Application of the Experimental and Theoretical Methods of Statistical Physics

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Summary

Research Objectives and Scope

This doctoral thesis applies experimental and theoretical methods of statistical physics to investigate three distinct natural phenomena: oscillatory dynamics and collective behavior in buoyancy-driven flows, statistical patterns in earthquake phenomena, and CO_2 flow dynamics in a natural mofette system. The research demonstrates how statistical physics approaches can identify fundamental patterns across different systems through a consistent methodological framework combining experimental measurements, theoretical modeling, and computational simulations.

The work addresses questions about oscillation and synchronization mechanisms, statistical universalities in avalanche-like phenomena, and complex temporal dynamics in natural gas emission systems. Each investigation employs the three-pronged approach of statistical physics: controlled experiments/real-world data collection, theoretical model development, and numerical simulations.

Oscillatory Dynamics and Collective Behavior in Buoyancy-Driven Flows

The first investigation examined oscillation and synchronization phenomena in candle flames and helium columns. Experimental studies on candle flame bundles revealed that oscillation frequency decreases with increasing bundle size in compact and hollow circular arrangements, while linear arrangements show no clear trend. Oxygen concentration significantly affects oscillatory behavior - higher concentrations can either initiate oscillations in normally stable single candles or suppress oscillations in bundles that normally oscillate at atmospheric oxygen levels.

For interacting candle bundles, experiments identified both in-phase and counter-phase synchronization depending on separation distance, with transitions occurring around 3-4 cm separation. The original theoretical model by Kitahata et al. was found inadequate as it predicted frequency increases with bundle size, contrary to experimental observations. An improved dynamical model was developed incorporating bundle size and oxygen concentration effects, with a novel coupling mechanism based on air movements rather than thermal radiation.

To isolate hydrodynamic mechanisms from chemical reactions, parallel experiments were conducted on helium columns using Schlieren visualization. These experiments showed remarkably similar behaviors: frequency decreasing with increasing nozzle diameter and increasing with flow rate. Interacting helium columns exhibited exclusively counter-phase synchronization, supporting the hypothesis that hydrodynamic instabilities are the primary drivers of these phenomena.

Computational fluid dynamics simulations using the FEniCS framework provided a two-dimensional numerical approach that successfully reproduced both individual oscillatory patterns and collective synchronization behaviors observed in experiments. The simulations validated that buoyancy-driven instabilities are sufficient to explain the experimental observations across different scales.

Statistical Patterns in Earthquake Phenomena

The second investigation analyzed statistical properties of avalanche phenomena across different scales. A unified energy-based approach was introduced for analyzing earthquake data, converting different magnitude scales to standardized energy measures. This enabled meaningful comparison between seismic regions (Japan, Southern California, Romania) despite their distinct geological settings and magnitude reporting systems.

Analysis confirmed that all three regions follow the Gutenberg-Richter law over eight orders of magnitude, with energy distributions fitting the Tsallis-Pareto probability density function. Recurrence time analysis revealed that intervals between consecutive earthquakes follow Gamma distributions with shape parameters $\alpha < 1$ for all regions, indicating temporal correlations between events.

The investigation extended to acoustic emissions from compressed metallic crystals ("microquakes"), which showed similar statistical patterns to tectonic earthquakes. Both systems follow comparable power-law distributions for energy release and exhibit similar temporal patterns, including aftershock sequences obeying Omori's law.

Computational modeling using the one-dimensional Burridge-Knopoff spring-block model successfully reproduced energy distribution statistics observed in both earthquakes and microquakes, with power exponents matching experimental values. However, the model failed to capture temporal correlations, generating uncorrelated Poisson recurrence times rather than the correlated patterns observed in physical systems.

An extended Local Growth Global Reset (LGGR) model was developed to analytically characterize earthquake statistics. The key innovation was introducing partial resets, allowing transitions to lower stress states rather than only complete resets to zero. By parameterizing transition rates through Burridge-Knopoff simulations, the model produces a Pareto type II distribution that accurately describes both real earthquake data and simulation results.

CO_2 Flow Dynamics in Natural Mofette Systems

The third investigation monitored CO_2 emissions from a natural mofette using a custom sensor array over seven months with high temporal resolution (1 Hz). Two distinct emission regimes were identified: steady-state oscillatory behavior with small fluctuations around slowly varying mean values, and intermittent eruption-like events characterized by sudden concentration bursts propagating through the measurement column.

Power spectral analysis revealed characteristic peaks at 12-hour and 24-hour periods corresponding to diurnal cycles, along with three distinct power-law scaling domains. Flow yield calculations showed correlations with environmental parameters: negative correlation with pressure change rates and positive correlation with temperature change rates.

A computational model was developed using a chamber grid approach to simulate subsurface gas transport, incorporating temperature-dependent resistivity and pressure-driven flow between chambers. The model successfully captured medium-term flow variations based on atmospheric parameters but could not reproduce eruption events, suggesting additional subsurface mechanisms beyond atmospheric influences.

The findings have practical implications for therapeutic mofette use, as sudden bursts can significantly increase CO_2 concentrations, potentially creating suffocation hazards.

Methods and Validation

Each investigation employed custom experimental setups: thermal radiation sensors and high-speed cameras for flame studies, Schlieren visualization for helium columns, and multi-sensor arrays for mofette monitoring. Theoretical approaches included dynamical systems models for oscillations, stochastic models for earthquake statistics, and fluid transport models for gas flow. Computational methods ranged from finite element fluid dynamics to spring-block simulations and chamber network models.

Validation was achieved through systematic comparison between experimental observations, theoretical predictions, and simulation results. The consistency across these different approaches provided confidence in the identified physical mechanisms and statistical patterns.

Conclusions

This thesis demonstrates how experimental and theoretical methods of statistical physics can reveal fundamental patterns across diverse natural phenomena. Key contributions include: identification of hydrodynamic instabilities as primary drivers of oscillation and synchronization in buoyancy-driven flows, demonstration of statistical universality in earthquake-like phenomena across multiple scales, development of improved theoretical models incorporating realistic physical constraints, and advancement of monitoring techniques for natural CO_2 emission in a mofette.

The work shows how statistical physics methodologies can connect apparently unrelated phenomena through common underlying principles, while highlighting the importance of careful experimental validation and appropriate theoretical modeling. The research contributes to fundamental understanding of complex systems and has practical applications in areas such as natural hazard assessment and environmental monitoring.