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DOCTORAL THESIS ABSTRACT

MATHEMATICAL DIFFICULTIES BEYOND DEFICIT: THE ORGANIZATION OF NUMERICAL COMPETENCIES, THE INTEGRATION OF COGNITIVE MECHANISMS, AND DEVELOPMENTAL TRAJECTORIES

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Introduction

The development of mathematical competencies in the early years of schooling represents a major determinant of later academic trajectories, being associated not only with performance in mathematics but also with achievement in other domains of learning. Longitudinal studies show that the level of numerical competencies at school entry robustly predicts later mathematical achievement (Peng et al., 2016) and overall academic performance (ten Braak et al., 2022), including in areas such as reading and science (Davis-Kean et al., 2022).

This predictive value reflects the role of early numerical competencies as a cognitive infrastructure for mathematical learning. Skills such as counting, understanding quantitative relations, and operating with numerical symbols support both the acquisition of later mathematical concepts and the efficiency of problem solving (Owen & Reed, 2025). In the absence of an adequate level of these competencies, the integration of new knowledge becomes more difficult, contributing to the accumulation of performance gaps.

The literature further suggests that initial differences tend to stabilize or amplify over time, reflecting cumulative developmental processes. This pattern is commonly described in terms of the Matthew effect, whereby early advantages and disadvantages shape subsequent learning trajectories (ten Braak et al., 2022), contributing to the persistence of performance differences throughout schooling (Owen & Reed, 2025). In the long term, these trajectories extend beyond the school context, being associated with educational attainment and later occupational opportunities (Davis-Kean et al., 2022).

Within this framework, the early identification of vulnerabilities and the understanding of the mechanisms that support or constrain mathematical development become central objectives for both research and educational practice. From a developmental perspective, mathematical difficulties can be understood not as simple deviations from a normative level of performance, but as expressions of trajectories characterized by early vulnerabilities and cumulative processes of differentiation. This perspective requires moving beyond a strictly categorical approach and adopting a relational and dynamic framework capable of capturing how numerical competencies and cognitive mechanisms interact and reorganize over time.

Mathematical difficulties (MD) represent a complex and heterogeneous phenomenon, characterized by a wide range of manifestations, from transient developmental delays to persistent forms associated with neurocognitive vulnerabilities (Dowker, 2024; Muñez et al., 2023). Prevalence estimates indicate that between 3% and 9% of children experience significant difficulties in mathematics (Kucian & von Aster, 2015; Wang et al., 2024), although the proportion of those at risk is considerably higher, reflecting the dimensional nature of mathematical performance. In line with contemporary perspectives, mathematical difficulties are no longer conceptualized exclusively as discrete diagnostic entities, but rather as positions along a continuum of performance, shaped by dynamic interactions among cognitive, educational, and developmental factors (Kroesbergen et al., 2023; Fletcher & Miciak, 2024).

1. THEORETICAL AND CONCEPTUAL FRAMEWORK

1.1. Cognitive Processes and Early Mathematical Development

Domain-general cognitive processes play an essential role in early mathematical development, being conceptualized not as direct determinants of performance, but as functional infrastructures that both support and constrain learning (Astle & Fletcher-Watson, 2020). Among these, working memory, executive functions, attentional control, and processing speed have been consistently identified as being associated with mathematical performance, particularly in the early stages of development (Peng et al., 2016; Cirino et al., 2015).

Working memory is involved in the maintenance and manipulation of information required for solving mathematical tasks (Baddeley, 2000), while *executive functions*—such as inhibition, cognitive flexibility, and monitoring—contribute to the selection and coordination of efficient strategies (Miyake et al., 2000). *Attentional control* facilitates the focusing of attention on relevant information and the filtering of distractions, whereas *processing speed* influences the efficiency with which basic cognitive operations are executed. Together, these processes support the integration and flexible use of numerical competencies across diverse learning contexts.

However, recent research indicates that the relationship between cognitive processes and mathematical performance is neither univocal nor stable. Although differences between children with mathematical difficulties and those with typical performance are frequently observed at the group level, these differences do not converge into a unitary cognitive profile and do not allow for consistent prediction at the individual level (Nelson & Powell, 2018; Cirino et al., 2015). This dissociation suggests that cognitive processes do not function as singular explanatory factors, but rather as functional resources whose relevance depends on context, task demands, and developmental stage (Astle et al., 2020).

In this sense, the role of cognitive processes can be understood as dynamic and contingent on the level of automatization of mathematical competencies. In the early stages, mathematical tasks place substantial demands on working memory and attentional control, as strategies are unstable and require active coordination. As competencies become more automatized, reliance on these processes decreases, and performance is increasingly supported by consolidated knowledge.

Accordingly, early mathematical development cannot be explained by a single cognitive deficit, but instead reflects complex functional configurations emerging from the interaction between cognitive processes and the contextual demands of learning. This perspective underscores the need for integrative approaches capable of capturing the variability and dynamic nature of the relationships between cognition and mathematical competencies.

1.2. Early Numerical Competencies as a Structural System

Early numerical competencies constitute the foundation of mathematical development and comprise a set of interrelated abilities that support the understanding and use of numerical concepts across diverse learning contexts. Traditionally, these competencies are organized into two broad categories: informal competencies—such as counting, quantity comparison, and the understanding of basic numerical relations—and formal competencies, which involve operating with numerical symbols and applying arithmetic procedures (Ginsburg & Baroody, 2003). Beyond this classification, contemporary literature emphasizes that mathematical development does not reflect the linear accumulation of isolated skills, but rather the progressive organization of a system of interdependent competencies. The relationships among these competencies—whether conceptual, procedural, or automatized—play a central role in supporting mathematical performance and facilitating transfer across different types of tasks (Rittle-Johnson et al., 2015). Accordingly, mathematical performance can be understood not only as a function of individual competencies, but as an outcome of how these are functionally integrated within a coherent structure.

This structural perspective implies that different dimensions of numerical competencies—such as symbolic and non-symbolic processing, conceptual and procedural knowledge, and levels of automatization—do not operate independently, but are organized within a network of dynamic relationships whose configuration evolves over the course of development. In this sense, mathematical development can be conceptualized as a process of structural reorganization, in which the role and relative importance of different competencies change as a function of learning experiences and educational demands.

However, most research has examined the structure of mathematical competencies through models centered on latent variables, which capture shared dimensions of performance but provide only a limited account of the direct relationships among competencies. Such approaches do not allow for the identification of how competencies influence one another, nor of their specific roles within the system. Consequently, there is a need to adopt analytical methods capable of capturing the relational organization of numerical competencies, thereby offering a complementary perspective on the structure of mathematical development.

Within this framework, conceptualizing numerical competencies as a structural system provides an appropriate basis for investigating mathematical difficulties not merely as low levels of performance, but as specific patterns of organization and functional integration of abilities. This perspective enables the examination of mathematical vulnerability in terms of the structure of relationships among competencies and opens the possibility of employing innovative analytical approaches, such as network analysis, to capture the dynamics of mathematical development.

1.3. Explanatory Models of Mathematical Difficulties

In recent decades, research on mathematical difficulties has been shaped by the development of distinct explanatory models aimed at capturing the nature and origins of these difficulties. Domain-specific models posit the existence of primary deficits in the numerical system, involved in the representation and processing of quantities, suggesting that mathematical difficulties—particularly in their severe forms—originate in dysfunctions of fundamental numerical mechanisms (Decarli et al., 2023; Dolfi et al., 2024).

In contrast, domain-general models emphasize the role of cognitive processes such as working memory, executive functions, and attentional control, conceptualized as factors that constrain the efficiency of mathematical learning. From this perspective, mathematical difficulties are interpreted as indirect effects of broader cognitive constraints that influence the acquisition and use of mathematical knowledge (Haberstroh & Schulte-Körne, 2022; Peng et al., 2016, 2018).

In response to the limitations of these unidimensional perspectives, integrative models—particularly the multiple deficit model (Pennington, 2006)—propose that mathematical difficulties arise from the convergence of multiple vulnerabilities, including both domain-specific deficits and dysfunctions in general cognitive processes. This approach accounts for the heterogeneity of observed profiles and suggests that similar performance trajectories may be underpinned by different combinations of risk factors.

Developmental approaches extend these frameworks by highlighting the dynamic and bidirectional nature of the relationships between mathematical competencies and cognitive processes. Mutualistic models propose that these components influence each other over the course of development, whereas cascade models suggest that initial differences may lead to diverging trajectories through cumulative, context-dependent effects (Kievit et al., 2019; Clark et al., 2013; Hu et al., 2023). These perspectives are consistent with the notion of the Matthew effect, according to which early advantages or disadvantages tend to amplify over time, contributing to the stabilization of performance differences.

Overall, contemporary explanatory models converge on the view that mathematical difficulties cannot be attributed to a single, isolated cognitive deficit. Rather, they reflect complex interactions between domain-specific and domain-general processes unfolding within a dynamic developmental framework, in which learning trajectories are shaped by both cognitive and contextual factors.

1.4. Limitations of the Literature and Research Gaps

Despite substantial theoretical and empirical advances in the understanding of mathematical difficulties, the literature continues to highlight a number of significant limitations that constrain the ability to coherently explain variability in mathematical performance and the associated developmental trajectories.

First, the relationships between cognitive processes and mathematical performance remain inconsistent. Although numerous studies report significant differences between children with mathematical difficulties and those with typical performance, the magnitude of these effects varies considerably, and the specific role of different cognitive components is not clearly delineated (Cirino et al., 2015; Nelson & Powell, 2018). Moreover, the cognitive processes that differentiate groups do not necessarily exhibit predictive value at the individual level, suggesting a dissociation between explanations at the group level and those relevant for individual variability.

Second, the structure of mathematical competencies is frequently investigated through models centered on latent variables, which capture shared dimensions of performance but provide only a limited account of the direct relationships among competencies. Such approaches do not allow for the identification of how mathematical abilities influence one another, nor of their specific roles within the system, thereby constraining the understanding of the functional organization of numerical competencies.

Third, longitudinal research on the stability and dynamics of mathematical difficulties remains relatively limited, particularly in the early years of schooling. Although there is evidence for the relative stability of mathematical performance, the mechanisms through which initial differences are maintained or modified over time remain insufficiently examined, especially with regard to the interaction between cognitive processes and the structure of numerical competencies.

In addition, within the Romanian educational context, these limitations are further compounded by the limited availability of psychometrically validated standardized instruments for the assessment of early mathematical competencies, which constrains both the comparability of findings and the possibility of conducting rigorous analyses of mathematical development.

Overall, these gaps point to the need for an approach that moves beyond the isolated analysis of cognitive variables or performance levels and instead integrates cognitive, structural, and developmental perspectives within a coherent analytical framework. Such an approach is essential for understanding mathematical difficulties as emergent processes, shaped by multiple interactions and by the dynamic nature of development over time.

1.5. The Need for an Integrative Approach

The limitations identified in the literature highlight the need for an integrative approach capable of simultaneously capturing the cognitive, structural, and developmental dimensions of mathematical difficulties. Traditional approaches, focused either on isolated cognitive processes or on global levels of performance, fail to adequately account for interindividual variability and the heterogeneity of developmental trajectories.

In this context, it becomes essential to integrate domain-general and domain-specific perspectives within a common analytical framework that allows for the investigation of how cognitive processes and numerical competencies influence one another and are functionally organized over time. Such an approach requires moving beyond static analyses and adopting methods capable of capturing both the relationships among system components and their developmental dynamics.

The present thesis addresses this need through a research program structured into five complementary studies, which examine mathematical difficulties from different yet interconnected analytical perspectives. The meta-analysis provides a rigorous synthesis of the relationship between cognitive processes and mathematical difficulties, the psychometric validation establishes the methodological foundation for the assessment of numerical competencies, and the predictive and network-based analyses enable the exploration of both relationships among variables and the structural organization of competencies. In addition, the longitudinal approach offers the necessary framework for examining the stability and transformation of these relationships over time.

By integrating these analytical levels, the thesis contributes to a reconceptualization of mathematical difficulties as emergent developmental trajectories, reflecting not only differences in performance levels but also distinct modes of organization and functional integration of numerical competencies and cognitive mechanisms. This perspective provides a coherent framework for understanding mathematical vulnerability and supports the development of more nuanced approaches to assessment and educational intervention.

2. AIMS AND GENERAL METHODOLOGY

The present thesis aims to investigate the early development of numerical competencies, their relationship with relevant cognitive processes, and the ways in which these dimensions contribute to the emergence and evolution of mathematical difficulties. In line with the integrative perspective adopted, the research simultaneously addresses cognitive factors, the structural organization of mathematical competencies, and their dynamics over time.

The general objectives of the thesis are to: (1) examine the relationship between domain-general cognitive processes and mathematical difficulties; (2) evaluate the psychometric properties of a standardized instrument for assessing early numerical competencies in the Romanian population; (3) investigate the structural organization of mathematical competencies and the interdependencies among them; and (4) analyze the stability and developmental dynamics of mathematical difficulties.

In line with these directions, the research program is structured into five complementary studies, each addressing a specific dimension of the investigated problem (Figure 1). **Study 1** consists of a meta-analysis of the relationship between working memory and mathematical difficulties, aiming to synthesize existing evidence and identify relevant moderating factors. **Study 2** focuses on the psychometric validation of the Test of Early Mathematics Ability – Third Edition (TEMA-3) for the Romanian population, using factor analytic approaches and item response theory models. **Study 3** investigates the relationships between domain-general cognitive processes and early mathematical performance, examining both group-level differences and their predictive value for individual variability. **Study 4** examines the relational organization of mathematical competencies through network analysis, providing insight into the interdependencies among abilities. **Study 5** analyzes the stability and mobility of mathematical difficulties within a longitudinal framework, highlighting the dynamics of developmental trajectories.

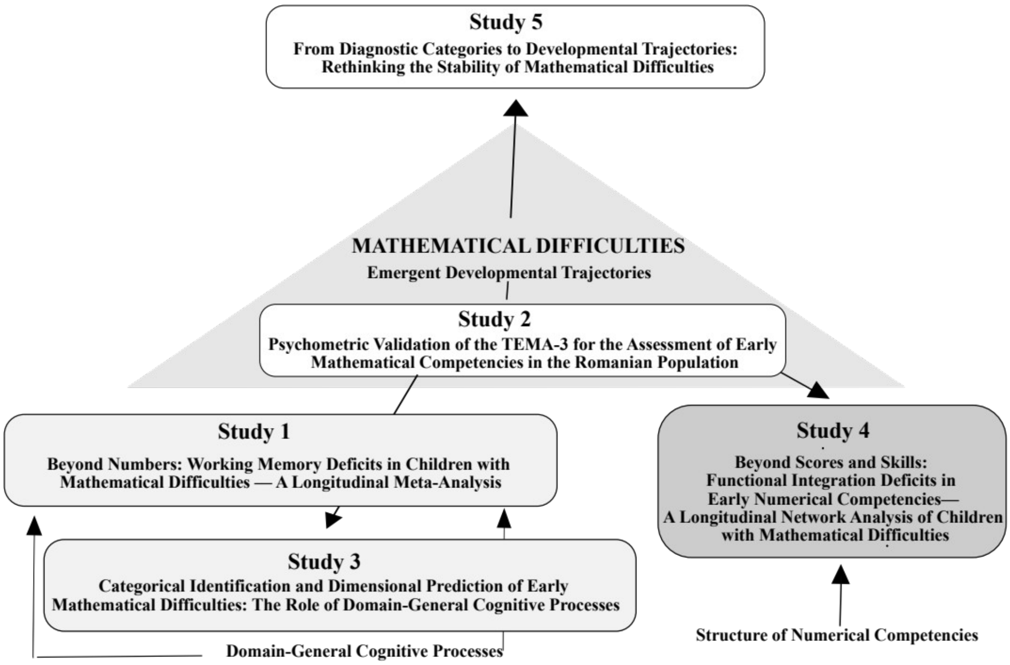

Figure 1
Integrative Conceptual Model of the Research Program (Studies 1–5)

Note. The figure illustrates the integrative structure of the research program, highlighting the relationships among the five studies and their contributions to conceptualizing mathematical difficulties as emerging developmental trajectories. Studies 1 and 3 examine the role of domain-general cognitive processes, Study 2 provides the psychometric foundation for assessing early numerical competencies, Study 4 investigates the structural organization of competencies using network analysis, and Study 5 addresses the longitudinal stability and dynamics of mathematical difficulties.

The overall methodology of the thesis integrates both cross-sectional and longitudinal designs, using samples of children in the early years of schooling. Early mathematical competencies were assessed through standardized instruments, while cognitive processes were measured through tasks targeting working memory, executive functions, attentional control, and processing speed. Statistical analyses included meta-analytic synthesis methods, psychometric modeling (CFA, IRT), regression analyses, and network analytic techniques, as well as longitudinal procedures for examining stability and relationships over time.

By combining these methodological approaches, the thesis provides a coherent analytical framework for investigating mathematical difficulties from an integrative perspective that moves beyond the isolated analysis of variables and enables the simultaneous examination of relationships among cognitive processes, the structure of competencies, and developmental dynamics.

3. EMPIRICAL STUDIES

The five studies that comprise the research program are designed as complementary investigations, examining mathematical difficulties at different yet interdependent analytical levels. Together, they provide an integrative perspective on the relationship between cognitive processes, the organization of numerical competencies, and the developmental dynamics of mathematical performance. In what follows, each study is presented in a concise manner, with emphasis on its objectives, methodology, and main findings.

3.1. Study 1. Meta-analysis

Beyond Numbers: Working Memory Deficits in Children with Mathematical Difficulties — A Longitudinal Meta-Analysis [1]

3.1.1. Theoretical Background

The relationship between working memory and mathematical performance has frequently been conceptualized as central to understanding mathematical difficulties. Classical models suggest that limitations in working memory may affect the maintenance and manipulation of numerical information, thereby influencing the acquisition and use of mathematical competencies (Baddeley, 2000; Geary, 2013). Within this framework, working memory has often been interpreted as a potential "core deficit" underlying mathematical difficulties.

However, the empirical literature reveals heterogeneous findings, both in terms of effect sizes and the role of different components of working memory. Studies indicate that the relationship between working memory and mathematical performance is dependent on context, task characteristics, and developmental stage, thereby calling into question unidimensional explanations (Peng et al., 2016; Cirino et al., 2015). In this regard, a

1 The study has been submitted for publication and is currently under review.
Kósa, A., Fodor, L. A. (2025). *Beyond numbers: Working memory deficits in children with mathematical difficulties — A longitudinal meta-analysis*. [Manuscript submitted for publication].

synthetic approach becomes necessary—one capable of integrating existing findings and clarifying the observed variability.

3.1.2. Objectives

The study aimed to synthesize empirical evidence on the relationship between working memory and mathematical difficulties and to estimate the magnitude of differences between children with mathematical difficulties and those with typical performance. It also sought to examine the variability of these effects as a function of working memory components, as well as according to the criteria used to define mathematical difficulties (i.e., cut-off thresholds) and participants' educational level.

3.1.3. Methodology

Studies were identified through systematic searches in international databases (e.g., Web of Science, Scopus, PsycINFO), using combinations of keywords related to working memory and mathematical difficulties. The selection process followed explicit eligibility criteria (school-aged populations, valid measures of working memory and mathematical performance, and sufficient data for effect size calculation), and was guided by PRISMA recommendations.

From an initially large pool of studies (n = 6,560), only those meeting the predefined methodological criteria were retained for the final analysis (36 studies, 38 samples). For each study, sample characteristics (age, educational level), types of measures used, and the indicators required for effect size estimation were coded. Analyses were conducted using random-effects models, examining both overall effects and differences across working memory components, as well as potential moderators.

3.1.4. Results

The results indicate the presence of a moderate working memory deficit in children with mathematical difficulties (overall effect size: $g \approx -0.50$ to -0.60; see Figure 2), consistently observed across the included studies. Component-level analyses show that deficits are more pronounced in the executive components of working memory compared to simple storage components.

Moderator analyses indicated that working memory deficits are present across all educational levels, ranging from $g = -0.38$ in the preschool period to $g = -0.65$ in Grades 5–6. The mixed-effects model, with educational level as a moderator, revealed significant residual heterogeneity ($QE_{(58)} = 128.34$, $p < .001$; $I^2 = 58.22\%$) and a significant moderator effect ($QM_{(6)}$ = 329.54, p < .001). However, the overlap of confidence intervals suggests that differences across levels are relatively moderate, indicating a stable presence of the deficit across development.

The severity of mathematical difficulties significantly moderated effect sizes. Children identified below the 16th percentile exhibited a large deficit (g = –1.03), significantly greater than those identified below the 30th percentile (g = –0.59). Moderator analyses confirmed that the use of stricter diagnostic thresholds is associated with larger effects ($QM_{(2)}$ = 131.63, p < .001), alongside substantial residual heterogeneity (I^2 = 94.20%). These findings indicate that the severity of the classification criterion substantially influences deficit estimation, contributing to the variability observed across studies in the literature.

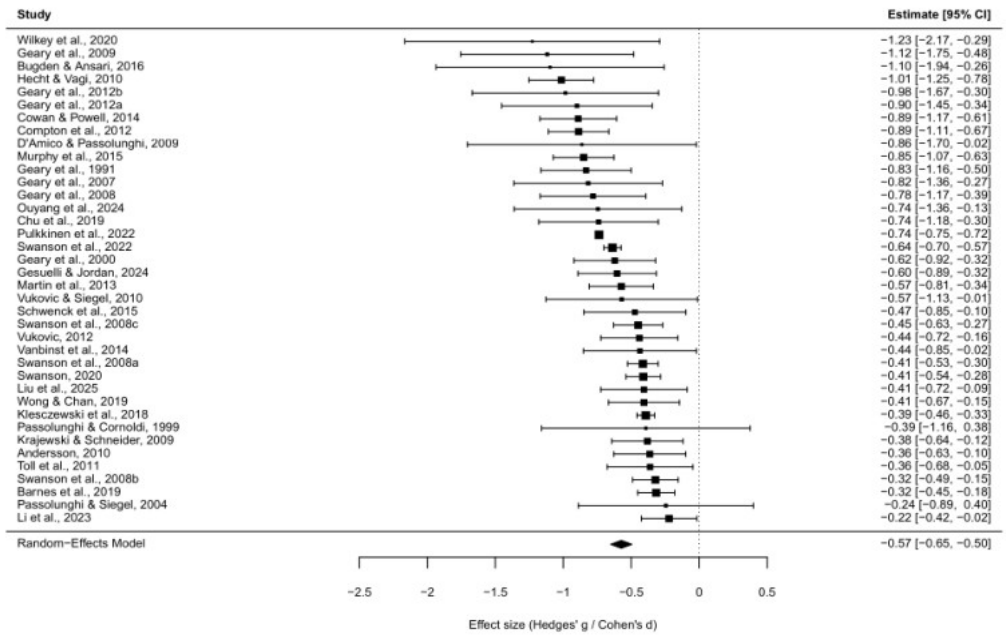

Figure 2

Effect Sizes of Working Memory Differences Between Children With Mathematical Difficulties and Typically Developing Children

Note. Lines represent Hedges' g and 95% confidence intervals; negative values indicate lower performance in children with mathematical difficulties. The diamond represents the pooled estimate (random-effects model); intervals crossing zero are not statistically significant.

3.1.5. Conclusions

The meta-analysis provides robust evidence that children with mathematical difficulties (MD) exhibit moderate and persistent working memory (WM) deficits (g = –0.57, 95% CI [–0.65, –0.50]) compared to typically developing peers. Based on 38 independent samples, the results indicate consistent deficits across all WM components—the central executive, the phonological loop, and the visuospatial sketchpad—as well as across different processing domains.

Although the direction of effects is consistent, their magnitude varies depending on study context and diagnostic criteria, suggesting that working memory deficits are not uniform but are shaped by methodological and developmental factors. Their persistence across developmental stages supports the interpretation of working memory as a stable cognitive vulnerability rather than a transient delay.

At the same time, publication bias analyses suggest that effect size estimates may be slightly inflated, warranting caution in their interpretation. Overall, the findings indicate that working memory constitutes a systemic constraint on mathematical learning, while also highlighting that the estimation of its impact critically depends on how mathematical difficulties are defined and operationalized.

3.2. Study 2. Psychometric Validation (TEMA-3)

Psychometric Validation of the TEMA-3 for the Assessment of Early Mathematical Competencies in the Romanian Population

3.2.1. Theoretical Background

The assessment of early numerical competencies requires standardized instruments that are sensitive to developmental variation and capable of discriminating between closely adjacent levels of performance. In the absence of robust psychometric evidence, the interpretation of results and the identification of educational risk remain limited (Outhwaite et al., 2024; ten Braak et al., 2022). The TEMA-3 is one of the most widely used instruments for assessing early mathematics; however, its application in a new cultural context requires factorial validation and an examination of item functioning.

The literature suggests that the structure of early numerical competencies can be described either by a dominant general factor or by specific dimensions; however, these propositions require empirical validation (Ryoo et al., 2015). Item response theory (IRT) models provide an appropriate framework for examining item properties and measurement precision across the ability continuum (Embretson & Reise, 2000).

3.2.2. Objectives

The study aimed to evaluate the psychometric properties of the TEMA-3 in the Romanian population by examining its factorial structure, item functioning, and measurement precision. It also aimed to compare IRT models and to assess measurement invariance across subgroups.

3.2.3. Methodology

The sample included children aged between 3 and 8 years (N = 534). Factor analyses were conducted based on tetrachoric correlations (KMO = 0.95), using robust estimators for categorical data (WLSMV). The dimensional structure was examined through exploratory and confirmatory factor analyses, including bifactor models.

Item functioning was analyzed using IRT models for dichotomous items, comparing the Rasch model and the two-parameter logistic (2PL) model. Local dependence was evaluated using Q3 statistics, and measurement precision was assessed through the test information function (TIF). In addition, differential item functioning (DIF) was examined as a function of language and gender.

3.2.4. Results

Factor analyses indicated excellent data suitability for modeling (KMO = .95; $\chi^2_{(2556)}$ = 322,653.5, p < .001). Although parallel analysis suggested a three-factor solution, exploratory and bifactor models revealed the dominance of a strong general factor (ωh = 0.80), which accounted for approximately 70% of the common variance (ECV = 0.70). Moderate to high inter-factor correlations and the pattern of factor loadings support a predominantly unidimensional structure with hierarchical organization.

Internal consistency was very high (α = 0.99), and the reliability of estimates within the IRT framework was also excellent (marginal reliability = 0.979), indicating high measurement precision.

Comparison of IRT models showed a significantly better fit of the 2PL model relative to the Rasch model ($\Delta\chi^2_{(71)}$ = 1267.05, p < .001), indicating systematic variation in item discrimination. Discrimination parameters were high (median $a \approx 5.35$), reflecting strong differentiation between adjacent levels of ability. Difficulty parameters covered a wide range ($b \in [-3.68, 1.67]$), suggesting appropriate test calibration for low to moderate levels of mathematical ability.

The test information function indicates maximum precision in the mid-range of ability ($I_{max} \approx 113$; $SEM \approx 0.094$), with adequate coverage also in the low-performance range (e.g., $I_{(\theta = -2)} \approx 58.18$), which is relevant for the identification of educational risk.

Local dependence analysis identified a small number of item pairs with elevated Q3 values (≈2% of pairs), indicating general adherence to local independence. DIF analyses revealed a minimal proportion of items exhibiting differential functioning (≈1.4% for both language and gender), supporting measurement equivalence across groups.

Overall, the results converge toward a predominantly unidimensional structure, with solid psychometric properties and good suitability for assessing early mathematical competencies in the studied population.

3.2.5. Conclusions

The results support the conclusion that the TEMA-3 primarily reflects a structure dominated by a general factor of early mathematical competence, suggesting that, in the early stages of development, individual differences are predominantly quantitative (level-based) rather than qualitative (profile-based). This finding is consistent with developmental perspectives that describe numerical competencies as initially weakly differentiated and progressively specialized over time.

IRT analyses indicate appropriate item functioning, with strong discrimination capacity and a coherent distribution of difficulty, supporting the interpretation of the instrument as sensitive to fine-grained variations in mathematical ability. At the same time, the distribution of test information shows that the instrument is optimally calibrated for the central and lower ranges of ability, which are most relevant for identifying educational risk.

The low level of local dependence and the minimal proportion of items exhibiting differential functioning support measurement equivalence across groups, while high reliability indices confirm the stability of the estimates. Overall, the results indicate that the TEMA-3 constitutes a robust psychometric instrument and provides an appropriate framework for investigating early mathematical competencies as an organized, predominantly unitary system.

3.3. Study 3. Cognitive Processes

Categorical Identification and Dimensional Prediction of Early Mathematical Difficulties: The Role of Domain-General Cognitive Processes [2, 3, 4]

3.3.1. Theoretical Background

Early mathematical performance shows longitudinal continuity and is associated with differentiated educational trajectories, highlighting the importance of early identification of mathematical vulnerability (ten Braak et al., 2022; Owen & Reed, 2025). At the same time, mathematical performance appears to be continuously distributed within the population, without clear discontinuities between typical and atypical functioning, calling into question rigid categorical distinctions (Peters & Ansari, 2019; Astle & Fletcher-Watson, 2020).

In this context, domain-general cognitive processes—working memory, executive functions, attentional control, and processing speed—have been frequently implicated in mathematical performance; however, findings are heterogeneous and dependent on how mathematical difficulties are operationalized. Therefore, integrating categorical and dimensional approaches becomes necessary to clarify whether cognitive vulnerability reflects qualitative differences between groups or gradual variation along the performance continuum.

3.3.2. Objectives

The study aimed to investigate the role of domain-general cognitive processes in early mathematical performance, using both categorical and dimensional analyses. Specifically, it examined whether these processes differentiate performance groups, whether they predict individual variation in mathematical performance, and the extent to which these findings are sensitive to the classification thresholds employed.

2 The correlational and predictive analyses of this study were presented at the international Spring Wind Conference, Budapest, May 3–5, 2024. Subsequently, the work was published as a conference proceeding:
Kósa, A. (2024). *Cognitive processes predicting mathematical performance in children with learning difficulties.* In D. Molnar, D. Molnar, & A. Sz. Nagy (Eds.), *XXVII Spring Wind Conference:* Conference Proceedings II (pp. 352–366). National Association of Doctoral Students.

3 The comparative analysis of this study was presented at the international *Connecting the Dots* conference, Cluj-Napoca, November 10–12, 2023. Subsequently, the work was published as a conference proceeding.
Kósa, A. (2025). Specificity of cognitive deficits in children with mathematical difficulties. In Sz. Rusu, B. Lazar, & Sz. Varga (Eds.), *15th Interdisciplinary Conference of Hungarian Doctoral Students: Connecting the Dots* (pp. 297–315). Presa Universitară Clujeană.

4 The study has been submitted for publication and is currently under review.
Kósa, A. (2026). *Categorical identification and dimensional prediction of early mathematical difficulties: The role of domain-general cognitive processes.* [Manuscript submitted for publication].

3.3.3. Methodology

The study employed a cross-sectional ex post facto design. The sample comprised 76 children in preparatory grade (39 girls, 37 boys), aged between 74 and 95 months (*M* = 81.34, *SD* = 4.37). All participants were native Hungarian speakers, had no reported neurological or developmental disorders, and demonstrated intellectual functioning within the normative range.

Mathematical performance was assessed using the TEMA-3, and mathematical difficulties were operationalized using multiple percentile cut-offs (16th, 25th, and 40th) to examine the sensitivity of the results to classification criteria. The cognitive processes assessed included verbal and visuospatial working memory, short-term verbal numerical storage, verbal cognitive flexibility, attentional efficiency, and processing speed (RAN). Analyses comprised group comparisons, ANCOVA, Pearson correlations, and hierarchical regression, complemented by bootstrap procedures, cross-validation, and relative importance analyses.

3.3.4. Results

At the categorical level, children classified with mathematical difficulties showed significantly lower performance than typically performing peers, particularly in verbal cognitive flexibility (Hedges' g = 0.60–0.79) and in the verbal components of working memory (g = 0.73–1.27). Differences were also observed in short-term verbal numerical storage, whereas visuospatial working memory and processing speed showed small or non-significant effects in most comparisons. More restrictive thresholds were associated with larger effect sizes; however, the overall configuration of cognitive differences remained stable across contrasts.

At the dimensional level, the full model was significant, $F_{(20, 55)} = 2.93$, $p < .001$, explaining 51.6% of the variance ($R^2 = 0.52$), but indicated high multicollinearity within the attentional block. After predictor reduction, the reduced model remained significant, $F_{(6, 69)} = 8.24$, $p < .001$, $R^2 = 0.42$.

The final parsimonious model retained two independent predictors of early mathematical performance—short-term verbal numerical storage and verbal cognitive flexibility—explaining 39% of the variance ($F_{(2, 73)} = 23.38$, $p < .001$, $R^2 = 0.39$; adjusted $R^2 = 0.37$). Their contributions were comparable (51% vs. 49%), indicating partially independent effects (Table 1). Model stability was supported by cross-validation ($R^2 = 0.395$; *RMSE* = 11.43).

Table 1
Final Regression Model for Predicting Mathematical Performance (N = 76)

Predictor	B	SE	t	p	95% Bootstrap CI
Short-term verbal numerical storage	3.19	0.79	4.03	< .001	[2.01; 4.48]
Verbal cognitive flexibility	2.35	0.60	3.91	< .001	[1.37; 3.40]

Note. B = unstandardized regression coefficient; SE = standard error. Bootstrap confidence intervals are percentile-based and were derived from 5,000 resamples.

In the subsample with severe mathematical difficulties (≤16th percentile; n = 30), the same predictive structure did not replicate. The model was not significant, $F_{(2, 27)} = 0.89$, $p = .423$, explaining only 6% of the variance ($R^2 = 0.06$), suggesting that cognitive predictors operate dimensionally across the full performance continuum but do not account for internal variability in severity within the low-performance range.

3.3.5. Conclusions

The results support a multifactorial and dimensional interpretation of early mathematical difficulties, in which performance is not determined by a single cognitive deficit but by selective and partially independent contributions of multiple domain-general processes. In particular, verbally mediated processes—short-term numerical storage and cognitive flexibility—emerge as central mechanisms supporting early mathematical performance.

At the same time, the study highlights an important dissociation between categorical differentiation and dimensional prediction: the cognitive processes that distinguish performance groups do not necessarily coincide with those that explain continuous variation in performance. This discrepancy suggests that findings based on group comparisons may overestimate the relevance of certain processes that lack independent explanatory value at the individual level. The fact that the predictive structure does not replicate within the low-performance range indicates that the mechanisms operating across the full continuum are not equivalent to those underlying the severity of mathematical difficulties. Cognitive vulnerability thus emerges as a gradual and heterogeneous phenomenon, dependent on context and on the level of analysis considered.

Overall, the results support the need to integrate categorical and dimensional approaches and contribute to the reconceptualization of mathematical difficulties as continuous variations in interdependent cognitive processes, rather than as discrete entities or expressions of a unitary deficit.

3.4. Study 4. Network Analysis

Beyond Scores and Skills: Functional Integration Deficits in Early Numerical Competencies—A Longitudinal Network Analysis of Children with Mathematical Difficulties [5,6,7]

3.4.1. Theoretical Background

Although early mathematical competencies are recognized as important predictors of later academic performance (Duncan et al., 2007; Geary, 2011), less is known about how these competencies are functionally organized and how specific relational configurations may reflect mathematical vulnerability from the very onset of schooling. Traditional approaches—based on global scores, predictive models, or latent variables—capture performance levels but provide limited insight into how competencies support and coordinate with one another within a developing system (Borsboom, 2017; Fried & Cramer, 2017).

From this perspective, network analysis provides an appropriate descriptive framework for examining mathematical competencies as a relational system, in which nodes represent distinct competencies and edges reflect the interdependencies among them (Epskamp et al., 2018). The study is based on the hypothesis that mathematical vulnerability may reflect not only lower levels of performance but also reduced functional integration, expressed through less differentiated structural patterns of relationships among competencies (Borsboom & Cramer, 2013).

3.4.2. Objectives

The study aimed to examine the relational organization of early mathematical competencies within a longitudinal framework and to investigate how this organization is associated with later mathematical performance. It also sought to compare the network

5 The correlational and predictive analyses of this study were presented at the international *Connecting the Dots* conference, Cluj-Napoca, November 8–10, 2024, under the title: *Does the Starting Point Matter? Predicting Mathematical Competencies Based on Skills and Knowledge Measured at School Entry.*

6 The correlational networks and cluster analysis were presented at the international *Special Treatment International Interdisciplinary Conference*, Hajdúböszörmény, Hungary, April 24–26, 2025. Subsequently, the work was published as a conference proceeding.
Kósa, A. (2025). From what to what? Correlational network and cluster analysis in the service of predicting mathematical competencies—identifying what matters. In K. Mező (Ed.), *Interdisciplinary research on special treatment* (1/2025, pp. 41–56). University of Debrecen, Faculty of Child and Special Needs Education.

7 The study has been submitted for publication and is currently under review.
Kósa, A. (2026). *Beyond Scores and Skills: Functional Integration Deficits in Early Numeracy—A Longitudinal Network Analysis of Children With Mathematical Difficulties* [Manuscript submitted for publication].

structures characteristic of children with mathematical difficulties and those with typical performance, with a focus on connectivity and centrality.

3.4.3. Methodology

The study employed a longitudinal observational design with two assessment points: school entry (T1) and the end of Grade 2 (T2). The final sample included 123 children (61 girls, 62 boys), with a mean age of 81.72 months at T1. Participants were classified based on the TEMA-3 into three performance profiles: mathematical difficulties (MD; $n = 32$), typical performance (TP; $n = 79$), and high performance (HP; $n = 12$).

At T1, distinct early mathematical competencies were assessed, including counting ability, number comparison, informal calculations and concepts, numerical literacy, number facts, and formal calculations and concepts. At T2, a curriculum-based instrument was administered, assessing numerical literacy, number comparison, number line representation, mathematical symbols, arithmetic fluency, calculation concepts, and problem solving.

Networks were constructed based on zero-order Pearson correlations and represented as undirected, weighted graphs. For descriptive interpretation, a threshold of $r > 0.50$ was selected, based on sensitivity analyses conducted across multiple thresholds ($r > 0.30$–0.70). The structural roles of competencies were evaluated using centrality indices, and differences between the MD and TP networks were tested using permutation-based structural comparison procedures.

3.4.4. Results

At the longitudinal level, overall mathematical performance at T1 was moderately associated with performance at T2 ($r = 0.59$, $p < .001$). Among specific competencies, counting ability and numerical literacy showed the strongest associations with later performance (both $r = 0.53$, $p < .001$; see Figure 3). Predictive models confirmed the relevance of these competencies: overall mathematical performance at T1 explained 35% of the variance in T2 performance ($R^2 = 0.35$), while early informal competencies accounted for 33% of the variance ($R^2 = 0.33$).

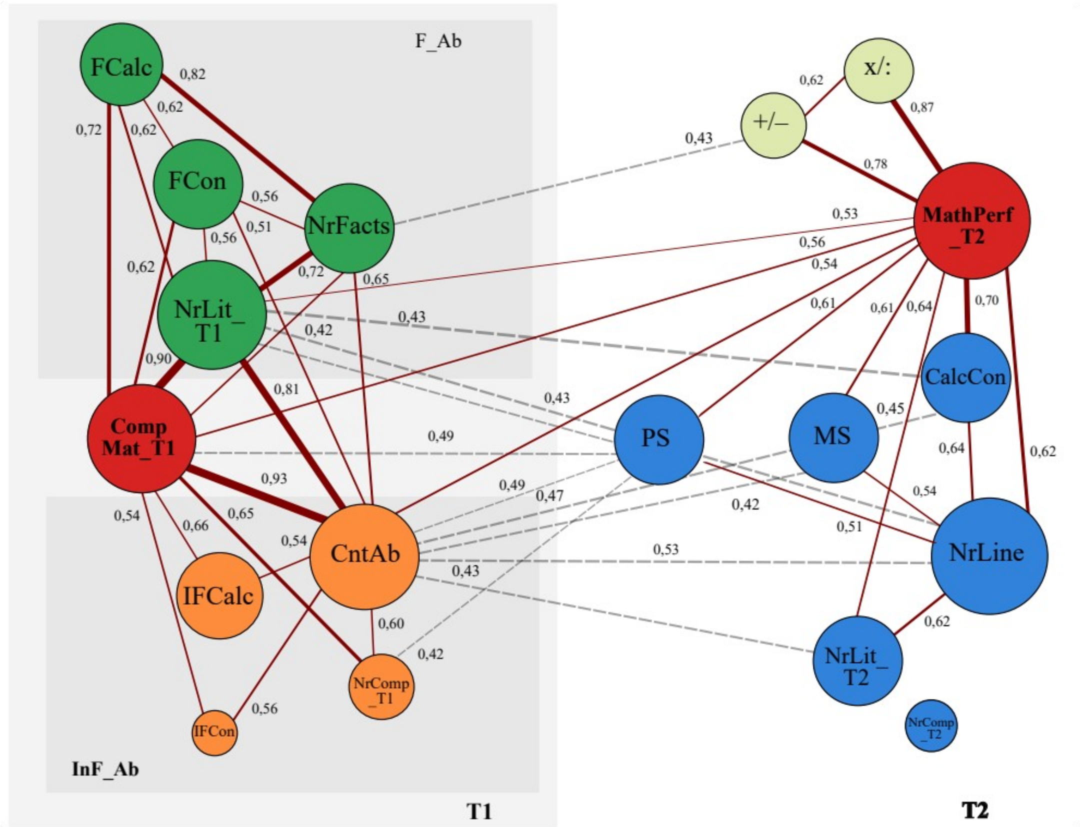

Figura 3

Integrated network of early (T1) and second-grade (T2) mathematical competencies based on Pearson correlations

Note. Nodes represent mathematical competencies; edge thickness reflects correlation strength. Only moderate or stronger associations are shown (solid lines: $r \geq .50$; dashed lines: $r \geq .40$).

T1 – First assessment point: CntAb = counting ability; NrComp_T1 = number comparison at T1; IFCalc = informal calculations; IFCon = informal concepts; NrLit_T1 = numerical literacy; NrFacts = number facts; FCalc = formal calculations; FCon = formal concepts; Math_Ab_T1 = mathematical ability at T1; InF_Ab = informal abilities; F_Ab = formal abilities.

T2 – Second assessment point: NrLit_T2 = numerical literacy at T2; NrComp_T2 = number comparison at T2; NrLine = number line; MS = mathematical symbols; +/- = addition–subtraction; x/: = multiplication–division; PS = problem solving; CalcCon = calculation concepts; MathPerf_T2 = mathematical performance in second grade.

Network analysis showed that, at T1, counting ability (*DC* = 8, *BC* = 42, *CC* = 0.07, *EC* = 0.84) and numerical literacy (*DC* = 6, *BC* = 8, *CC* = 0.06, *EC* = 0.81) occupied structurally central positions. At T2, centrality reorganized around more advanced procedural and conceptual competencies, with overall mathematical performance emerging as the dominant integrative node (*DC* = 10, *BC* = 73, CC = 0.08, *EC* = 0.55). This pattern suggests a functional redistribution of structural roles over the course of development.

Comparison of performance profiles revealed significant structural differences between the MD and TP networks. Global edge similarity was moderate ($r = 0.41$, $p < .05$), but comparison tests indicated significant differences in structural invariance ($M = 3.22$, *p*

$< .001$) and global network strength ($S = 0.44$, $p = .0004$). The low correlation between node centrality values ($r = 0.18$, $p = .0002$) indicates substantial differences in the structural roles of competencies.

From a descriptive perspective, the MD network was denser and remained fully connected across the examined thresholds, but exhibited a more diffuse distribution of centrality, without clearly dominant nodes. In contrast, the TP network was sparser but more hierarchically differentiated, with centrality selectively concentrated in competencies such as counting ability, numerical literacy, and arithmetic facts. This pattern suggests that mathematical vulnerability is associated not only with lower performance but also with weaker functional differentiation of the competency system.

3.4.5. Conclusions

The results support a relational and longitudinal perspective on early mathematical development. Later performance is associated not only with initial levels of competencies but also with how these competencies are structurally organized within an integrated system. Central early competencies, particularly counting ability and numerical literacy, appear to function as coordinating nodes between informal and emerging formal domains.

At the same time, differences between developmental profiles suggest that mathematical vulnerability may be more appropriately understood as a form of reduced functional integration rather than merely lower performance. The network of children with mathematical difficulties appears less differentiated and less specialized, indicating an altered structural organization of competencies. Overall, the study extends models focused exclusively on scores and supports the use of relational indicators as complementary markers for understanding early mathematical development and for the early identification of mathematical risk.

3.5. Study 5. Diagnostic Stability

From Diagnostic Categories to Developmental Trajectories: Rethinking the Stability of Mathematical Difficulties [8, 9]

3.5.1. Theoretical Background

The stability of mathematical difficulties represents a central issue in the literature, as the distinction between persistent difficulties, transient vulnerabilities, and normative developmental variation remains insufficiently clarified (Geary, 2011; Morgan et al., 2009). Although categorical models assume the existence of relatively stable profiles, recent research suggests that early mathematical performance is more appropriately understood as part of developmental trajectories that are sensitive to context, instruction, and the classification criteria employed (Dowker, 2024; Haberstroh & Schulte-Körne, 2022; Jordan et al., 2009).

Within this framework, early numerical competencies—particularly informal ones—may function both as risk factors and as protective factors, influencing the continuity or change of performance profiles (Toll & Van Luit, 2014). The study examines the diagnostic stability of mathematical difficulties not as a fixed property, but as the result of the interaction between initial competencies, subsequent development, and school-based assessment practices.

3.5.2. Objectives

The study aimed to examine the diagnostic stability of mathematical difficulties across three assessment points and to analyze the predictive value of early numerical competencies for later performance. It also sought to investigate the mediating role of informal and formal competencies in the developmental trajectories of mathematical performance.

3.5.3. Methodology

The study employed a longitudinal observational design with three assessment points: preparatory grade (T1), Grade 2 (T2), and Grade 3 (T3). The sample included 119 children

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8 The study was presented at the international interdisciplinary conference *Learning and Society,* Eger, Hungary, October 3–5, 2025, under the title: *In the Labyrinth of Numbers: Risk and Protective Factors in Early Mathematical Development—Longitudinal Evidence on the Dynamics of Mathematical Difficulties*.

9 The study has been submitted for publication and is currently under review.
Kósa, A. (2026). *From diagnostic categories to developmental pathways: Rethinking the stability of mathematical difficulties*. [Manuscript submitted for publication].

(60 girls, 59 boys), classified based on the TEMA-3 into three profiles: mathematical difficulties (MD; n = 32), typical performance (TP; n = 77), and high performance (HP; n = 12).

At T1, early numeracy competencies were assessed, including both informal and formal dimensions. At T2 and T3, curriculum-based measures were administered to assess school mathematical performance. Analyses included group comparisons, repeated-measures *ANOVA*, diagnostic stability indices (κ), logistic and multinomial regressions, *ROC* analyses (*AUC*), mediation models, and multigroup structural equation modeling.

3.5.4. Results

Mathematical performance increased significantly from T1 to T3 across all groups, $F_{(2,\ 232)} = 95.85$, $p < .001$, $\eta^2 = 0.23$, while initial differences between profiles were maintained, with no time × group interaction, $F_{(4,\ 232)} = 0.72$, $p = .58$. This pattern indicates parallel but differentiated developmental trajectories.

Diagnostic stability analyses revealed pronounced early mobility and a progressive consolidation of classifications over the course of schooling (Figure 4). Between T1 and T2, 63% of children initially classified with mathematical difficulties moved into the typical performance range, whereas stability increased between T2 and T3, with 76.5% of children maintaining their classification. Kappa indices reflected the same pattern: $\kappa = 0.14$ for T1→T2, $\kappa = 0.24$ for T1→T3, and $\kappa = 0.52$ for T2→T3.

Early categorical classifications showed limited predictive value, whereas continuous models demonstrated greater and more consistent predictive power. In parallel, mediation analyses indicated that performance at T2 significantly mediates the relationship between early and later performance (total effect $\beta = 0.67$, $p < .001$), supporting the presence of cascading developmental processes.

With regard to specific predictors, early counting skills and number comparison abilities functioned as protective factors against persistent difficulties. Counting remained the most robust predictor of later performance, including in multivariate models and LASSO analyses, suggesting a central role of informal numeracy in supporting subsequent mathematical development.

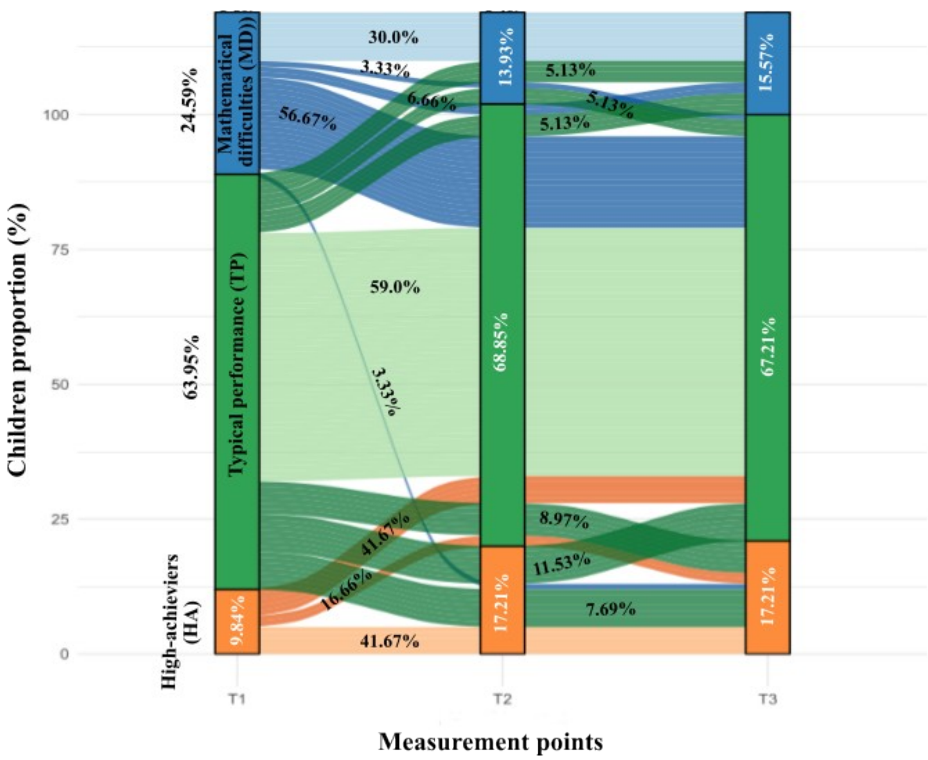

Figure 4

Developmental Trajectories of Mathematical Performance Across Diagnostic Groups

Note. The diagram illustrates student transitions between mathematical performance categories (HP – high performance, TP – typical performance, MD – mathematical difficulties) across the three assessment points (T1–T3). The width of the flows is proportional to the number of students transitioning between categories, and the percentages indicate the corresponding proportions at each measurement point.

3.5.5. Conclusions

The results support a developmental and dimensional interpretation of mathematical difficulties. Diagnostic categories appear only partially stable, reflecting both the continuity of individual differences and substantial mobility during the early years of schooling. This dynamic suggests that early mathematical difficulties should not be interpreted exclusively as fixed states, but rather as trajectories subject to reorganization.

At the same time, the study shows that early informal numeracy—particularly counting and number comparison—constitutes a fundamental basis of mathematical development and an important marker for the early identification of risk. Overall, the results support the need for developmentally oriented screening and intervention practices that are sensitive to variability and to the early plasticity of mathematical learning.

4. SYNTHESIS OF FINDINGS AND INTEGRATION OF CONTRIBUTIONS

The set of five studies provides an integrated perspective on mathematical difficulties (MD), conceptualized as emergent developmental phenomena situated at the intersection of cognitive mechanisms, the structural organization of competencies, and educational assessment practices. Taken together, the findings converge into a coherent framework that allows for a reinterpretation of mathematical difficulties beyond traditional approaches focused exclusively on performance levels.

At the cognitive level, the meta-analytic results indicate that working memory deficits—particularly in executive components—represent stable vulnerabilities, emerging early and persisting throughout early development. The integration of a longitudinal perspective and the systematic examination of diagnostic thresholds show that the magnitude of these deficits varies as a function of classification criteria, thereby clarifying an important source of variability in the literature. These findings support a multifactorial conceptualization of MD and highlight the central role of executive control mechanisms in mathematical development.

At the psychometric level, the validation of the TEMA-3 in the Romanian context confirms the predominance of a general early mathematical ability factor and provides a robust framework for assessing performance across the full ability continuum. The analyses indicate an appropriate distribution of item difficulty and strong discrimination across adjacent ability levels, while the higher measurement precision in the lower range of the distribution supports the instrument's use in identifying educational risk. The equivalence across language versions and the stability of reliability indices further support its use in bilingual educational contexts.

Analyses of domain-general cognitive processes reveal a dissociation between the mechanisms that differentiate groups and those that account for individual variation. Although executive functions and working memory distinguish children with low versus typical performance, their predictive value in multivariate models is limited and selective. At the same time, these relationships prove sensitive to classification thresholds, supporting the interpretation of mathematical difficulties as dimensional variations in performance, contingent on methodological decisions.

At the structural level, network analysis indicates that early mathematical development is organized as a system of interdependent competencies. Differences between children are reflected not only in performance levels but also in the way competencies are integrated. Networks associated with MD are characterized by a more diffuse organization and reduced structural differentiation, suggesting suboptimal functional integration rather than the absence of specific competencies. Within this framework, the role of early competencies is interpreted in terms of structural position and relative connectivity, and their evolution reflects progressive reorganization processes across development.

At the developmental level, longitudinal analyses reveal moderate diagnostic stability combined with substantial early mobility. Early categorical classifications show limited predictive value, whereas continuous models more accurately capture developmental trajectories. The findings indicate that mathematical performance evolves in ways that reflect both initial competence levels and educational context, while informal numerical skills—particularly counting and number comparison—emerge as central protective factors. Mediation models suggest cascading developmental processes, in which intermediate performance mediates the relationship between early and later competencies.

Overall, the findings converge toward a conceptualization of mathematical difficulties as dynamic developmental trajectories shaped by the interaction between cognitive vulnerabilities, the structural organization of competencies, and the educational context. This perspective supports moving beyond rigid categorical models and argues for dimensional, process-oriented approaches focused on prevention and adaptive intervention.

5. THEORETICAL AND PRACTICAL IMPLICATIONS

The results of the thesis support a reconceptualization of mathematical difficulties as emergent developmental phenomena, reflecting the interaction between cognitive processes, the structural organization of competencies, and the educational context. At the cognitive level, working memory—particularly executive components—emerges as a stable vulnerability, yet one that is dependent on task context and classification criteria. At the structural level, differences between children are manifested not only in performance levels but also in the organization and integration of competencies, suggesting that mathematical difficulties can be understood as variations in the structure of the competency system. At the developmental level, the observed moderate stability and mobility indicate that these difficulties are more appropriately described as developmental trajectories rather than fixed states. Overall, the findings support moving beyond traditional dichotomies (categorical vs. dimensional, domain-general vs. domain-specific) and argue for an integrated, process-oriented perspective.

From a practical perspective, the findings highlight the importance of flexible early assessment that combines categorical classifications with dimensional indicators of performance. The validation of the instrument used supports the need for standardized measures sensitive to the lower range of the distribution, which are particularly relevant for identifying risk. The role of working memory suggests the need to adapt instructional demands to children's cognitive resources, through task structuring and the reduction of cognitive load. At the same time, the importance of early numerical competencies—especially counting and numerical literacy—indicates the need for interventions that support their integration across varied learning contexts. The observed diagnostic mobility supports continuous monitoring and adaptive interventions oriented toward individual developmental trajectories. Overall, the results support a developmental educational approach in which assessment and intervention are conceived as dynamic processes, sensitive to variability and the plasticity of early mathematical learning.

6. LIMITATIONS OF THE THESIS AND DIRECTIONS FOR FUTURE RESEARCH

The interpretation of the findings should be considered in light of several methodological and conceptual limitations. First, some of the analyses, particularly those based on correlations and networks, are descriptive in nature and do not allow for causal inferences. Second, the results are sensitive to decisions regarding the operationalization of variables and the classification criteria employed, which affect group composition and comparability with other studies. Third, the assessment of cognitive processes and mathematical competencies relies on tasks that may simultaneously involve multiple cognitive components, limiting the strict separation of underlying mechanisms. In addition, the instrument used to assess early numerical competencies demonstrates higher precision in the central and lower ranges of the distribution, which may reduce sensitivity at very high levels of performance. Furthermore, domain-general cognitive processes and domain-specific mathematical competencies were examined within separate empirical frameworks, limiting the direct testing of their interactions within unified models. Finally, the generalization of the findings should be approached with caution, given the characteristics of the sample and the educational context in which the data were collected.

The findings of the thesis open several relevant directions for future research. A first direction concerns the use of more complex longitudinal designs capable of clarifying the directionality of relationships between cognitive processes and mathematical performance. A second direction involves refining the assessment of cognitive processes through more differentiated batteries and multimethod approaches. Furthermore, integrating domain-general cognitive processes and domain-specific mathematical competencies within unified analytical models would allow for the direct testing of how cognitive vulnerabilities support the structural organization of numerical competencies.

With regard to mathematical development, combining network analysis with latent models and trajectory-based approaches could provide a more nuanced understanding of the reorganization of competencies over time. In addition, the systematic examination of contextual factors—linguistic, educational, and institutional—could help explain variability across studies and contribute to the adaptation of interventions. Overall, these directions support the development of research frameworks capable of capturing mathematical difficulties as dynamic, emergent processes sensitive to the interaction between cognitive, structural, and contextual factors.

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