, *augmented Tchebycheff*, and *modified Tchebycheff* [Ehrgott, 2006; Kathrin and Tind, 2007; Miettinen, 1999] that can be used as a decomposition technique. In this Section we explore the performance and scalability of these techniques to many-objective problems.

3.1.1 Scalarization Techniques - Prerequisites

If we aggregate all f_i objectives of a MOP, under some conditions, a Pareto optimal solution of the multiobjective problem can be found. By repeatedly solving the scalar problem a subset of efficient solutions to problem (2.1) can be found.

Regardless of the scalarization technique used, it must meet the following conditions [Wierzbicki, 1986]:

- an optimal solution of the scalar problem needs to be an efficient solution to problem (2.1),
- all efficient solutions of problem (2.1) needs to be found using the scalarization technique.

3.1.2 Results

We explore the scalability of the above scalarization techniques to many-objective problems. For this purpose we use Differential Evolution (DE) [Storn and Price, 1997] as an underlying evolutionary technique. We intend to observe the behavior of the scalarization techniques not to analyze the effectiveness of DE. Mating similar parents yields better results for many-objective problems [Ishibuchi et al., 2012; Kowatari et al., 2012]. Therefore we considered a crowding based DE, *DE/best/1* variant, and *DE/current-to-best*. All scalarization techniques behave in the same manner regardless of the DE version used.

As basis for comparison the WFG [Huband et al., 2006], DTLZ [Deb et al., 2005], and ZDT [Zitzler et al., 2000] test problem suites are used. All problems are real valued unconstrained and require the minimization of objective functions. These test problems provide different difficulties such as: different geometry for the Pareto optimal front - linear, convex, concave, disconnected, DTLZ and WFG offer scalability with the number of objectives.

For assessing the quality of the approximated Pareto set we compute the Inverted Generational Distance (IGD) and Hypervolume (HV) indicators. The reference point for the Hypervolume is $(ref_i) = 7, i \in \{2, ..., m\}$.

For 2 and 3 objectives experiments indicate that *modified* and *augmented Tchebycheff* techniques performe better than *Weighted Sum* and *Weighted Tchebycheff*. For many-objective problems we can observe a bigger increase in performance for *augmented, modified Tchebycheff*, and L_p techniques. L_p norm with p sufficiently large $p = \{100, 1000\}$ represents a good choice for many-objective problems.

3.2 Exploring Lorenz Dominance

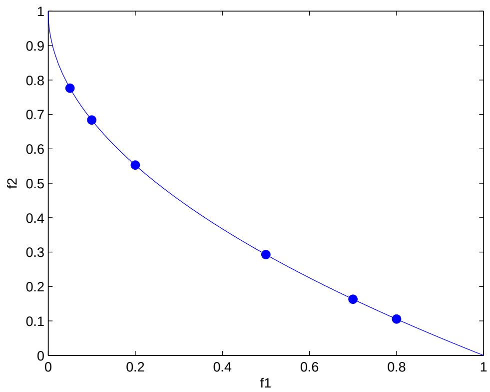
A problem frequently encountered in classical multi-criteria optimization is the existence of a large (often infinite) set of optimal solutions. The decision making process based on selecting a unique preferred solution becomes difficult. The Lorenz dominance [Kostreva and Ogryczak, 1999], also called equitable dominance, could offer a solution to this problem. The Lorenz dominance relation, is a refinement of the Pareto dominance relation and it is used in Decision Theory and fair optimization problems.

3.2.1 Lorenz Dominance

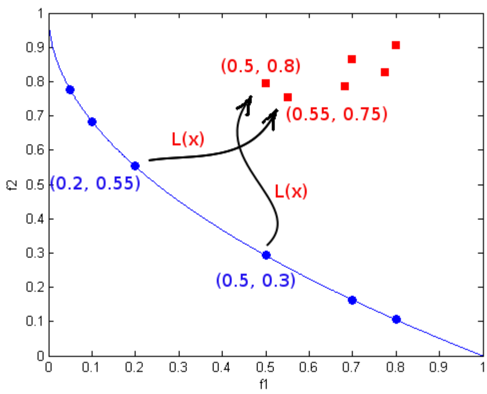
Optimization with multiple equitable criteria was introduced by Kostreva and Ogryczak [Kostreva and Ogryczak, 1999; Kostreva et al., 2004]. By replacing Pareto dominance ($\prec_P$) with the concept of Lorenz dominance ($\prec_L$) the search algorithm will favor only equitable solutions.

$L : X \to \mathbb{R}^m$ is the Lorenz vector valued function, $L(x) = (l_1(x), ..., l_m(x))$, where:

$$\begin{cases} l_1 = f_1(x), \\ l_2 = l_1 + f_2(x), \\ ... \\ l_m = l_{m-1} + f_m(x). \end{cases} \tag{3.1}$$



(a) $F(x)$.



(b) $F(x)$ and $L(x)$.

Figure 3.1: Fig 3.1a presents vector $F(x)$ for 6 solutions that lie on the true Pareto front. Fig 3.1b present the Lorenz vectors $L(x)$ for the 6 solutions from Figure 3.1a - it can be observed that the equitable solutions *Pareto dominate* the other solutions after the Lorenz transformation is applied.

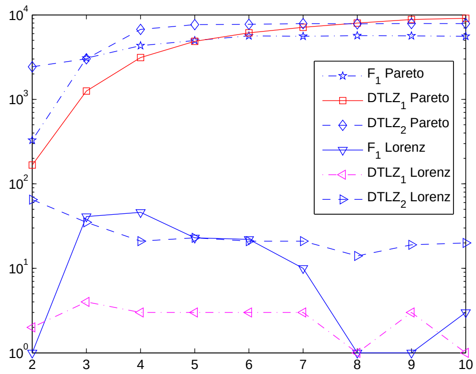


Figure 3.2: Pareto and Lorenz dominance. Evolution of the number non-dominated solutions with increasing number of criteria (10^5 randomly generated solutions).

and $f_1(x) \geq f_2(x) \geq ... \geq f_m(x)$ represent the components of vector $F(x) = (f_1(x), ..., f_m(x))$ sorted in decreasing order. If the optimization problem is a maximization one then $F(x)$ needs to be sorted in ascending order. Figure 3.1 presents $F(x)$ and $L(x)$ vectors for 6 solutions that lie on the true Pareto front for test problem ZDT1.

3.2.2 Pareto and Lorenz Dominance in Randomly Generated Solutions

Figure 3.2 illustrates the evolution of the number of non-dominated Lorenz and Pareto solutions for 10000 randomly generated solutions for the F_1, DTLZ1, and DTLZ2 test problems. It can be observed that the number of Pareto non-dominated solutions increases drastically for a number of objectives greater than two.

Figure 3.3 depicts the number of optimal solutions detected by GDE3 and L-GDE3 algorithms for various number of objectives. For all three problems, WFG1, WFG2 and WFG3, when Pareto dominance is used the number of optimal solutions detected by the algorithm decreases exponentially with the increasing number of criteria. In case of two and three objectives the GDE3 algorithm, that uses Pareto dominance, detects a reasonable number of optimal solutions. This number decreases in case of four objectives and tends to zero when the number of objectives is higher than five. By contrast, the performance of the Lorenz based algorithm, L-GDE3, is relatively constant even when the number of objectives is higher than four.

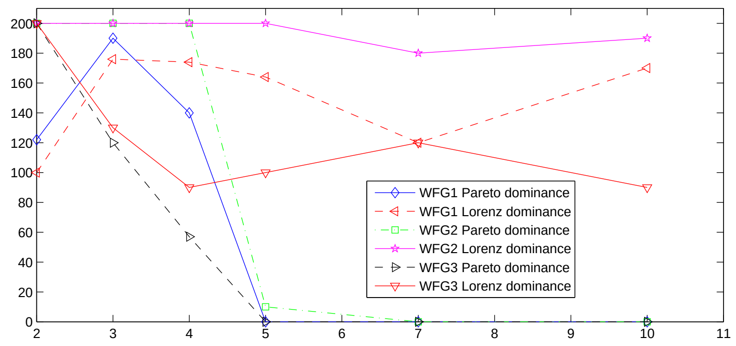


Figure 3.3: Number of optimal solutions detected by GDE3 and L-GDE3 algorithms. A population of 200 solutions is considered.

3.3 Resonance Search Technique - a New EMOA Model.

We consider a new model according to which the Pareto set is approximated by several instances of a *Dynamic Fitness Landscape* (DFL). Each DFL instance is described by an aggregated function. This aggregated function serves for fitness assignment and guides the exploration of DFL instance. Several implementations of this model are possible. We consider a very simple technique able to offer some indications concerning the potential of the DFL model.

The proposed approach, called *Resonance Search* (RS) is a multi-population evolutionary technique. Each population uses one or several search techniques, mainly based on scalarization.

Populations evolve independently. After a predefined number of generations, interactions between populations take place by changing individuals or by migration of some clones. Population sizes and search mechanisms may change dynamically during the search process.

3.3.1 Resonance Search Model

Scalability improvement of EMOAs to many-objective problems may be obtained by replacing Pareto dominance, as the main search engine, with other fitness assignment mechanisms based on scalarization and indicators expressing the quality measure of solution sets.

Another idea is to use the dynamic fitness approach. Several search mechanisms within the same (sub-)population may be considered in a dynamic way. In this situation the search process is guided by several interconnected (cooperating or competing) mechanisms. The search model in a (sub-)population may be variable (it may change according to a fixed or a flexible schedule).

Resonance Search (RS) relies on these ideas and considers a set of m $\mu-$populations (possibly of unequal size) at each generation. Each $\mu-$population size may vary during the search process.

Each $\mu-$population (also called internal population) is a member of a global generational population. Each $\mu-$population may have several search mechanisms. Internal populations evolve independently using different search mechanisms. After a predefined number of generations, internal populations exchange individuals.

The search process within a $\mu-$population, as well as the topology of the exchange mechanism are dynamic. $\mu-$populations interact in a coherent manner producing a *resonance* process: solutions in a $\mu-$population improve the search process in other $\mu-$populations.

In order to address many-criteria problems and to avoid drawbacks due to non-domination the main RS search mechanism relies on decomposition and scalarization.

RS typically considers two aggregation functions that are used to guide the search within each $\mu-$population.

For ensuring an efficient exploration of the search space two strategies may be considered:

(i) each individual in a $\mu-$population is evaluated using one of the two aggregation functions available;

(*ii*) each individual is evaluated using a combination of the two aggregation functions.

Both strategies enable the search to cope with non-convex or with convex-concave fitness landscapes.

Typically the strategy (*i*) is used in reported experiments. In this case two fitness landscapes are explored in parallel by a $\mu-$population at each iteration.

Our proposed model maintains a global population composed of several independently evolving micro-populations (or, simply, populations). Each $\mu-$population evolves independently for an *epoch* (k generations) using two decomposition techniques: Weighted Sum and Weighted Tchebycheff, having the same weighting system. After k generations $\mu-$populations exchange individuals (k represents the migration period). This is intended as a supplementary diversity preserving mechanism. It also serves for aggregating partial results obtained by several $\mu-$populations. Within each $\mu-$population the search continues with the same weighting system until the end of the current epoch.

Within RS two decomposition techniques (Weighted Sum and Weighted Tchebycheff method) coexist in a given $\mu-$population. Some individuals of the $\mu-$population are evaluated using Weighted Sum and the others are evaluated with the Weighted Tchebycheff approach.

Within each $\mu-$population the rate of individuals evaluated by Weighted Sum and by Weighted Tchebycheff is constant during the search process. This rate is

$$R = \frac{WS}{WT},$$

where $WS(WT)$ represents the number of individuals evaluated with respect to the Weighted Sum (Weighted Tchebycheff) method.

Numerical experiments indicate that, for a small number of criteria, RS perform comparable to the state of the art algorithms NSGA2, SPEA2, and MOEA/D. Moreover RS is able to deal well with many-criteria optimization problems. No archive is compulsory to keep the non-dominated solutions but it can be useful in some situations. RS performs well for 21 criteria. Current experiments indicate this is not the limit of proposed techniques. Further research will investigate RS ability to cope with a combination of concave-convex landscapes. The limits concerning the number of criteria will also be explored.

Chapter 4

Applications of EMOAs in Web Service Composition

Service Oriented Architecture (SOA) implementations are more and more popular, diverse and widespread in enterprise distributed environments. This fact is due to their technical advantages over more traditional methods of distributing computing. These advantages include: delivering application functionality as services across several platforms, providing location independence, authentication/authorization support, dynamic search and connectivity to other services.

When the existing problem has multiple criteria, if a single-objective algorithm is considered, it is necessary to define a priori the importance of each criteria and use some form of aggregation to combine all criteria in a single objective. The difficulty of this approach lies in defining the weights (importance) of each criteria. Usually the correlation between objectives is complex and dependent of the alternatives available, the different objectives are non-commensurable (e.g. minimize latency, maximize efficiency) thereby it is difficult to aggregate them into one objective. It may be very difficult to combine different objectives into a single goal function before alternatives are known. By optimizing a single function only a partial view of the result is obtained.

The *QoS* optimization problem is multiobjective by its nature. The user might prefer to see several good solutions (Pareto optimal) and decide which is best for himself while the standard optimization algorithms offer only one solution. For example it is more natural to let the user decide if he wants to pay a specific known price than aggregating the objectives and ask him to specify *a priori* how important is the price for him without knowing the precise price value. By using multiobjective optimization it is not necessary to define a priori an aggregation function. This is not a trivial task and different users may have different preferences, e.g., price versus quality requirements.

4.1 Web Service Composition Based on QoS Parameters

Different services that provide same functionality may have different *Quality of Service (QoS)* properties. For example, a service may be cheaper but have a higher response time, while the other may be more expensive but less available. Given a composite service described by a business workflow that includes a set of abstract services, where each abstract service can be realized by several concrete services, the QoS optimization problem is to find the optimal combination of concrete services (having the best QoS).

Executing an activity means invoking a service. For each activity, which is assimilated to an *abstract service*, several *concrete services* exist. Each concrete service has different *QoS* properties. For describing the *QoS* we use the following parameters: *response time, rating, availability* and *cost*.

The *QoS* indicator of the composite service is obtained by aggregating the *QoS* of the component services. Different aggregation rules can be found in the literature [Canfora et al., 2005; Li et al., 2010; Yao and Chen, 2009]. The aggregation operations depend on the composite service architecture. Table 4.1 shows how the aggregate *QoS* attribute is computed for each control structure.

The problem stated previously is well known in domains like *Service Oriented Computing (SOC)* and *Search-based Software Engineering (SBSE)* [Harman, 2007; Canfora et al., 2005; Comes et al.,

Table 4.1: *QoS* aggregation rules for different control structures.

QoS property	Flow	Sequence	Switch	While
Response time (T)	$\max_{i\in 1..m}\{t_i\}$	$\sum_{i=1}^{m} t_i$	$\sum_{i=1}^{m} p_i \cdot t_i$	$k \cdot t$
Reliability (R)	$\prod_{i=1}^{m} r_i$	$\prod_{i=1}^{m} r_i$	$\sum_{i=1}^{m} p_i \cdot r_i$	l^k
Availability (A)	$\prod_{i=1}^{m} a_i$	$\prod_{i=1}^{m} a_i$	$\sum_{i=1}^{m} p_i \cdot a_i$	a^k
Cost (C)	$\sum_{i=1}^{m} c_i$	$\sum_{i=1}^{m} c_i$	$\sum_{i=1}^{m} p_i \cdot c_i$	$k \cdot c$

2010; Liu et al., 2009; Vanrompay et al., 2008]. Various solutions are proposed based on different approaches such as: integer programming, genetic and hill climbing algorithms [Bahadori et al., 2009; Canfora et al., 2005; Zeng et al., 2004; Parejo et al., 2008].

4.2 An Adaptive EMOA for Web Service Composition

Despite the fact that the *QoS* optimization problem is multiobjective by nature few approaches based on multiobjective algorithms can be found in the literature [Li et al., 2010; Ross, 2006; Taboada et al., 2008; Wada et al., 2008; Yao and Chen, 2009]. In most cases single-objective algorithms are used to solve this problem. The user might prefer to see several good solutions (Pareto optimal) and decide which is the best for himself. Objective aggregation offers only one solution. It is more natural to let the user decide the importance of each objective than aggregating the objectives and ask the user to specify *a priori* his preferences (this is not a trivial task). For solving the *QoS* web composition problem few applications based on multiobjective optimization algorithms can be found in the literature

There are many variants of EAs which have different control parameters: population size, operators used, crossover and mutation probabilities, etc. Selecting appropriate values is mainly done based on empirical studies, often a "trial and error" fashion is used for adjusting these values. Typically one parameter is adjusted at a time, which may lead to sub-optimal choices, since often it is not known how different parameters interact. Such an approach is time consuming. In the last couple of years there has been an increasing interest in designing methods that self-adapt these parameters [Chakhlevitch and Cowling, 2008; Eiben et al., 1999; Neri and Tirronen, 2010].

The QoS-based service optimization is a combinatorial multiobjective optimization problem. Using a decomposition technique many-criteria problems may be decomposed in a class of single-objective problems [Benders, 1962; Kathrin and Tind, 2007]. Decomposition techniques have good performance for combinatorial problems, another advantage is the small computational load. One drawback to decomposition evolutionary approach is the dependency between the problem type and algorithm parameters. The same algorithm must solve different instances of this problem. One instance is represented by the business workflow that describes the web services composition (the interconnections of the composing web services, see [Pop et al., 2011] for more details). Different web services are described by different workflows, thus the search space changes for each workflow making it a very dynamical problem. A set of parameters that work for one particular instance of the workflow may not yield good results for another workflow. It would be very difficult to tune parameters for each particular workflow. A self-adaptive technique seems the obvious choice for this kind of problem.

The hybridization between a decomposition based multiobjective optimization algorithm and an adaptive technique is explored in. An algorithm based on a decomposition technique [Benders, 1962; Kathrin and Tind, 2007] seems appropriate for solving this problem. The new approach is validated on some well known multiobjective test problems, and then applied to the web service composition problem. We compare our results with some state of the art algorithms from the literature.

4.2.1 Proposed Approach

As the *Qos*-based web service optimization problem is a combinatorial one we use $MOEA/D$ algorithm to solve it. To cope with the dynamic nature of the problem we endow $MOEA/D$ with an adaptation mechanism. $MOEA/D$ is based on *DE*. Some very simple and yet powerful adaptation techniques for *DE* have been propose [Neri and Tirronen, 2010]. We propose two adaptive variants of $MOEA/D$ obtained by considering the *DE* adaptive mechanisms *SaDE* [Qin et al., 2009] and *CoDE* [Wang et al., 2011]. The new models are called $MOEA/D_C$ (Algorithm 2) and $MOEA/D_S$ (Algorithm 3).

Algorithm 2: Adaptive $MOEA/D_C$

input : N, T - number of sub-problems, neighborhood size
output: EP - external population that holds the non-dominate solutions
1 Initialization: $EP = \varnothing$, generate weight vectors and compute $B(\lambda)$;
2 **for** $i \leftarrow 1$ **to** N **do**
3   generate 3 offsprings using a random combination between a trial vector generation strategy and the control parameters;
4   update the neighboring solutions;
5   update z^* and EP;
6 If stopping criteria is satisfied output the EP. Otherwise go to step 2;

Algorithm 3: Adaptive $MOEA/D_S$

input : N, T, LP - number of sub-problems, neighborhood size, learning period
output: EP - external population that holds the non-dominate solutions
1 Initialization: $EP = \varnothing$, generate weight vectors and compute $B(\lambda)$, $Cr_m = 0.5$;
2 **for** $i \leftarrow 1$ **to** N **do**
3   generate 2 offsprings based on the strategies *rand/1/bin* and *best/2/bin*;
4   update the neighboring solutions;
5   update z^* and EP;
6 after LP generations update Cr_m and the probabilities p_i for the trial vector generation strategies;
7 If stopping criteria is satisfied output the EP. Otherwise go to step 2;

In $MOEA/D_C$ *DE* trial vector generation strategy *DE/rand/1/bin* used in $MOEA/D$ is replaced with *CoDE* strategy - three trial vectors are created and the best one is kept (lines 3-4). In $MOEA/D_S$ *DE* trial vector generation strategy is replaced with *SaDE* strategy, after LP generations parameter Cr_m and the probabilities for using each trial vector generation strategy from *SaDE* candidate pool are computed (line 6). By using an adaptive scheme we avoid the drawback of manual tuning algorithm parameters for each specific workflow.

The genome we use for our problem is depicted in figure 4.1. It consists of an array of integer values and has the length equal to the number of abstract services. Each gene stores the index of the concrete service that realizes the corresponding abstract service.

We compare the proposed adaptive algorithm with the classic version of $MOEA/D$ then apply it to the *QoS*-based service composition problem, some state of the art multiobjective algorithms are considered for comparison. The results show the potential of this approach. Better performance is obtained (with respect to multiobjective quality indicators) when the adaptive approach is applied to standard test problems and some business workflows of high complexity.

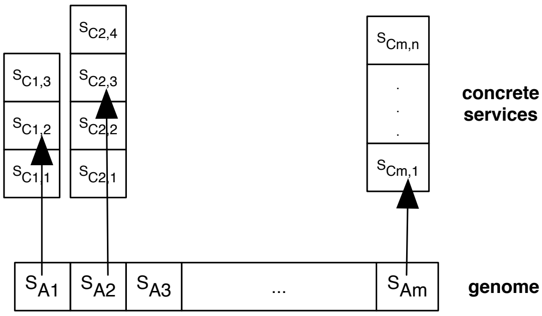


Figure 4.1: Genome encoding for service composition.

4.3 Equitable Solutions

Compared to single-objective algorithms, Multiobjective Evolutionary Algorithms (EMOAs) have some advantages: *(i)* the aggregation of criterion functions is not necessary and *(ii)* the user has the possibility to select *a posteriori* one of the Pareto optimal solutions.

A *Decision Maker* (DM) must pick the suitable solution from a final solution set found by the algorithm. In many cases the DM is not an expert thus this final step is not easy for him. By replacing Pareto dominance with an equitable dominance relation in the search process the size of the final solution set is greatly reduced, helping the non expert DM in the final step.

We propose to address the *QoS* optimization problem using the concept of *Lorenz dominance*. Lorenz solutions are equitable and well balanced. Therefore, an equitable approach based on Lorenz dominance could simplify the Decision Maker's choice. Evolutionary detection of Lorenz solution seem to be an appealing one. For this purpose some state-of-the art EMOAs are slightly modified for addressing the *QoS* optimization problem and detect the Lorenz front. Lorenz solutions drastically reduce the number of solutions in the Pareto set, and thus the decision costs.

Lorenz dominance is a refinement of Pareto dominance and it is natural in fair optimization problems. Using Lorenz dominance in the search process can relieve the Decision Maker from the burden of choosing a good solution from a large set of final solution (that may contain a large number of solutions). Lorenz equitable solution concept has been investigated for the *QoS* optimization problem.

Several multiobjective optimization algorithms are considered for solving the *QoS* optimization problem considering Lorenz dominance. *L-NSGA2* seems to have the best performances according to performance metrics, but it is also the slower algorithm. Therefore, DE approaches seem to be a better choice for addressing complex problems. The results show that the Lorenz dominance concept reduces the number of solutions and the decision making is easier.

4.4 EMOAs for Web Service Composition

Some multiobjective algorithms are better than others and this depends on the nature and complexity of the optimization problem. For this purpose we do a set of comparative tests between *NSGA2* [Deb et al., 2002], *SPEA2* [Zitzler et al., 2001], *MOEA/D* [Zhang and Li, 2007], *GDE3* [Kukkonen and Lampinen, 2005], and *POSDE* [Chang et al., 1999] multiobjective optimization algorithms on workflows with different complexities and real *QoS* data.

Several concrete services (implementations) are available for a certain function or abstract service. Their *QoS* parameters do not change during the optimization phase. This is a simplification hypothesis made by the majority of the related approaches.

Abstract *BPEL* workflows of various complexity are randomly generated. The complexity is adjusted by increasing the number of abstract services from 10 to 30. A fixed number of abstract services m is considered for each tested scenario. The number of concrete services that could realize each abstract service n is varied from 10 to 90. The *Quality of Service* parameters for each concrete service are not randomly generated but represent real data measured for real web service parameters and are taken from an available database that contains more than 2500 real web services (http://www.uoguelph.ca/~qmahmoud/qws/).

For all combinations of $m \in \{10, 20, 30\}$ abstract services and $n \in \{10, 20, ..., 90\}$ concrete services average values of the normalized Hypervolume indicator show that in average all tested algorithms behave similarly. 50 independent runs are considered.

We can conclude that all algorithms behave similarly relative to the Hypervolume indicator. Let us consider a business workflow with 10 abstract services and 80 concrete services. If we look at the standard deviation of the average we can see that *POSDE* (a basic multiobjective version of Differential Evolution) and *NSGA2* are more stable/robust than the other algorithms. This happens for all combinations of abstract and concrete services.

Set Coverage quality indicator measures the percentage of solutions found by algorithm A that Pareto dominate the solution set found by algorithm B. From the average values we can see that in general when there is a difference on the solution set *POSDE* and *NSGA2* are the algorithms that give the best results, a large percentage of the found solution set Pareto dominates the solution set given by the other optimization algorithms.

We wanted to see if there is a statistical difference between the approximations sets obtained by tested algorithms. Because the data does not follow a normal distribution we performed a *Kruskal-Wallis* analysis over the Hypervolume values obtained by each method over 50 independent runs. Test results show that there is a statistical difference between algorithms results so we individually compared each algorithm with the others algorithms using *Wilcoxon rank-sum* test. We compare each algorithm based on mean Hypervolume values and Wilcoxon test results for $m \in \{10, 30\}$ abstract services and $n \in \{30, 80\}$ concrete services. In general, *NSGA2*, *GDE3*, and *POSDE* algorithms outperform the other approaches.

Chapter 5

Evolutionary Equilibrium Detection in Strategic Interactions

Game Theory tries to shed light on the decision making process between interacting decision-makers. Despite skepticism Game Theory models have characterized and explained a vast array of economical, biological, political situations. When one tries to find a solution to an explored game, he tries to find its respective equilibrium - a game is in equilibrium if all actions chosen by the decision-makers are mutually consistent.

5.1 Epsilon-Berge Equilibrium

The ϵ-Nash equilibrium [Radner, 1980] is a weakening of the strict rationality - in this case it is enough to be near Nash equilibrium. But ϵ can be interpreted in several ways [Dumitrescu et al., 2009]: measuring uncertainty of a strategy choice, a supplementary cost of attending the equilibrium strategy, or a perturbation of the players rationality.

Berge-Zhukovskii (BZ) equilibrium [Zhukovskii and Chikrii, 1994] is a solution concept in which every player's strategy is stable against the deviations of all other players. This alternative equilibrium can be useful for games having several Nash equilibria, or for games where Nash equilibrium does not ensure the best alternative for the players (i.e. trust games [Osborne, 2004]).

A computational method for detecting the ϵ-Berge-Zhukovskii ($\epsilon - BZ$) equilibrium is presented in this section. The intuition behind is the same as in the case of the ϵ-Nash equilibrium: the *epsilon* gives a perturbation to the players strategies. A generative relation for finding $\epsilon - BZ$ equilibrium is used, it induces a non-domination concept which is used in a evolutionary multiobjective optimization algorithm for equilibria detection.

5.1.1 Epsilon-Berge-Zhukovskii equilibrium

Inspired by the notion of ϵ-Nash equilibrium, the ϵ-Berge-Zhukovskii equilibrium analyzed. ϵ relaxation gives flexibility to the standard Berge-Zhukovskii equilibrium.

Definition 2 *A strategy profile $s^* \in S$ is ϵ-Berge-Zhukovskii equilibrium if the inequality*

$$u_i(s^*) \geq u_i(s_i^*, s_{N-i}) - \epsilon, \epsilon > 0$$

holds for each player $i = 1, ..., n$, and $s_{N-i} \in S_{N-i}$.

We denote by BE_ϵ the set of all ϵ-Berge-Zhukovskii equilibria of the game.

5.1.2 Evolutionary Detection of Epsilon-BZ Equilibria

Solving a multiplayer game in which players seek to maximize their payoffs has many common features with the multiobjective optimization problem (MOP).

For detecting the ϵ-Berge-Zhukovskii equilibrium any Pareto based multiobjective algorithm is suitable. The only modification needed is the replacement of the Pareto dominance relation in s_s and s_v (if the algorithm uses Pareto ranking for maintaining diversity) with a dominance relation for the $\epsilon - BZ$ equilibrium - $\prec_{B_\epsilon}$.

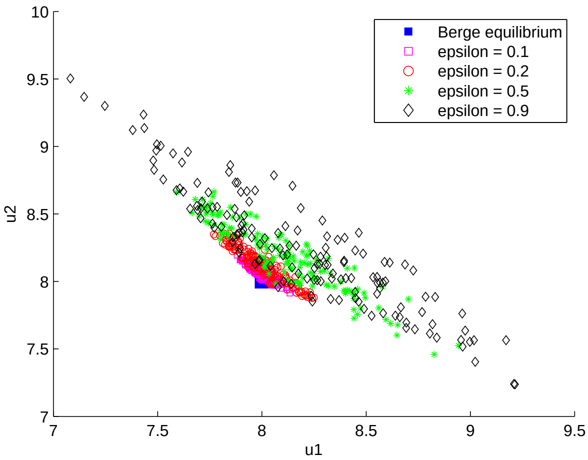


Figure 5.1: Payoffs for the Voluntary Contribution Mechanism game, $\epsilon \in \{0, 0.1, 0.2, 0.5, 0.9\}$.

For example let us consider the Voluntary Contribution Mechanism (VCM) is a good example that people are not totally self-interested. They spend time making something for the common good. Theoretical studies [Bochet et al., 2006] and experiments [Fischbacher et al., 2001; Muller et al., 2008] are made concerning player behavior. A model of the VCM is described as game G_2:

$$u_i(s) = 10 - s_i + 0.4 \sum_{i=1,n} s_i, s_i \in [0, 10], i = 1, ..., n.$$

In this game the *BZ* equilibrium means that all players play strategy 10, which means they spend all for the public good. Detected ϵ-BZ equilibria for the two-player version of the VCM game are presented in Figures 5.1,

5.2 Pareto-optimal Nash Equilibrium

Equilibrium detection in non-cooperative games is an essential task. Decision making processes can be analyzed and predicted using equilibrium detection. *Nash equilibrium* (NE) [Nash, 1951] has some limitations: if a game has multiple Nash equilibria a selection problem can appear, it does not always give the highest payoffs. Refinements of *Nash equilibrium* such as: Aumann equilibrium [Aumann, 1959], coalition proof Nash equilibrium [Bernheim et al., 1987], modified strong Nash equilibrium [Ray, 1989; Gaskó et al., 2011], etc., address this limitations.

Pareto-optimal Nash equilibrium is a refinement for Nash equilibrium. For a problem with multiple NEs it selects the one that is Pareto non-dominated with respect to the other NEs of the game. We propose an evolutionary method able to detect this equilibrium.

Definition 3 *Let $s^* \in S$ be a Nash equilibrium. s^* is a Pareto-optimal Nash equilibrium, if there is no $s \in S$ such that:*

$$u_i(s) \geq u_i(s^*), \forall i \in N.$$

For two strategy profiles $s^*, s \in S$ we denote by $pn(s^*, s)$ the number of strategies for which some players can benefit deviating.

We may express $pn(s^*, s)$ as:

$$pn(s^*, s) = card\{i \in N,\ u_i(s) > u_i(s^*), s \neq s^*\} +$$

$$+ card\{i \in N, u_i(s_i, s^*_{-i}) > u_i(s^*)), s^*_i \neq s_i\},$$

where $card\{R\}$ denotes the cardinality of the set R.

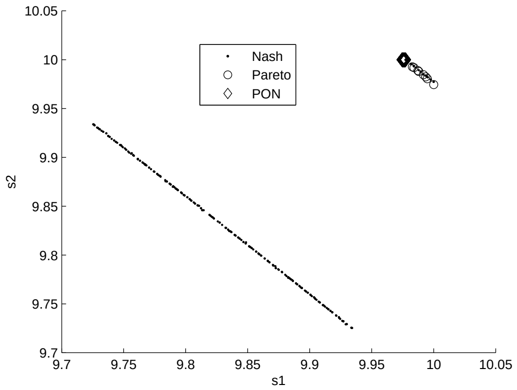


Figure 5.2: Detected Pareto front, Nash equilibrium, and Pareto-optimal Nash equilibrium strategies for Game 2.

Definition 4 *Let $s^*, s \in S$. We say the strategy s^* is better than strategy s with respect to Pareto-optimal Nash equilibrium, and we write $s^* \prec_{PN} s$, if and only if the inequality*

$$pn(s^*, s) < pn(s, s^*),$$

holds.

Definition 5 *The strategy profile $s^* \in S$ is a Pareto-optimal Nash non-dominated strategy, if and only if there is no strategy $s \in S, s \neq s^*$ such that s dominates s^* with respect to $\prec_{PN}$ i.e.*

$$s \prec_{PN} s^*.$$

For detecting the Pareto-optimal Nash equilibrium we use the Differential Evolution algorithm [Storn and Price, 1997]. The only modification needed is the replacement of the Pareto dominance relation in s_s and s_v (operators) with a dominance relation for the Pareto-optimal Nash equilibrium - $\prec_{PN}$. Parameter settings used in numerical experiments are the following: population size $n = 100$, number of generation =1000, $Cr = 0.7$, $F = 0.25$.

Let us consider the two person game G_2 [Dumitrescu et al., 2010], having the following payoff functions:

$$\begin{cases} u_1(s_1, s_2) = s_1(10 - sin(s_1^2 + s_2^2)), \\ u_2(s_1, s_2) = s_2(10 - sin(s_1^2 + s_2^2)), \\ s_1, s_2 \in [0, 10]. \end{cases}$$

Obtained strategies are depicted in Figure 5.2, obtained payoffs are depicted in Figure 5.3. The Pareto-optimal Nash equilibrium reduces the set of Nash equilibria.

5.3 Sub-Optimality and Infra Equilibria

We investigate strategic interactions where some constraints are imposed to player strategies. Resulting equilibria are identified as infra-equilibria of the original game and may be considered as second-best options with respect to the initial game. Infra equilibria are characterized by generative relations and a computational method for detecting them is proposed.

Let us consider a game G and a strategic situation where one or more players have some constraints applied to their strategy profile. This may be related to the cost of reaching the strategies or to other practical constraints. It is natural to associate to a profile strategy a cost that can not be included in the payoff function.

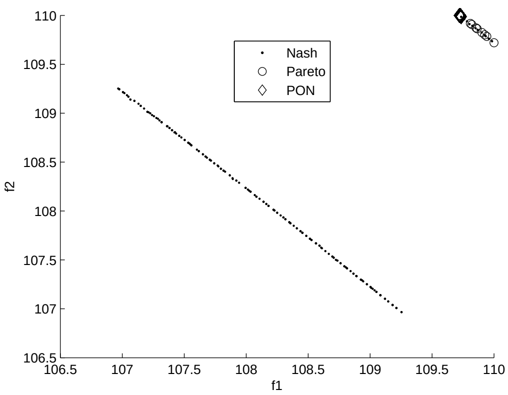


Figure 5.3: Detected Pareto front, Nash equilibrium, and Pareto-optimal Nash equilibrium payoff for Game 2.

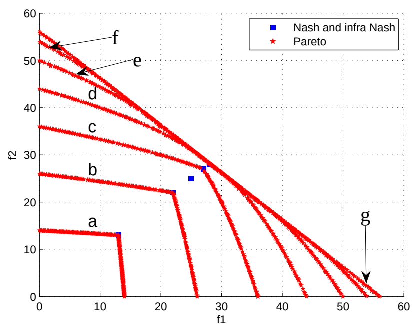


Figure 5.4: Infra Nash payoffs for Experiment 1.1, several restricted strategy spaces are considered: (a)$x_1, x_2 \in [0,1]$; (b)$x_1, x_2 \in [0,2]$; (c)$x_1, x_2 \in [0,3]$; (d)$x_1, x_2 \in [0,4]$; (e)$x_1, x_2 \in [0,5]$; (f)$x_1, x_2 \in [0,6]$; (g)$x_1, x_2 \in [0,15]$.

5.3.1 Infra Games

If on a strategic game G we impose some restrictions on the available strategies we obtain a game G' that we call an *infra game* (with respect to G).

We consider situations where the NE of the game belongs to the restricted strategy space. When a Nash equilibrium of G is inaccessible for G' we may consider the equilibrium of G' as a different type of equilibrium (or stable situation) exploring the set of accessible strategies and representing a second-best solution concept (for G). Nash equilibrium may be described as the set of non-dominated strategies with respect to a generative relation [Lung and Dumitrescu, 2008]. But here we are interested in approximating the equilibrium situations of game G as some of its strategies become inaccessible (for cost or for other reasons) or are simply restricted.

One idea is to look for Nash non-dominated feasible strategies of G'. If we find an accessible non-dominated strategy and this situation is computationally stable we may consider it as representing an equilibrium situation and we call it an *infra-Nash equilibrium* of the game G.

Imposing several restrictions on the strategies of the game G we may obtain a class of infra Nash equilibria. Infra equilibrium may be associated to practical decision making or economical situation. For instance when applied for describing exploitation with regeneration infra equilibria may permit an estimation for the limits of exploitation. An interesting type of sub-optimal exploitation may be associated to Pareto optimal infra Nash equilibrium (infra Nash lying on the Pareto set). The exploitation without regeneration may push the infra Nash equilibrium outside the Pareto set.

For detecting an infra equilibrium evolutionary multi-criteria optimization techniques are considered. The Pareto dominance relation is replaced by the generative relation of the infra equilibrium. Symmetric and asymmetric versions of the Cournot game are considered. Results indicate the stability of the detected *infra-Nash* equilibria. In all cases the infra equilibria tend to the real

Nash equilibrium as the strategy set tends to the strategy set of the game G.

In Figure 5.4 the effect of restriction can be observed in the space of payoffs. Seven different cases are studied. In the restricted version of the Cournot game each firm can produce only a restricted quantity q_i. In each restricted case, (a)-(d), the infra Nash equilibrium is the maximum possible quantity. The algorithm finds correctly all infra equilibria.

Chapter 6

Evolutionary Equilibrium Detection in Dynamic Games

In general when optimizing a problem its characteristics do not change during the optimization process, but there are many problems do not exhibit a static behavior. If an optimization problem changes its characteristics over time then it is considered a *dynamic problem*. Dynamic nature of the problem may be reflected in the change of the fitness function, problem constraints, or problem instance [Yang et al., 2007].

Within Game Theory a discrete-time dynamic games capture changes within game parameters (payoffs). Such a game represents a more realistic real-world decision model. Within a one-shot non-cooperative game players make their decisions simultaneously, the game consisting only from one decision-making process. In some cases a static non-cooperative game is considered just a special case of a dynamic game that represents a more complex and realistic notion with multiple applications in economics, engineering and others.

Dynamic Games are mathematical models of the interaction between independent decision makers (players, agents) who are controlling a dynamical system [Haurie and Krawczyk, 2000].

Although there is an extensive amount of research on evolutionary optimization techniques for dynamic environments [Jin and Branke, 2005; Plessis and Engelbrecht, 2013; Cheng et al., 2013; Yang et al., 2012; Nguyen et al., 2012; Cruz et al., 2011] to the best of our knowledge there are no previous attempts to adapt the search of an evolutionary algorithm to dynamic games.

6.1 Dynamic Equilibrium Tracking

In order to compute the GNEs in a constrained dynamic environment an Evolutionary Algorithm for Equilibria Detection (EAED) [Suciu et al., 2013b] can be endowed with a mechanism for tracking and adapting to environment changes.

An EAED uses a selection for survival operator that guides the search using a generative relation [Lung and Dumitrescu, 2008] (an offspring replaces its parent only if it is better than it in *Nash* sense).

We address the equilibrium detection of the *Nash* equilibrium in a dynamic game using Differential Evolution [Storn and Price, 1997] algorithm because it is computationally effective. This simple and efficient algorithm requires less payoff function evaluations than a standard Evolutionary Algorithm would. Using the generative relation to guide the search towards the *Nash* equilibrium of a game is a computationally expensive step, due to the selection for recombination operators a standard Ea would require a significantly larger number of payoff functions evaluations. We call this method *Dynamic Equilibrium Tracking Differential Evolution Algorithm* (DET-DE).

A sentinel is used to detect a change in the game. The sentinel is randomly generated at the beginning of the search and it does not change during the evolutionary process. In each generation its fitness is evaluated, if this value differ from the last generation then a change in the environment has occurred. Based on the new and old values the algorithm estimates the amplitude of the change and adapts the search accordingly.

The general outline of the *DET-DE* algorithm is presented in Algorithm 4.

Algorithm 4: Dynamic Equilibrium Tracking Differential Evolution Algorithm (*DET-DE*)

```
Randomly generate initial population of game situations;
repeat
    Create offspring by mutation and recombination (DE/rand/1/bin);
    Evaluate offspring (compute payoff functions for all players);
    if the offspring is better (in Nash sense) than parent then
        Replace the parent by offspring in the next generation;
    if change detected then
        Apply uniform mutation with the adapted value of p_m and step N(0, σ) according to
        Steps 1-3;
        Increase F and decrease C_r;
    Evolve using adapted DE (using Step 4. for varying F and C_r);
until termination condition is met ;
```

6.1.1 Symmetric Cournot Model

A discrete-time dynamic Cournot oligopoly model can be easily designed by inducing a change in the parameter controlling the payoffs, that is b. In this case the payoff parameter is the same for all players. Thus we can consider adding a normal perturbation to b at each epoch $j+1$, according to equation (6.1). Here $\Phi(0,\sigma)$ represents a random number generated from a normal distribution with mean 0 and variance σ. Thus σ controls the amplitude of the change from period j to period $j+1$.

$$b_{j+1} = b_j + \Phi(0,\gamma), \tag{6.1}$$

Numerical experiments are conducted with 2, 5, 10 players. Each algorithm has a population of $N = 100$ individuals and it is run for $G = 10000$ generations. After 200 generations the value b changes (according to (6.1)), 200 generations represent an *epoch/period* for the game. We run 10 independent simulations for $\sigma = \{0.5, 1, 5\}$, $p_{min} = 0.01$, $p_{max} = 0.07$. The seeds for the random number generator are different for each run. Basic DE parameters are: $C_r = 0.8$, and $F = 0.2$ (at each change F is set to 0.5 and is decremented each generation with 0.02 to 0.2). For DET-DE the probability of mutation, when a change is detected, is directly proportional with the amplitude of the change.

The mean and standard deviation of the payoffs for the best solution found in each epoch are computed. The offline error is computed as the average of the minimum distance to the current NE for each generation. A Wilcoxon signed rank test has been performed to determine if the difference between the average values of the offline errors between DET-DE and AE-DE is statistical significant (with a p value lower than 0.05 for all the cases).

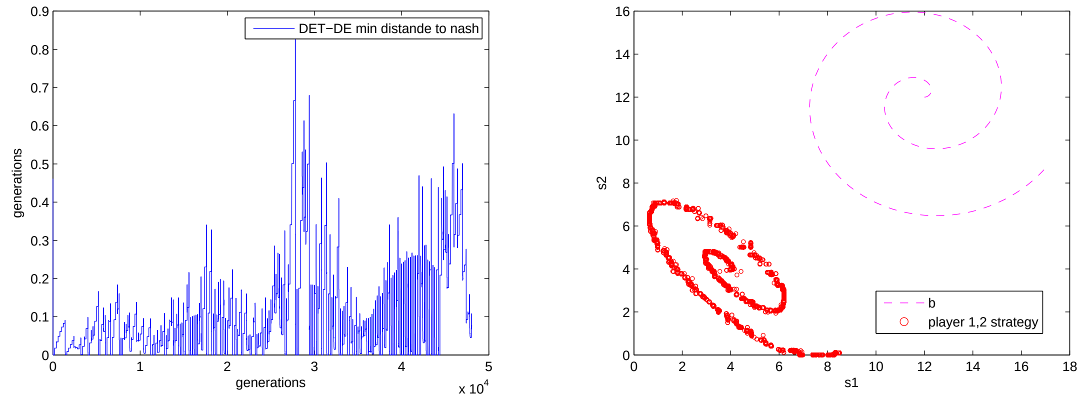
6.1.2 Symmetric Cournot Game - Predefined Dynamics

Until now the dynamic nature of the Cournot game consisted in the random variation of the payoff parameter b. But what if the variation of b is predefined. Would *DET-DE* be able to find a good approximation of the *Nash* equilibrium?

Spiral dynamics For a Cournot game with 2 players the dynamics of parameter b is described by the following equations:

$$\begin{cases} b_1(t) = x\cos(x) + 10, \\ b_2(t) = x\sin(x) + 10, \end{cases} \tag{6.2}$$

where $b_i(t)$ represents the value of the payoff parameter at generation t for player i. The game is asymmetric because the payoff parameter is different for each player.



(a) Mean distance to NE. (b) Strategies in each epoch.

Figure 6.1: Two player spiral dynamic Cournot game: mean distance to *Nash* equilibrium and b values in each round (Figure 6.1a), strategies for each player and b values in each round (Figure 6.1b).

It can be observed *DET-DE* algorithm is able to find the *Nash* Equilibrium of the game in each epoch, the average Euclidean distance to the NE is in average less than 0.4 in case of a 2 player game, Figure 6.1a.

6.2 Dynamic Generalized Equilibrium Detection for Berge-Zhukovskii Equilibrium

While, as central concept in Game Theory, the *Nash* equilibrium [Nash, 1951] has been widely studied and numerous algorithms for its computation or approximation have been proposed. Other types of equilibria did not benefit from the same kind of attention, one of the reasons being their intractable character. A computational approach that is efficient not only in computing but also in tracking the set of Berge-Zhukovskii equilibria in a dynamic setting is presented in this Section.

A generalized non-cooperative game the strategy space changes in the following way: for all player $i \in N$ the **common strategy set** $\mathcal{S}$ is formed by the decisions $s = (s_1, ..., s_n)$ where $s_i \in K_i(s)$, $K_i : S \to \mathcal{P}(S_i)$.

A strategy s^* is a Berge-Zhukovskii (BZ) equilibrium, if the payoff of each player i does not decrease considering any deviation of the other $N - \{i\}$ players.

The main importance of the BZ equilibrium can be observed in social dilemmas (prisoner's dilemma, centipede game) where the cooperation of the players is essential, as its definition leads, in some games, to a form of apparent cooperation. Formally we have the following definition:

Definition 6 *A strategy profile $s^* \in S$ is a Berge-Zhukovskii equilibrium if the inequality $u_i(s^*) \geq u_i(s_i^*, s_{N-i})$ holds for each player $i = 1, ..., n$, and $s_{N-i} \in S_{N-i}$.*

6.2.1 Dynamic game

The dynamic game considered is a symmetric constrained Cournot oligopoly. We consider the following constraints related to the quantity of goods to be produced (players strategies are inter-connected):

$$\sum_{i=1}^{n} \alpha_i \cdot s_i > u, \alpha_i \in [0, 1], s_i \in [l_1, l_2], i = 1, ..., n. \quad (6.3)$$

If coefficients α_i, $i = 1, ..., n$ vary in time, a dynamic generalized Cournot oligopoly model is obtained. A simple model that can be used is $\alpha_{i+1} \in U(0, 1)$ where $U(0, 1)$ represents a random uniform number between 0 and 1. Any other variation can be considered.

We consider 20 independent runs for each algorithm with each run consisting of 50 epochs. A population of $N = 150$ individuals is used. For the algorithms initial parameter values are: $C_r = 0.8$ and $F = 0.2$. After 200 generations (1. epoch) the constraints of the game change by generating new α_i values following a uniform distribution. *DET-DE* is able to find a better approximation of the *BZ* equilibrium than *AE-DE*.

Chapter 7

Repeated Spatial Games

Real-life interactions between players who belong to the same group are better modeled by repeated games. Through repeated interactions one can learn from his opponents, the actions of a player may influence the actions of the group and conversely. For example if the game is played in a population context and all players play strategy s^*, the best action is to "follow the crowd". The population is in equilibrium if all players use strategy s^*. One may pose a question: what happens in a heterogeneous population? If players have different strategy preferences the population converges to Nash equilibrium or it moves away? *Evolutionary Game Theory* (EGT) [Fisher, 1930; Gintis, 2009] tries to answer these questions.

In an evolutionary framework players are grouped together. The structures usually used to describe interactions between players are: lattices, small world networks, scale-free graphs, evolving networks, and random graphs. Real life interactions are better modeled thorough repeated games. Prisoner's Dilemma game is a simple interaction model, it shows the difference between common interest (cooperation), and private interest (defection). We study the dynamics that appear in repeated spatial game and propose a new model based on multidimensional games.

7.1 Evolutionary Dynamics in a Repeated Cournot Oligopoly

We study whether agents through repeated interactions converge to Nash equilibrium. This is very similar to the Nash implementation problem [Moore and Repullo, 1990] where through some rules the interacting decision makers are guided towards the desired equilibrium concept. In our case the spatial structure and the simple update rules are able to guide players to a fairly good approximation of the Nash equilibrium.

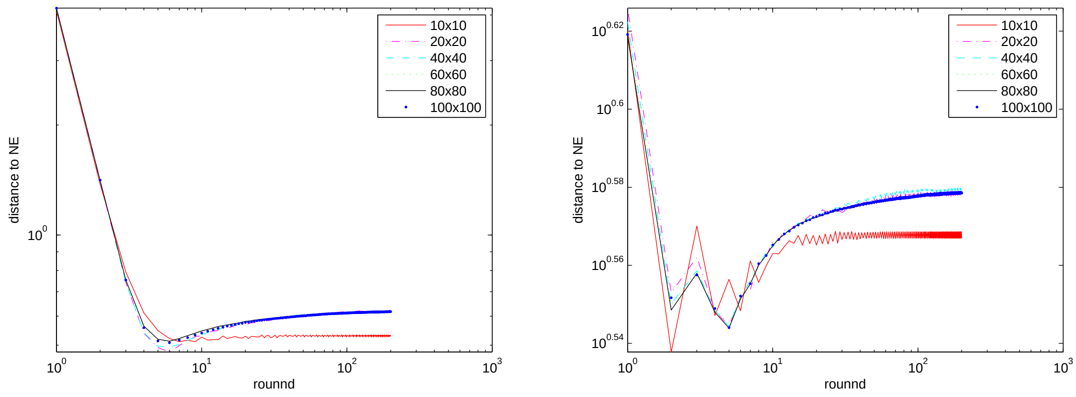
Interactions between economic agents in our spatial model are modeled by the Cournot game. The Cournot game is used to model the outcome of competition among different companies, outcome that depends on the number of firms, characteristic of demand and cost function. We consider a $n = 2$ and $n = 9$ player version of the game. The Nash equilibrium of this game can be easily computed: $NE = \frac{a-c}{n+1}$.

A simple lattice is used to describe player interactions. Each player is represented by a point in the lattice and has *8* neighbors with who will play Counrot game for t rounds.

The game can be played either with all neighbors at the same time or with each one individually. After each round players change their strategy (q_i - number of units produced) by imitating the neighbor with the highest payoff. This update rule is similar to the successful *Tit for Tat* strategy.

Figure 7.1 presents the average distance to the NE in each round. The average distance over 20 independent games is computed for all individuals in the population (lattice). For the duopoly version $NE = 5$ and for the 9 player version $NE = 1.6$. As we can observe in the first $10 - 20$ rounds there is a phase in which the population converges near the NE and after that is relatively stable. The average distance is smaller for population of 100 individuals (10×10 lattice) and increases with the population size. Also for the duopoly version of the game (Figure 7.1a) this average distance is considerably smaller than the 9 player version (Figure 7.1b).

Players converge to a strategy close to the Nash equilibrium of the game. The size of the population does not affect the speed of convergence but the type of game played does. For the



(a) Cournot duopoly. (b) $n = 9$ player Cournot game.

Figure 7.1: Evolution of average distance to Nash equilibrium in each round, the game is played for 200 rounds (average results over 20 independent games for different population sizes).

duopoly case the average distance to Nash equilibrium is smaller than the case where the game is played simultaneously.

7.2 Evolutionary Dynamic for Inter-Group Cooperation

An important question in Evolutionary Game Theory [Fisher, 1930] is the emergence of cooperation. Darwinian evolutionary theory favor defectors, selfish individuals. Despite this, cooperative individuals can benefit from cooperation.

Emergence of cooperation has been studied only for interactions in groups [Szabó and Fáth, 2007]. Our aim is to study the interactions between groups. In order to analyze emergence of cooperation between heterogeneous groups we concentrate on interactions between groups, ignoring intra-group interactions. A hypergraph interaction model is proposed to describe interactions between players and groups of players. When playing n-person Prisoner's Dilemma game an interaction model based on a hypergraph is natural. Interaction between different groups can be described in this way. The proposed hypergraph model ensures a higher flexibility in the sense of the interactions between players (the players can interact in each round with other random opponents).

7.2.1 Proposed Model for Inter-Group Cooperation

For model representation a hypergraph [Berge, 1985] structure is used. A hypergraph is a generalized notion of a graph: in a hypergraph an edge can connect any number of vertices.

Definition 7 *Let $X = \{x_1, x_2, .., x_n\}$ be a finite set, and $E = (E_i, i \in I)$ be a family of subsets of X, satisfying the conditions:*

- $E_i \neq \phi, i \in I$;
- $\cup_{i \in I} E_i = X$.

The system $H = (X, E)$ is a hypergraph, where X is the set of vertices, and E is the set of hyperedges [Berge, 1985].

Interacting groups are represented by a (unique) hypergraph. Vertices's of the hypergraph represent interacting players, a hyperedge represents a group of players. Each player has two possible actions: *cooperate* or *defect*.

PD game is played for a predefined number of rounds. Players strategies (C or D) are randomly initialized. We want to explore the emergence of cooperation between groups, for this reason we investigate only the case when the game is played between hyperedges/groups and not between players from the same hyperedge/group.

In a round each player plays PD against his opponents (selected according to a specific rule). The rule does not change in a game. After each round an update step is performed. Players keep their strategy if they have the highest total payoff with respect to their opponents. A losing player adopts the strategy of the most successful wining opponent (the player with the highest total payoff).

Interaction inside groups is extensively studied in the literature. Inter-group cooperation seldom happens, and when it does it is much harder to maintain. Our aim is to study the interactions between different groups. We propose an interaction model based on the concept of hypergraph where different groups are represented by hyperedges. The hypergraph model can be useful in describing real-life interactions, and social networks. The hypergraph representation is a natural one for describing intergroup interactions. A hypergraph model allows groups to overlap (as a player may belong to several hyperedges). In a more elaborated model each player can have a random number of opponents from several groups.

We consider the interactions based on four simple rules for choosing opponents in a Prisoner's Dilemma game. Numerical experiments indicate that when the initial number of defectors is small (one or two defectors) cooperation between groups is not affected. Cooperation is independent of the number of groups and parameter b. When the number of defectors is greater than 2, noncooperation between groups can be observed. The explanation of these results is resides in dominance of *Defect* strategy. If a defector plays PD game with some cooperators from other groups the cooperators will adopt the defectors strategy. As this is true for all rounds, the number of defectors increases exponentially. After a few rounds the population is composed only of defectors.

7.3 Repeated Multidimensional Games: the Importance of Identity in Emergence of Cooperation

Standard Game Theory makes some simplifying assumptions: players are rational, know that their opponents are rational, and have common knowledge about the game played [Osborne, 2004; Gintis, 2009]. So a players only goal is to maximize its own gain. We believe that this is only partially true. Real players, besides maximizing their payoff, take into account other criteria such as: reputation, morality, etc. These criteria capture the identity of a player and weigh heavily in their decision.

Standard game theoretic models allow interactions between only players that have the same rationality type. This is an unrealistic restriction, in real life situations players within a game rarely think and act in the same manner. Thus these game models are overly simplified and do not model real life decision making adequately.

Our goal is to develop a model that overcomes these limitations. For these purpose we study the Prisoner's Dilemma game within the context of multidimensional games (games with vector payoffs). The standard model is extended by adding a second criterion that models the identity of a player. In this manner a psychological factor can be considered in the decision making process, besides actual payoffs. Real life players are diverse, thus our goal is to allow interactions between players with different identities.

We aim at achieving cooperation without any external interactions. Our goal is to create a more realistic interaction model between real life agents. By introducing the second criterion in the Prisoner's Dilemma game we want to better model the irrational nature of real life players.

7.3.1 Multicriteria Prisoner's Dilemma

A more realistic model can be obtained if in the decision making process other criteria, than payoff associated to played strategy, are taken into account. These other criteria could be morality, altruism individuality, conscience, etc. Because these criteria are not correlated they cannot be aggregated with the payoff. Thus we obtain a situation that resembles a multiobjective optimization problem. But as we know [Farina and Amato, 2002; Hughes, 2005] a multiobjective problem based

Table 7.1: Multicriteria Prisoner's Dilemma

		Player2	
		C	D
Player1	C	$[(R, a_1); (R, a_2)]$	$[(S, c_1); (T, c_2)]$
	D	$[(T, b_1); (S, b_2)]$	$[(P, d_1); (P, d_2)]$

on Pareto dominance works well only for $2 - 4$ objectives. For simplicity we model the above mention criteria as a second objective in the payoff vector u_i. This additional objective does not take into consideration the actual payoff, i.e. a players identity can be modeled this way. The fist objective represents the obtained payoff and the second one models players identity.

Various real life situations [Ostrom, 1990] can be modeled with two player dilemma games [Szabó and Fáth, 2007]. Learning is incorporated in these models through the repetitive nature of the game. *Iterated Prisoner's Dilemma* (IPD) is the preferred model [Nowak and Robert, 1992; Nowak and May, 1993; Nowak et al., 2010; Szabó et al., 2000; Szabó and Fáth, 2007]. Various studies concern themselves with the emergence of cooperation. The outcome is all players adopt strategy *defect* for the standard IPD game.

We study the emergence of cooperation in a multidimensional iterated spatial Prisoner's Dilemma game.

Real life agents do not make their decision based only on actual payoffs. Besides considering the concrete payoffs they also take in consideration other criteria as solidarity, conscience, morality, etc. Beyond actual payoffs identity determines their choices. For example an agent with a cooperator identity is less likely to defect even though rational thinking implies defection regardless of the strategy of the opponent.

One question regarding real agents is whether they are willing to act in a way that is favorable for the community or they act only to maximize their own gain. All players benefit by mutual cooperation, but at the same time temptation to defect against a cooperator is high. However fair play suggests that the best strategy would be to cooperate. With the help of the second criterion this fair decision making can be modeled. In case of a *cooperator* (a player that plays strategy *Cooperate*) the identity-payoff for cooperating is considerably larger than for defecting (a player that plays strategy *Defect*).

Multicriteria payoffs are depicted in Table 7.1. The payoffs for the first criterion (R, S, T, and D) are the actual gains and are equal to the payoffs of the standard Prisoner's Dilemma. The payoffs for the second criterion (a_1, a_2, b_1, b_2, c_1, c_2 and d_1, d_2) correspond to the identity payoffs. With the help of this second criterion a human player can be modeled more accurately; these values capture the fairness or morality of a player. A possible relation between the values of the identity payoff is the following:

$$\begin{aligned} a_1 > c_1 > d_1 > b_1, \\ a_2 > b_2 > d_2 > c_2. \end{aligned} \tag{7.1}$$

While actual payoffs suggest defection, the identity payoffs are higher for cooperation. So even if cooperation does not necessarily imply high actual payoffs, it offers a moral gain. A player gets the highest payoff when defecting against a cooperator, however the identity payoff in this situation is obviously very low.

What is the equilibrium of the game? The Nash equilibrium of the standard PD game is mutual defection. For the multidimensional version this question is not trivial. Considering only the first criterion (the actual payoffs), the most successful strategy is defection. Adding an identity payoff that favors cooperation for both players leads to no multidimensional ideal Nash or Pareto-Nash equilibrium. Each solution from the population is dominated by a single solution that is dominated by a single solution and so on. The multidimensional Prisoner's Dilemma with identity payoff has no equilibrium.

7.3.2 Multicriteria Spatial Model

Emergence of cooperation in iterated spatial Prisoner's Dilemma game has been widely studied [Nowak and Robert, 1992; Nowak and May, 1993; Nowak et al., 1994, 2010; Axelrod, 2006; Szabó and Fáth, 2007].If players interactions are modeled by a lattice structure then each player is an element of the lattice. Usually players initial strategy is randomly chosen. The game is played with all neighbors for t rounds. One possible update strategy is to imitate the neighbor with the highest payoff at the end of each round. The outcome of such a game is a population in which all players play strategy *defect*.

For our experiments we consider a $n \times n$ lattice, the number of players is n^2. In each round players play the multidimensional PD game with all neighbors. At the end of each round players update their strategy in the following manner:

a. The payoff for each player is computed:

$$(u_1, u_2)_i = (\sum_{j=1}^{8} u_{j,1}, \sum_{j=1}^{8} u_{j,2}),$$

where u_1 represents the actual payoff and u_2 is the moral gain.

b. Player i compares its payoff with its neighbors payoffs, Pareto dominance is used to differentiate between payoffs. Player i will adopt the dominant neighbor strategy if its payoff is dominated in Pareto sense.

c. In case of indifference, neither payoff dominates each other, or there are more than one neighbor with the same better payoff then one of their strategy is chosen randomly.

We want to be able to model different identity types, for this purpose the second criterion is weighted, thus the payoff for each player will be:

$$(u_1, u_2)_i = (\sum_{j=1}^{8} u_{j,1}, \sum_{j=1}^{8} \lambda \cdot u_{j,2}),$$

where $\lambda \in [0, 1]$.

In contrary to the standard single criterion PD game, the identity payoff favors cooperation. Using a model based on multidimensional games cooperation is reached solely based on the identity criterion. When all players have the same identity cooperation emerges disregarding the payoffs values (as long as the inequalities (7.1) are fulfilled). If the game is played by a heterogeneous population, i.e. between players with different identities, according to our experiments players will chose the strategy that best reflects their identity.

Chapter 8

Conclusions and Future Work

Almost all real life optimization problems have multiple conflicting objectives that need to be simultaneously optimized. Evolutionary multi-objective optimization algorithms represent a good choice for solving such problems, they are able to find a good approximation of the Pareto front in a single run whereas the shape, continuity or other mathematical properties of the true front do not hinder the search. Pareto dominance based algorithms are the preferred choice for solving such problems, but they have a main disadvantage that they work for problems with only $2-4$ objectives.

Game Theory models have some limitations when applied in real life. Assumptions made about players are sometimes too harsh - all players are rational and have common knowledge about the game. Real life agents do not make their decision based only on actual payoffs. Besides considering the concrete payoffs they could also take into consideration other criteria such as: solidarity, conscience, morality, etc. The main solution concept in Game Theory, Nash equilibrium, does not always yield the best outcome, i.e. Prisoner's Dilemma game. Other equilibrium concepts have been proposed and investigated.

8.1 Summary of Results

In Chapter 3 we propose a new multi-population optimization algorithm based on decomposition, called *Resonance Search*, for multiobjective and many-objective optimization problems. Decomposition approaches represent a good choice for solving many-objective optimization problems as it is much easier to solve a single objective problem. By solving an optimization problem in this way the exponential increase of non-dominated solutions can be avoided - by decomposing the problem only preferred regions are explored.

Resonance Search is a flexible scheme that uses several decomposition techniques involving a number of $\mu-$populations, each population evolves in isolation for a number of generations and performs an independent search over a specific region of the fitness landscape, populations periodically exchange individuals in order to promote diversity.

We also explore *augmented Tchebycheff*, *modified Tchebycheff*, and L_p techniques as alternatives to *Weighted Sum* and *Weighted Tchebycheff* decomposition techniques [Suciu et al., 2013a].

Lorenz dominance is also studied as an alternative to Pareto dominance, using the Lorenz concept of dominance to guide the search a preference based optimization is realized [Nagy et al., 2012a,b].

A multiobjective problem that arises in the field of Service Oriented Computing is the composition of web services based on their *Quality of Service* parameters. Most existing solutions solve this problem using a single objective approach. As a consequence, in Chapter 4 we explore the applications of multiobjective optimization algorithms for this problem [Pop et al., 2011; Suciu et al., 2012]. An adaptive evolutionary optimization technique is proposed [Suciu et al., 2013c], better performance is obtained (with respect to multiobjective quality indicators) when the adaptive approach is applied to standard test problems and some business workflows of high complexity.

Chapter 5 explores the game equilibrium detection problem using an evolutionary approach. The concept of ϵ-*Berge-Zhukovskii* equilibrium is proposed. It represents a flexible concept that approximates the *BZ* equilibrium - a powerful equilibrium concept especially in trust games. Evolutionary detection of ϵ-*Berge-Zhukovskii* equilibrium is explored. Based on the idea that equilibrium search and multi-objective optimization can be considered as the same class of problems, an evolutionary algorithm for multiobjective optimization is adapted for ϵ-*Berge-Zhukovskii* detection. Numerical experiments validate the proposed method.

Nash equilibrium is not always the best solution in all non-cooperative games. In games having several Nash equilibria a selection problem may appear, which Nash equilibrium is best from the detected equilibria. Pareto-optimal Nash equilibrium is a refinement of the Nash equilibrium. An evolutionary method based on generative relations is considered in order to detect this equilibrium [Gaskó et al., 2012]. We also explore the evolutionary equilibria detection in games where some constraints are imposed on player strategies.

An optimization problem/game may not be static - its main characteristics could change over time. Chapter 6 investigates the equilibrium detection problem in dynamic games using an evolutionary approach [Suciu et al., 2013b, 2014; Gaskó et al., 2013]. The dynamic nature of the game raises a few difficulties: detecting changes in the environment and appropriately reacting to these changes. We propose a simple method *Dynamic Equilibrium Tracking (DET-DE)* able to find a game equilibrium in a dynamic environment. In order to deal with changes in the environment *DET-DE* uses a simple mechanism to estimate the amplitude of a change and adapt the parameters of an uniform mutation operator - used to induce the necessary amount of diversity in the population.

DET-DE is evaluated by means of numerical experiments on a simple discrete-time dynamic form of the Cournot oligopoly. Symmetric, asymmetric, constrained and unconstrained versions of the Cournot game are explored. Results are compared with a DE variant that does not use the adaptive features of the *DET-DE*, called *AE-DE*. A Particle Swarm algorithm, adapted to compute *Nash* equilibria within a dynamic fitness landscape, is also used for comparison in case of the constrained version. In most cases our approach is able to find a better approximation than compared algorithms. Also in case of predefined game dynamics, *DET-DE* is able to track the *Nash* equilibrium.

A *generalized Berge-Zhukovskii equilibrium problem* is introduced and studied in a dynamic environment. Using a generative relation for the *BZ* equilibrium the proposed method is able to find the *Generalized BZ equilibrium* of the game in a dynamic environment. The performance of the proposed method is evaluated by computing the Berge-Zhukovskii Generational Distance and Inverted Generational Distance. Numerical experiments show that *DET-DE* is able to find a better approximation of the *GBZE* equilibrium than *AE-DE*.

Through repeated interactions players can learn from their opponents, a player's actions influence the actions of the group. Local interactions within the group can maintain strategies different than the Nash one indefinitely. Chapter 7 explores the evolutionary dynamics that arises in spatial iterated games. An economic game is used to model the interactions between players in a spatial framework. We study whether agents, through repeated interactions, converge to Nash equilibrium, and, as it turns out, players converge to a strategy close to the Nash equilibrium of the game. Population size does not affect the convergence speed but the type of game does. For the duopoly case the average distance to Nash equilibrium is smaller than the case where the game is played simultaneously.

A *hypergraph interaction model* is proposed to describe interactions between players and groups of players. When playing a n-person Prisoner's Dilemma game an interaction model based on a hypergraph seems more natural. The interaction between different groups can be described in this way. The proposed hypergraph model ensures a higher flexibility in the sense of the interactions between players (the players can interact in each round with other random opponents). Numerical experiments indicate that, when the initial number of defectors is small (one or two defectors), cooperation between groups is not affected. Cooperation is independent of the number of groups and parameter b from Prisoner's Dilemma game. When the number of defectors is greater than two, noncooperation between groups can be observed.

Standard single criterion games take into consideration only the actual payoffs of the game.

However, in human decision making there are many other factors that play an important role. Human players, besides actual payoffs, take in consideration other criteria such as morality, cooperation, altruism. These factors are hard to quantify and cannot be added to the actual payoffs. A new model including player identity in iterated spatial games is proposed. By introducing the second criterion in the Prisoner's Dilemma game we want to better model the irrational nature of real life players. Numerical experiments indicate that players play according to their identity.

8.2 Future Work

Future work will consider extending the proposed Resonance Search model to include other scalarization techniques and explore different exchange topologies between populations. Other problems to e addressed: including user preference in the form of weight vector values, different preference point for scalarization techniques used, and reduction of redundant objectives in many-objective optimization problems. The performance of a scalarization technique is greatly influenced by its parameters: reference point, ρ and p for *augmented, modified Tchebychef* and L_p, respectively. We will consider the adaptation of these parameters.

Another research direction will consider intelligent exploration of the fitness landscape. For some hard multiobjective optimization problems with strong local optima. Such approach could greatly increase the efficiency of optimization algorithms.

Future work will address equilibrium detection for other dynamic games than the already tested ones. Also, more efficient ways in adapting to game changes and tracking game equilibria will be investigated.

Multicriteria iterated games offer a more realistic model. In an iterated multicriteria spatial game the emergence of cooperation is highly correlated with player identities. Future work will investigate how different spatial topologies influence the emergence and stability of cooperation and if different punishing rules reinforce cooperation regardless of player identity or other criteria.

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